sound pressure & particle velocity

$$\frac{1}{\beta}\frac{\partial p}{\partial t} + \nabla\cdot\bar{u} = 0$$

given velocity field → work out the pressure

$$-\nabla p = \rho_0 \frac{\partial \bar{u}}{\partial t}$$

momentum

- given pressure can calculate particle velocity

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

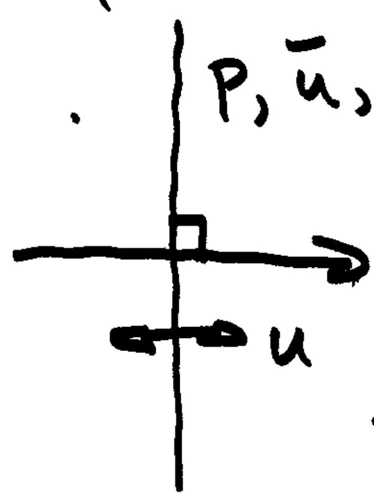
$c_0 = 331.5$ m/s at 0°C

$$c = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{\gamma P_0}{\rho}}$$

One - Dimensional Solutions

3.3.1 Plane Waves

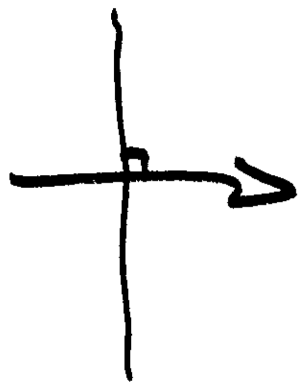
- properties are instantaneously constant over an infinite plane $\perp$ to the direction of wave prop



longitudinal wave
- particle motion is $\parallel$ to direction of wave prop

3.3.1.1 Propagation the x-direction

- no variation in the y or z directions



$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

$$p(x,t) = \underbrace{p_1(ct - x)}_{+ve} + \underbrace{p_2(ct + x)}_{-ve}$$

complex harmonic form

$$p(x,t) = \tilde{p}(x)\, e^{j\omega t}$$ sub into wave eqn

$$\frac{d^2\tilde{p}}{dx^2} + k^2 \tilde{p} = 0$$ Scalar Helmholtz Eqn.

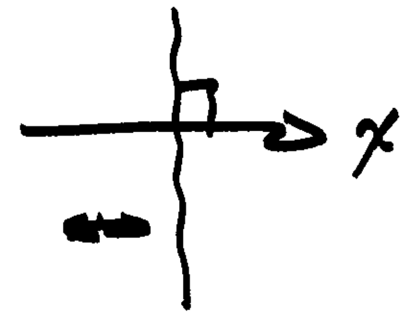
$$k = \frac{\omega}{c}$$

$$\underline{\tilde{p}(x)} = A_1 e^{-jkx} + B_1 e^{+jkx}$$

steady solution

$$p(x,t) = \tilde{p}(x)\, e^{j\omega t}$$

$u_x(x,t)$



linearized momentum eqn.

$$-\nabla p = \rho_0 \frac{\partial \bar{u}}{\partial t} \qquad \bar{u} = u_x \bar{i} + u_y \bar{j} + u_z \bar{k}$$

1-D

$$-\frac{\partial p}{\partial x}\cancel{\bar{i}} = \rho_0 \frac{\partial u_x}{\partial t}\cancel{\bar{i}}$$

$$p(x,t) = \tilde{p}(x)\, e^{j\omega t}$$

$$u_x(x,t) = \hat{u}_x(x)\, e^{j\omega t}$$

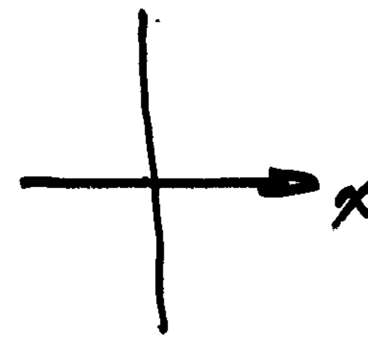
$$-\frac{d\tilde{p}}{dx} = (j\omega)\,\rho_0\, \tilde{u}_x$$

$$\boxed{\hat{u}_x = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dx}}$$

one component of
the particle velocity
vector

$$\tilde{u}_y = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dy}$$

$$\tilde{u}_z = \_\_\_ \quad \_$$



Particle Velocity associated with

$$\tilde{p}_+ = A_1 e^{-jkx}$$

$$\tilde{u}_{x_+} = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}_+}{dx} = -\frac{1}{j\omega\rho_0}(-jk)A_1 e^{-jkx}$$

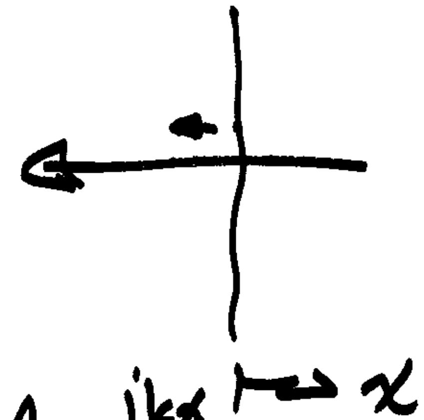
$$= \frac{k}{\omega\rho_0}A_1 e^{-jkx} \qquad k = \frac{\omega}{c}$$

$$= \frac{A_1}{\rho_0 c}e^{-jkx} \quad \tilde{p}_+$$

$$\tilde{u}_{x_+} = \frac{\tilde{p}_+}{\rho_0 c}$$ characteristic impedance for a freely propagating plane wave

$$\frac{\tilde{p}_+}{\tilde{u}_{x_+}} = \rho_0 c \qquad 415 \quad [\text{Rayls}]$$

$$\tilde{p}_- = A_2 e^{jkx}$$



$$\tilde{u}_{x-} = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}_-}{dx} = -\frac{1}{j\omega\rho_0}(jk)A_2 e^{jkx}$$

$$= -\frac{\underbrace{A_2 e^{jkx}}_{\tilde{p}_-}}{\rho_0 c}$$

$$= -\frac{\tilde{p}_-}{\rho_0 c}$$

$$\frac{\tilde{p}_-}{\tilde{u}_{x-}} = -\rho_0 c$$

use these results to calculate the particle velocity for any given sound field

$$\hat{u}_x = -\frac{1}{j\omega\rho_0}\frac{d\hat{p}}{dx} \qquad \text{always true}$$

$$\bar{u} = -\frac{1}{j\omega\rho_0}\nabla\hat{p}$$

$$\hat{p}(x) = \underbrace{A_1 e^{-jkx}}_{\hat{p}_+} + \underbrace{A_2 e^{jkx}}_{\hat{p}_-}$$

for this case

$$\hat{u}_x = \frac{\tilde{p}_+}{\rho_0 c} - \frac{\tilde{p}_-}{\rho_0 c}$$

true for plane waves in a lossless medium

$$= \frac{A_1}{\rho_0 c} e^{-ikx} - \frac{A_2}{\rho_0 c} e^{+ikx}$$

Typical Velocity

- plane wave prop in the +ve x-dir

$$|\hat{p}_+| \simeq \underline{1}\ \text{Pa}$$

$$\underline{94\ \text{dB}}$$

$$|\tilde{u}_{x_+}| = \frac{|\tilde{\hat{p}}_+|}{\rho_0 c}$$

$$= \frac{1}{415} \simeq \underline{2.4\ \text{mm/s}}$$

$$c = 340\ \text{m/s}$$

$$|\tilde{u}_+| << c$$

ξ displacement

for harmonic fields

$$j\omega\tilde{\xi} = \tilde{u}$$

$$|\tilde{\xi}| = \frac{|\tilde{u}|}{\cancel{j}\omega}$$

particle displacement

$$|\tilde{\xi}| = \frac{|\hat{u}|}{\omega}$$ at $\underline{1}$ kHz $\omega \simeq 6000$

$\omega = 2\pi f$

94 dB
@ 1 kHz $= \dfrac{2.4 \times 10^{-3}}{6 \times 10^{3}}$

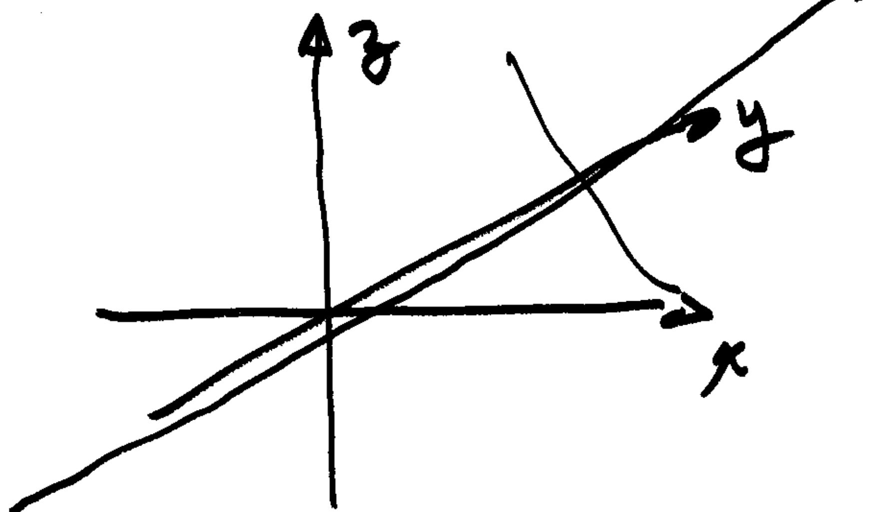
$O(10^{-6})$ micron

74 dB $O(10^{-7})$

54 $O(10^{-8})$

34 dB $O(10^{-9})$ nanometer

3.3.1.2 Arbitrary Direction wrt coordinate system



$$\tilde{p}(x,y,z,t) = A\,e^{j(\omega t - k_x x - k_y y - k_z z)}$$

component wave numbers

$$\nabla^2 \tilde{p} + k^2 \tilde{p} = 0$$

$$\underbrace{k^2 = k_x^2 + k_y^2 + k_z^2}_{\text{if this is true}}$$

e.g., +ve x
+ve y
+ve z