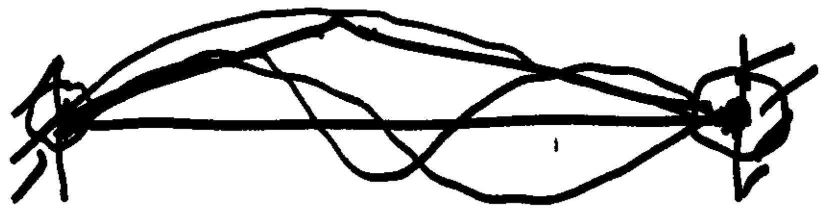
Normal Modes of Finite strings



characteristic eqn

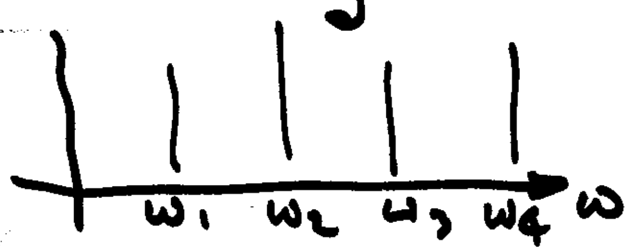
$$\sin \underbrace{k_n L}_{\phi} = 0$$

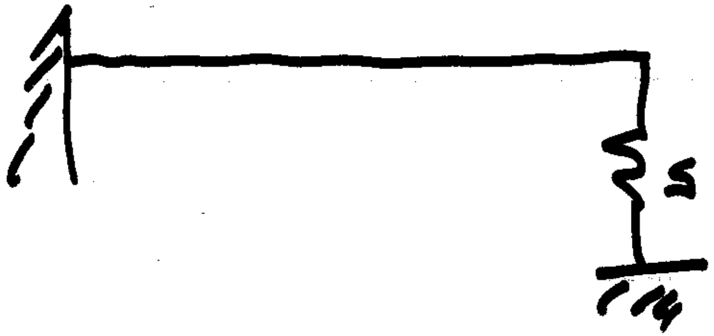
$$L = \frac{n}{2}\lambda_n$$

k_n's $\rightarrow$ ω_n's

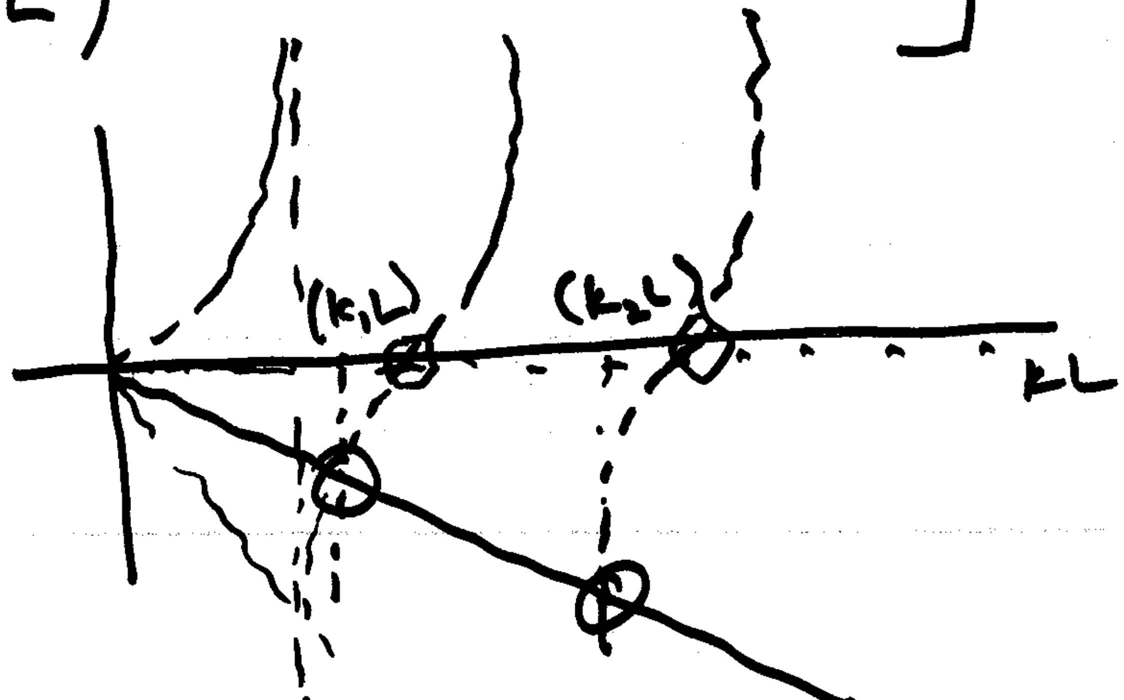
<u>mode shapes</u>

$$y(x,t) = \sum_{n=1}^{\infty} \underbrace{a_n}_{\text{initial conditions}} \sin k_n x \, e^{i\omega_n t}$$





$$-\left(\frac{T}{sL}\right)(k_n L) = \tan k_n L$$



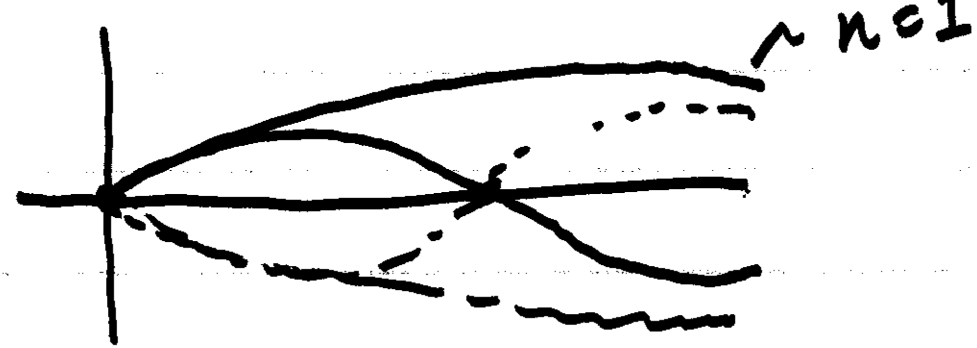
k_n's $\downarrow$ $\frac{\omega_n}{c}$ $\rightarrow$ natural freq's

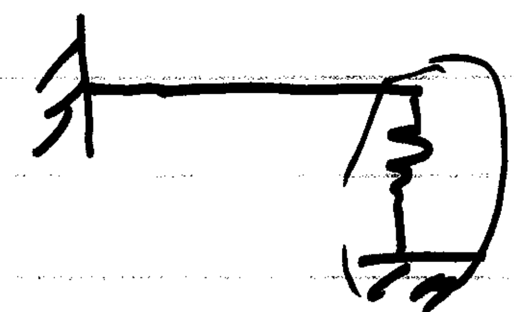
$k = \frac{\omega}{c}$

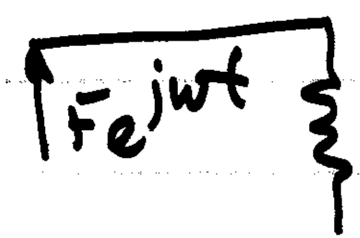
$\frac{\omega_n}{2\pi} = f_n$ [Hz]

Mode shape

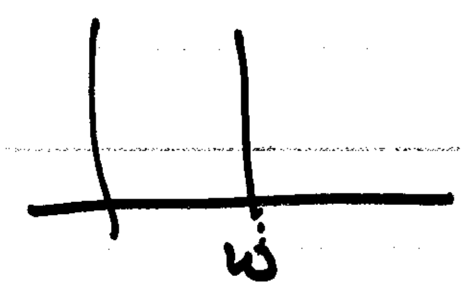
$$y_n(x,t) = -2jA_n \underline{\sin k_n x}\, e^{j\omega_n t}$$ nth mode







$$y(x,t) = \sum_{n=1}^{\infty} \text{modes}$$



summary

- Derivation of a wave equation (modeling)
  - restoring force
  - equation of motion

  ] wave equation

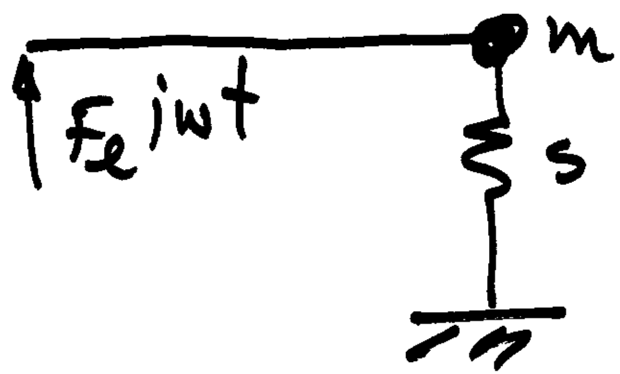
  - inertia
  - stiffness

- wave propagation

  $c = \sqrt{T/\rho_L}$ harmonic solution $e^{j\omega t}$

  k = spatial frequency = wave number e^{-jkx}

- Boundary Conditions
  - fixed
  - stiffness
  - mass
  - force

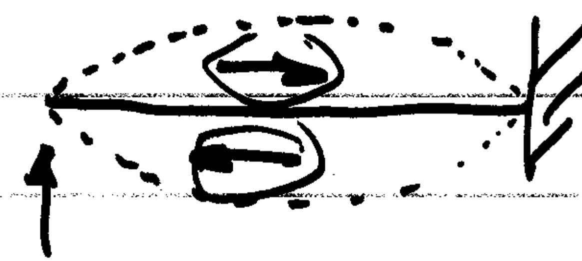


Forced Vibration response is at ω

$$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)}$$

Impedance $\left(\dfrac{\text{Force}}{\text{Velocity}}\right)$

Standing Waves



$e^{j(\underbrace{\omega t - kx}_{\phi_w})}$

Free Vibration
- natural frequencies
- modes & mode shapes

characteristic equation

Characteristic Impedance

$\phi = \omega t$

$e^{i\phi}$ $\quad \frac{\partial \phi}{\partial t} = \omega$

$\rho_L c$

phase speed vs. particle velocity

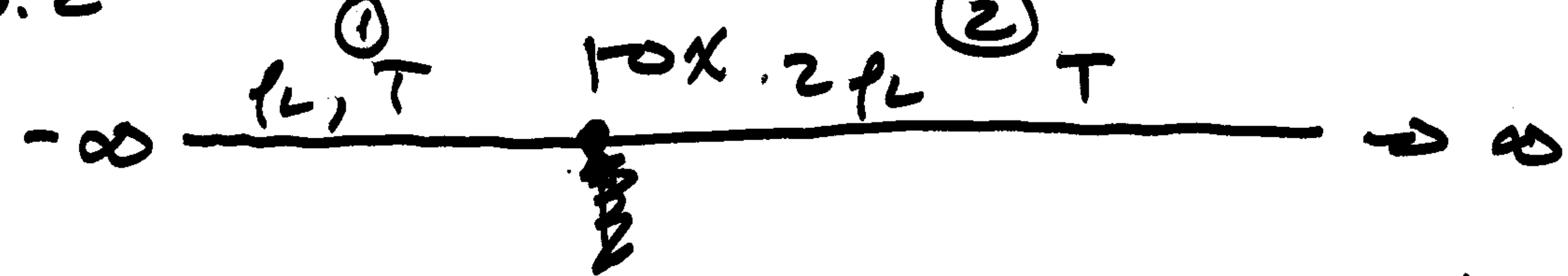
c $\quad u$ $\quad \frac{\partial y}{\partial t}$

Homework Hints

2.4.1 $$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

2.8.1 $$y = 4\cos(3t - 2x) \qquad \cos(\omega t - kx)$$

2.8.2



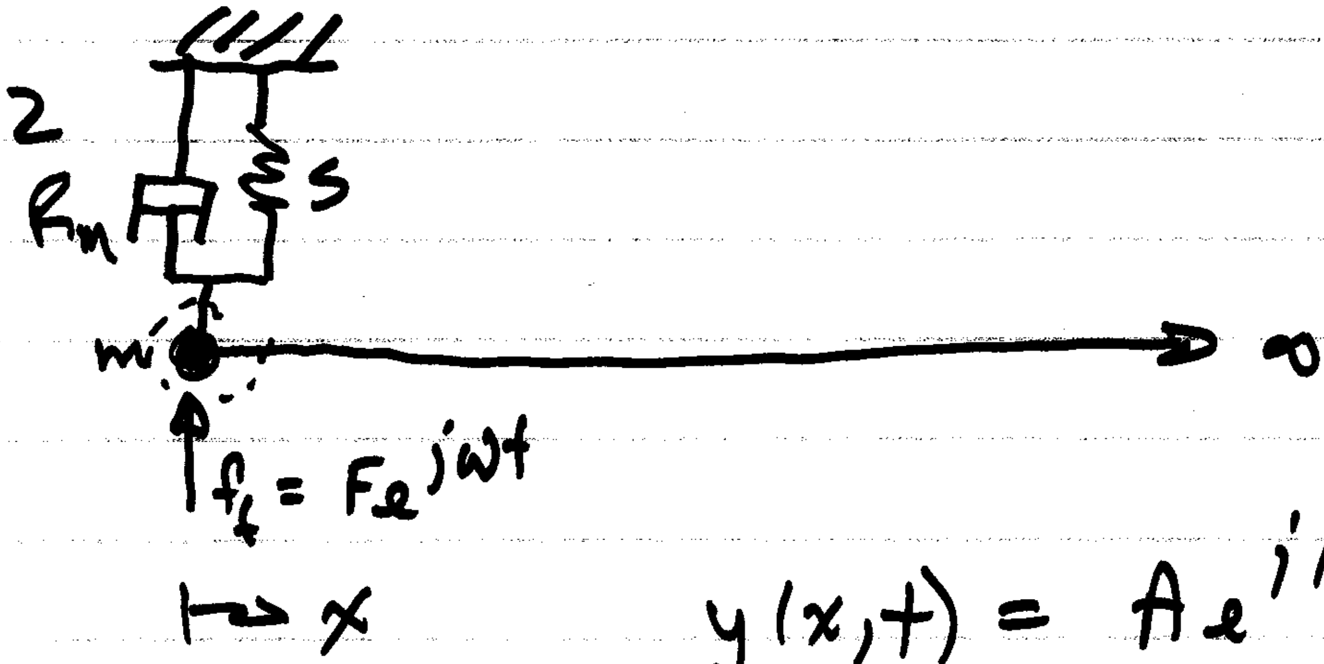
① $y_1(x,t) = A e^{j(\omega t - k_1 x)} + \hat{B} e^{j(\omega t + k_1 x)}$

② $y_2(x,t) = \hat{C} e^{j(\omega t - k_2 x)}$

2 b.c.'s — displacement continuity at $x=0$

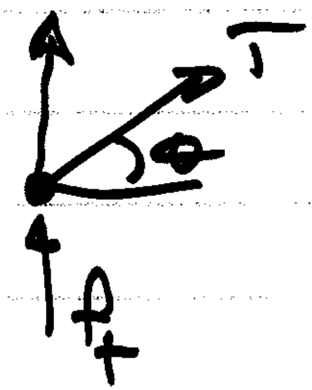
$\sum f_y = 0$ at $x=0$

2.9.2



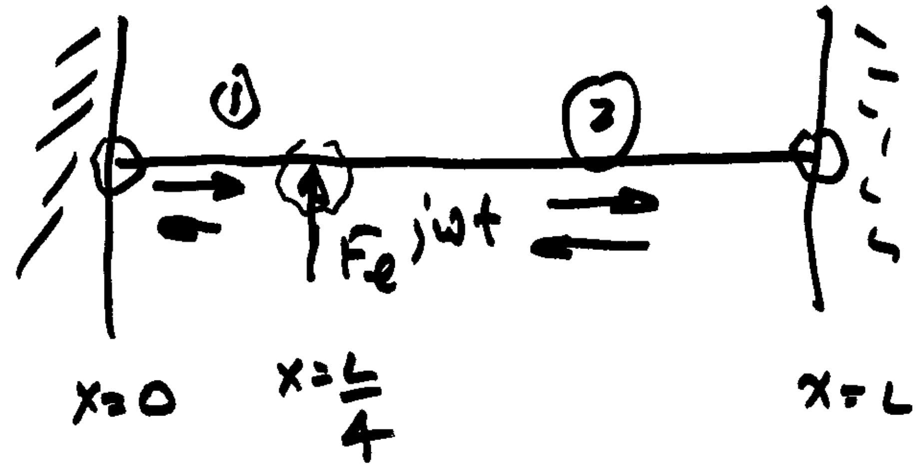
$$y(x,t) = Ae^{j(\omega t - kx)}$$

FBD



$$\Sigma f_y = ma$$

2.9.3



$\cot\frac{kL}{4}$

$\cot\frac{3kL}{4}$

Drive point impedance

$$y_1 = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

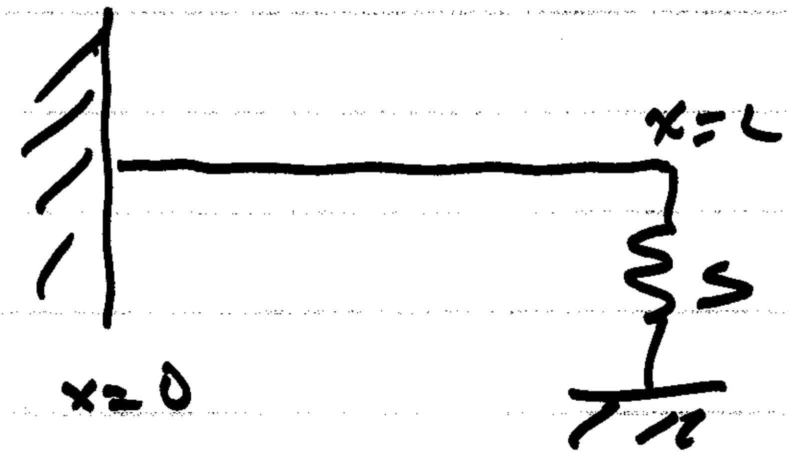
$$y_2 = C \qquad D$$

$$Z_{mo} = \frac{F_e{}^{j\omega t}}{\text{velocity}}$$

$$\left(1 + e^{2jkL}\right)$$

$$e^{jkL}\left(e^{-jkL} + e^{+jkL}\right) \quad 2\cos kL$$

2.11.1



apply 2 b.c.'s → characteristic eqn

solve for $\underbrace{k_1 + k_2}_{\text{stick into solution}}$

$T = sL$