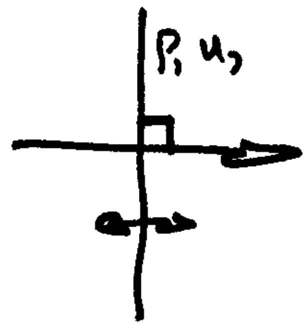


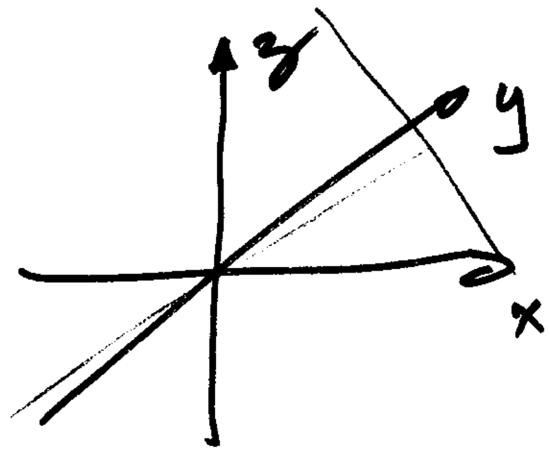
plane wave

$$\tilde{p}(x) = A e^{-jkx} + B e^{+jkx}$$

linearized momentum eqn

$$\hat{u}_x = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dx}$$

$$\hat{u}_x(x) = \frac{A}{\rho_0 c} e^{-jkx} - \frac{B}{\rho_0 c} e^{+jkx}$$



$$p(x,y,z,t) = A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

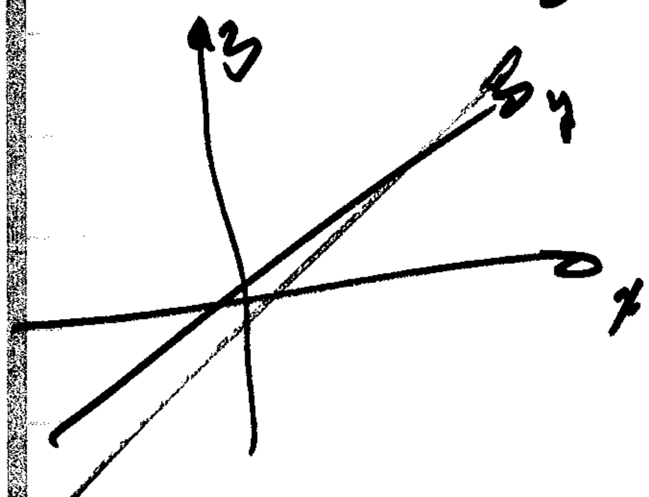
$$\nabla^2 p + k^2 p = 0 \qquad k = \frac{\omega}{c} \text{ fixed by problem frequency.}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

must be true if the plane wave is "sound"

Only 2 of the k_x, k_y & k_z can be chosen independently if the solution is to represent "sound"

$$k_z^2 = k^2 - k_x^2 - k_y^2$$



$$\tilde{p}(x,y,z,t) = A e^{j(\omega t \pm k_x x \pm k_y y \pm k_z z)}$$

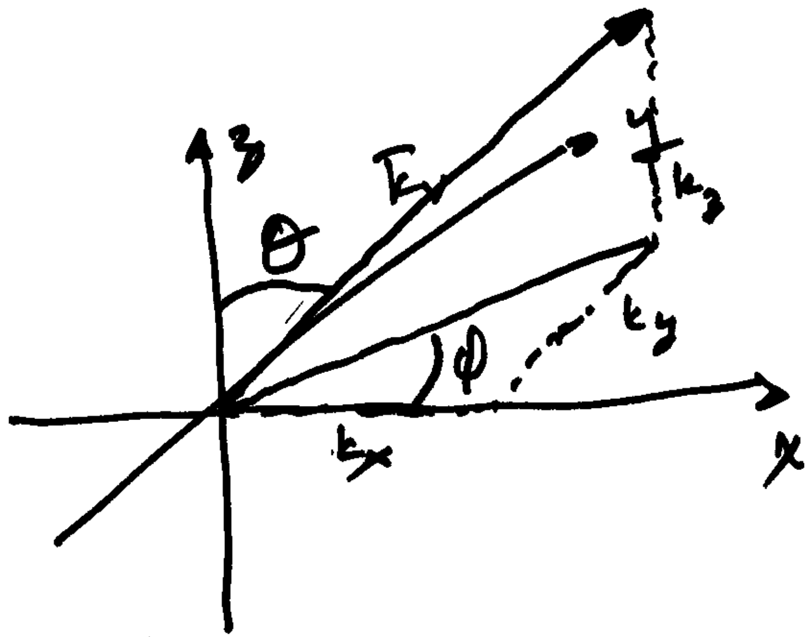
any combination is possible

8 possibilities

always required $k^2 = k_x^2 + k_y^2 + k_z^2$

k_x, k_y & k_z

- vector direction of wave propagation
- rate of change of phase with position in the coordinate directions



$$\bar{k}_v = k_x \bar{i} + k_y \bar{j} + k_z \bar{k}$$

$$|\bar{k}_v| = k = \frac{\omega}{c}$$

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

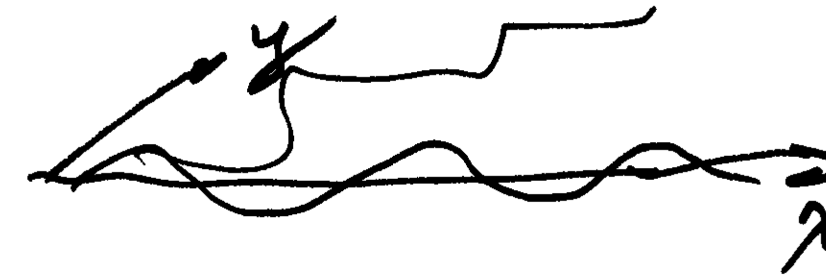
θ = polar angle

φ = azimuth angle

$$k_z = k \cos\theta$$
$$k_x = k \sin\theta \cos\phi$$
$$k_y = k \sin\theta \sin\phi$$

$$A e^{j(\omega t - \bar{k}_v \cdot \bar{x})}$$

$$k_z^2 = \underset{\substack{\uparrow \\ \text{fixed by the freq of the problem}}}{k^2} \underbrace{- k_x^2 - k_y^2}_{\text{chosen}}$$



$$k_z = \pm \sqrt{k^2 - k_x^2 - k_y^2}$$

(i) if $k_x^2 + k_y^2 > k^2$ $\quad < \sqrt{\ } < 0$

$$k_z = \pm j \underbrace{\sqrt{k_x^2 + k_y^2 - k^2}}_{\alpha}$$

$$k_z = \pm j\alpha \qquad e^{\pm j \underbrace{k_z}_{\alpha} z} \rightarrow \underbrace{e^{\pm \alpha z}}_{\text{pure exponential growth or decay}}$$

$e^{-jk_x x}$ evanescent or non-propagating solution

(ii) $k_x^2 + k_y^2 < k^2$

$\rightarrow k_z$ real

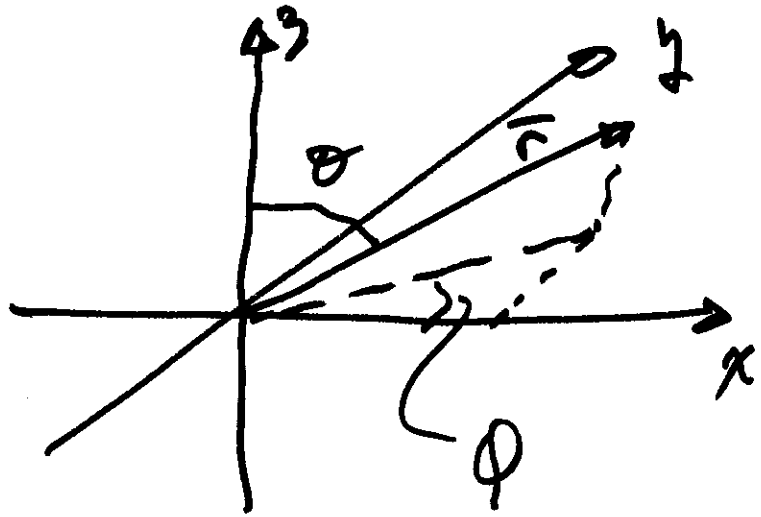
$e^{\pm j k_z z}$] oscillation in the z-direction

propagating solution

$$k^2 = \underbrace{k_x^2 + k_y^2}_{\text{only 1 can be chosen arbitrarily}} + \cancel{k_z^2} \to 0$$

$$k_y = \pm\sqrt{k^2 - k_x^2}$$

3.3.2 Spherical Waves



(r, θ, ϕ)

......

spherically symmetric waves

no variation with θ or φ

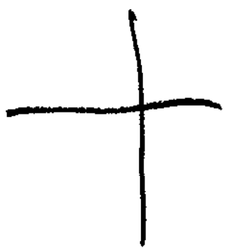
omnidirectional source

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

In a spherical system

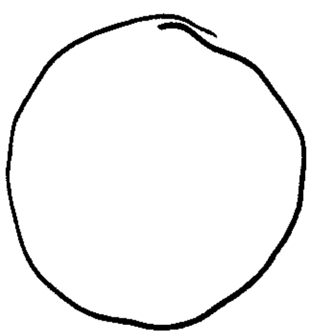
$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \cancelto{0}{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}}\left(\cancelto{0}{\sin\theta \frac{\partial}{\partial \phi}}\right)$$

Spherical symmetry case



$\tilde{p}$ is a function of $\underset{\sim}{r}$ only

1-D



All acoustic quantities in this case are instantaneously constant on a ~~surf~~ spherical surface of radius r

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t}{}^2$$

$$\underbrace{\frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r}}_{\frac{1}{r}\frac{\partial^2 (rp)}{\partial r^2}} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$\frac{\partial^2 (rp)}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2} = 0$$

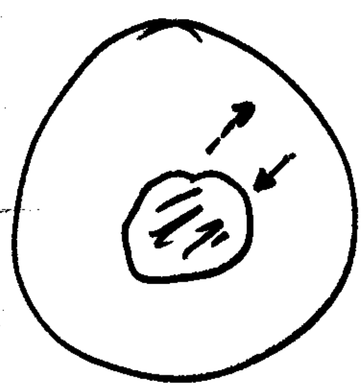
for the harmonic case

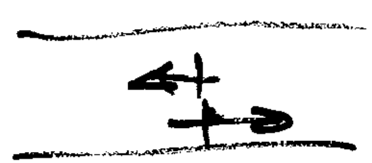
$$\frac{d^2(r\tilde{p})}{dr^2} + k^2(r\tilde{p}) = 0 \qquad k = \frac{\omega}{c}$$

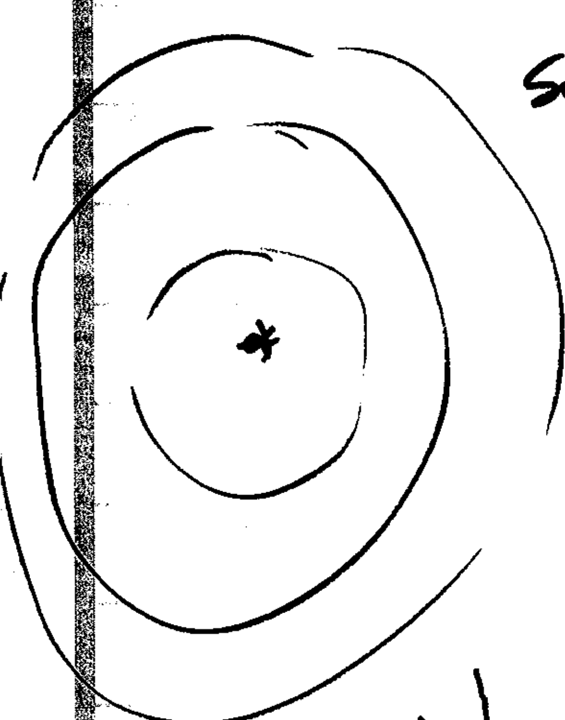
$$r\tilde{p} = A e^{-ikr} + B e^{+ikr}$$

$$\tilde{p}(r) = \underbrace{\frac{A}{r} e^{-ikr}}_{\text{outwards}} + \underbrace{\frac{B}{r} e^{+ikr}}_{\text{inwards}}$$

General harmonic soln for spherically symmetric waves







solution in free space

$$\tilde{p}(r) = \frac{A}{r} e^{-ikr}$$

outward-going soln

$$|\tilde{p}| \propto \frac{1}{r}$$