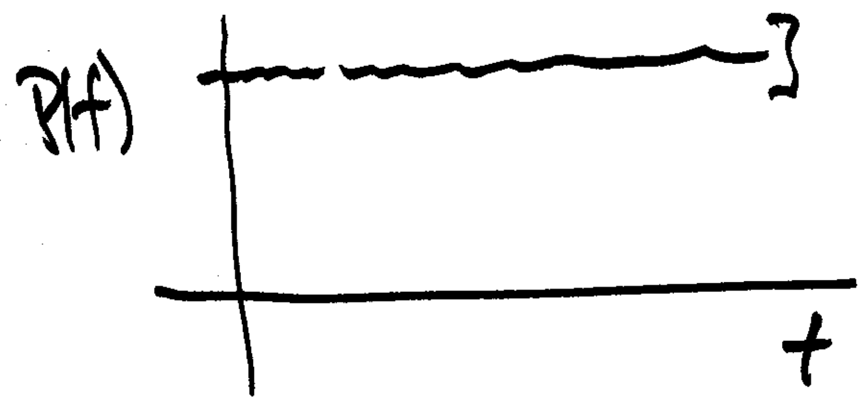
# Sound Waves

- propagating pressure fluctuations in an elastic medium




$p(t) = P(t) - P_0$

$p$ & $u$ ~~Eqn~~ Eqn of state

Eqn of continuity

$p = \beta s$    $\dfrac{\partial s}{\partial t} + \nabla \cdot \vec{u} = 0$

$\gamma P_0$

$$\nabla \cdot \bar{u}$$

$$\bar{u} = u_x \bar{i} + u_y \bar{j} + u_z \bar{k}$$

$\nabla$ grad operator

$$\nabla = \bar{i} \frac{\partial}{\partial x} + \bar{j} \frac{\partial}{\partial y} + \bar{k} \frac{\partial}{\partial z}$$

$$\nabla \cdot \bar{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

(iii) pressure vs particle velocity

combine (1) $p = \beta s$ with (2)

$$\boxed{\frac{1}{\beta}\frac{\partial p}{\partial t} + \nabla \cdot \overline{u} = 0} \quad (3)$$

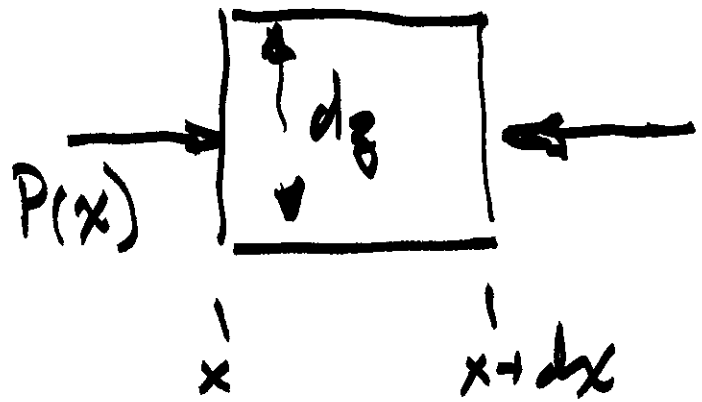
<u>linearized</u> continuity equation

## 3.2.2 Pressure - Velocity Relation (II)

(i) Eqn of motion

Apply $f = ma$ to a fixed mass of fluid that is moving with the fluid

1-D




$$P(x+dx) = P(x) + \frac{dP}{dx}\Big|_x dx + \ldots\ldots$$

Net force acting in the x-direction

$$df_x = P(x)\,dy\,dz - P(x+dx)\,dy\,dz$$

$$= \underbrace{-\frac{dP}{dx}\Big|_x}_{\text{Force/unit volume}} \underbrace{dx\,dy\,dz}_{\text{volume } dV}$$

In 3-D $\quad d\bar{f} = -\nabla P \, dV$

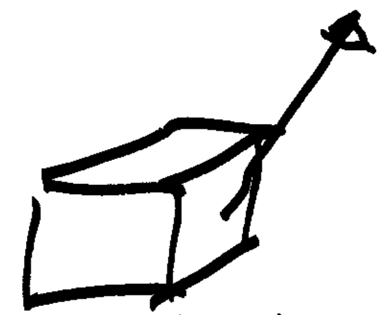
$\nabla P$ = gradient of the pressure

$$= \frac{\partial P}{\partial x}\bar{i} + \frac{\partial P}{\partial y}\bar{j} + \frac{\partial P}{\partial z}\bar{k}$$

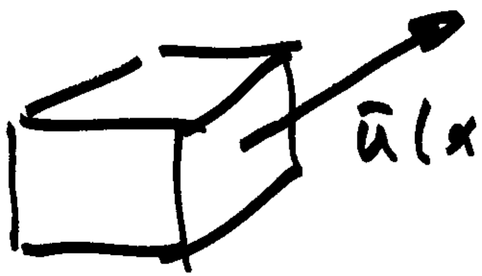
direction - steepest change
in pressure

## (iii) Acceleration of a moving element of fluid

$$f = ma$$



$$\bar{u}(x+dx, y+dy, z+dz, t+dt) = \bar{u}_f$$



$$\bar{u}(x, y, z, t) = \bar{u}_i$$

$$\bar{u}_f = \overset{\overline{u}_i}{u(x,y,z,t)} + \left.\frac{\partial \bar{u}}{\partial x}\right|_{x,y,z,t} dx + \cdots \overset{dy}{} \quad - - \quad \overset{dz}{}$$

$$+ \left.\frac{\partial \bar{u}}{\partial t}\right|_{x,y,z,t} dt$$

$$dx = u_x \, dt$$

$$dy = u_y \, dt$$

$$dz = u_z \, dt$$

$$\bar a = \frac{\bar u_f - \bar u_i}{dt} = \left[\underbrace{u_x \frac{\partial \bar u}{\partial x} + u_y \frac{\partial \bar u}{\partial y} + u_z \frac{\partial \bar u}{\partial z}} + \frac{\partial \bar u}{\partial t}\right]$$

convective acceleration
- very small (in linear acoustics)

$$\bar a = \cancelto{0}{(\bar u \cdot \nabla)\bar u} + \frac{\partial \bar u}{\partial t}$$

neglect because terms are non-linear

$$d\bar f = -\nabla P \, dV$$

$$\bar a = \frac{\partial \bar u}{\partial t}$$

mass of element

$$m = \rho \, dV$$

$$d\bar{f} = m\bar{a}$$

$$-\nabla P \, dV = \rho \, dV \frac{d\bar{u}}{dt}$$

(iv) Pressure-Velocity Relation

$$P = P_0 + p \qquad \underline{\nabla P = \nabla p}$$

$$\rho = \rho_0(1+s)$$

$$s = \frac{\rho - \rho_0}{\rho_0}$$

$$s << 1$$

$$\boxed{-\nabla p = \rho_0 \frac{\partial \bar{u}}{\partial t}} \quad (4)$$

linearized momentum equation

Euler Equation

## 3.2.3 Linear Wave Equation

$$(3) \qquad \frac{1}{\beta}\frac{\partial p}{\partial t} + \nabla\cdot\bar{u} = 0 \qquad \frac{\partial}{\partial t}$$

$$(4) \qquad \nabla p + \rho_0 \frac{\partial \bar{u}}{\partial t} = 0 \qquad \frac{1}{\rho_0}\nabla\cdot$$

eliminate $\bar{u}$

subtract (3) from (4)

$$\nabla^2 p - \left(\frac{\rho_0}{\beta}\right)\frac{\partial^2 p}{\partial t^2} = 0$$

$$c = \sqrt{\frac{\beta}{\rho_0}}$$

$$\boxed{\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0}$$

c = speed of sound

$$= \sqrt{\frac{\beta}{\rho_0}} = \sqrt{\frac{\gamma P_0}{\rho_0}}$$

linearized wave equation

- governs propagation of small amplitude pressure fluctuations in a stationary, ideal fluid.

$\hookrightarrow u < \frac{1}{10}c$

## 3.7.4 Sound speed

c = speed of wave propagation

$$= \sqrt{\frac{\gamma P_0}{\rho_0}} = 331.6 \text{ m/s at } 0°C$$

$$\simeq 340 \text{ m/s at room temp } 20°C$$

in an isothermal atmosphere

$\left(\frac{P_0}{\rho_0}\right)$ is a constant

$c \neq$ function of height (if temp is constant) c = directly proportional to absolute temp to the ½ power

In air $c = c_0 \sqrt{\frac{T_K}{273}}$ $\quad T_K$ [°K]

$= c_0 \sqrt{1 + \frac{T_C}{273}}$ $\quad T_C$ [°C]

## 3.3 One-Dimensional Solutions

- depends only on 1 space variable
  - plane
  - spherical  spherically symmetric
  - cylindrical  cylindrically symmetric