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~~Thu28~~

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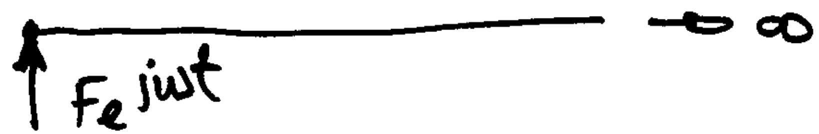
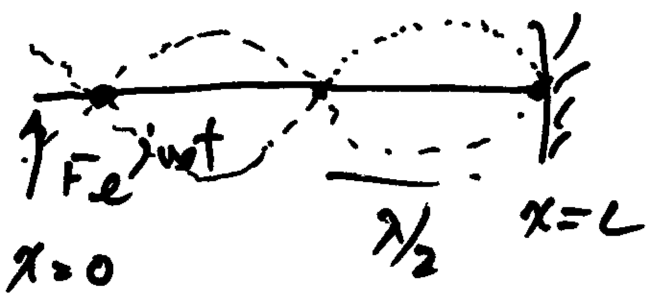


$$u(x,t) = \frac{F}{\rho_L c} e^{j\omega t} e^{-jkx}$$



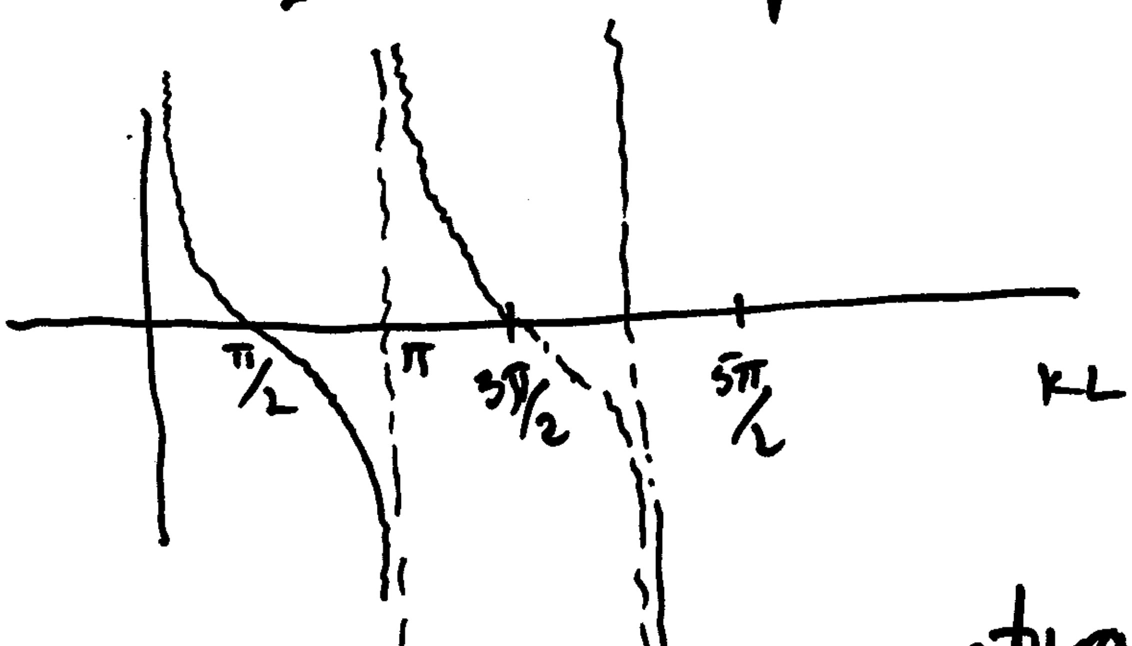
$$Z_{m0} = \rho_L c$$

$$y(x,t) = \left(\frac{F}{kT}\right) \frac{\sin k(L-x)}{\cos kL} e^{j\omega t}$$

$$Z_{mo} = \underbrace{-j\,\rho_L c}_{\text{medium}}\;\underbrace{\cot kL}_{\text{geometry}}$$

imaginary

$\text{Im}\{Z_{mo}\} = 0$ defines the natural frequencies

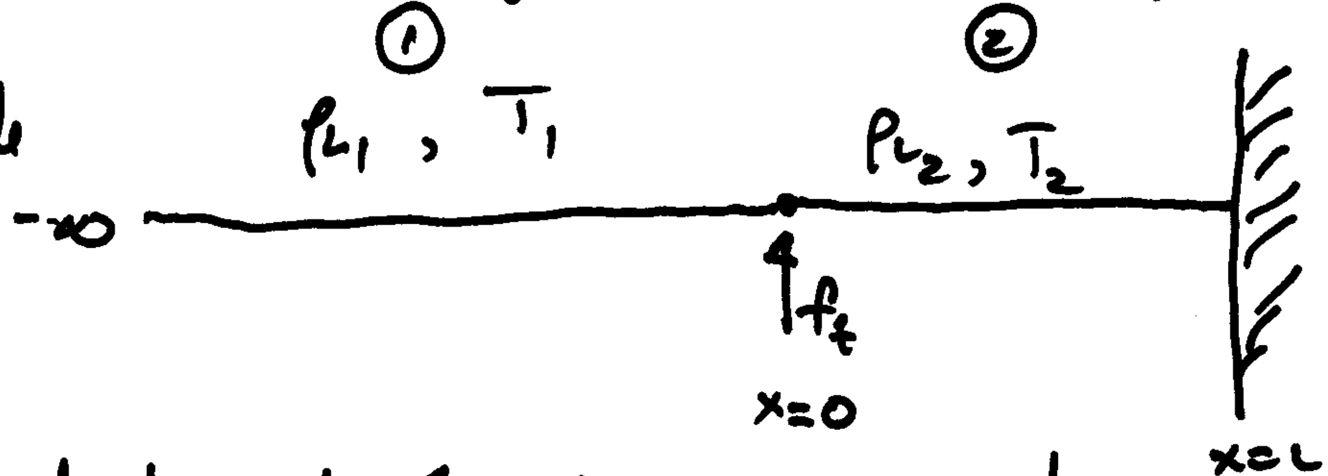


frequencies of maximum response

$(kL)_1 = \pi/2 \quad (kL)_2 = \frac{3\pi}{2}$ → natural freqs

2.4.3 Strings with multiple segments

Example



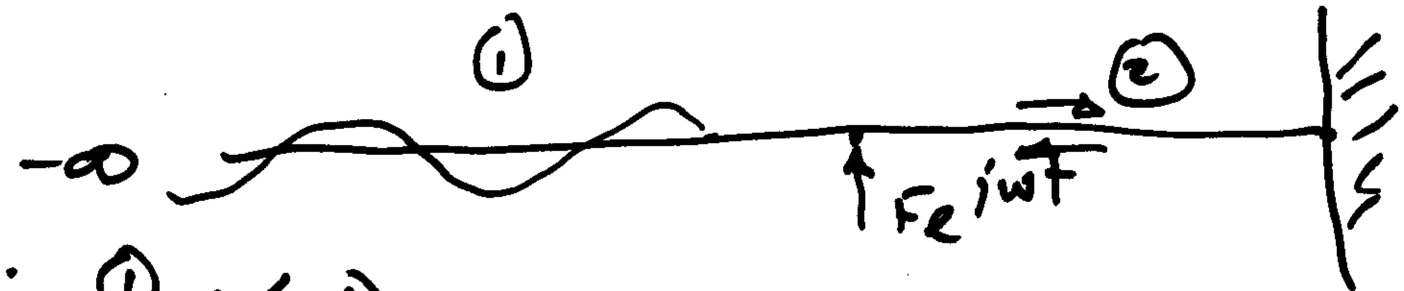
$f_t = F e^{j\omega t}$

must treat the two segments as distinct strings
- coupled together through b.c.'s

wave Eqn for each segment

$$\frac{\partial^2 y_1}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 y_1}{\partial t^2} = 0 \qquad \frac{\partial^2 y_2}{\partial x^2} - \frac{1}{c_2^2}\frac{\partial^2 y_2}{\partial t^2} = 0$$

$c_1 = \sqrt{T_1/\rho_{L_1}}$ $\qquad$ $c_2 = \sqrt{T_2/\rho_{L_2}}$



In the region ① $x \le 0$

$$y_1(x,t) = \cancel{A e^{j(\omega t - k_1 x)}} \to 0 + B e^{j(\omega t + k_1 x)} \qquad k_1 = \frac{\omega}{c_1}$$

+ve   -ve

In region ② $0 \le x \le L$

$$y_2(x,t) = C e^{j(\omega t - k_2 x)} + D e^{j(\omega t + k_2 x)}$$

$$k_2 = \omega / c_2$$

3 unknowns

3 b.c.'s - 2 displacement condition
- force b.c.

slope at the drive point is not necessarily continuous - string have no flexural stiffness

b.c.'s (i) $x = L$ $\quad y_2(L, t) = 0$ (1)

(ii) $x = 0$ $\quad \underbrace{y_1(0,t) = y_2(0,t)}_{\text{displacement continuity}}$ (2)

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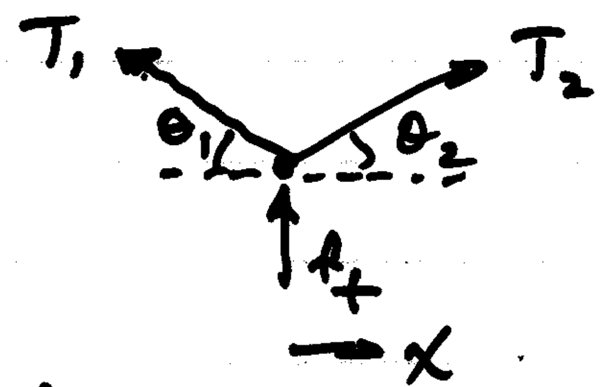
(iii) force b.c. at $x=0$



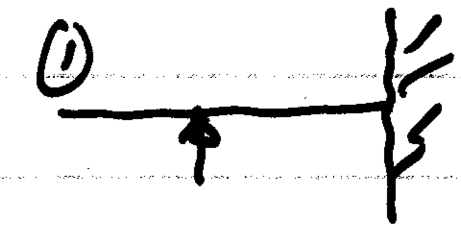
$$\sum f_y = 0 = f_t + T_2 \sin\theta_2\Big|_{x=0} + T_1 \sin\theta_1\Big|_{x=0}$$

$$f_t + T_2 \frac{\partial y_2}{\partial x}\Big|_{x=0} - T_1 \frac{\partial y_1}{\partial x}\Big|_{x=0} = 0 \qquad (3)$$

$$f_t = F e^{j\omega t}$$

Substitute assumed solutions into (1), (2) & (3)

3 linear eqns in unknowns B, C & D
:
solve

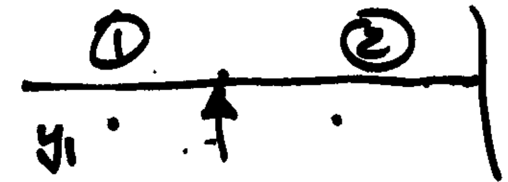


$$Z_{mo} = \frac{f_t}{u_1|_{x=0}} = \rho_{L_1} c_1 - j \rho_{L_2} c_2 \cot k_2 L$$

use either y_1 or y_2 to calculate either u_1 or u_2

in electrical terms
- series connection (since the velocity is shared)

Notes:

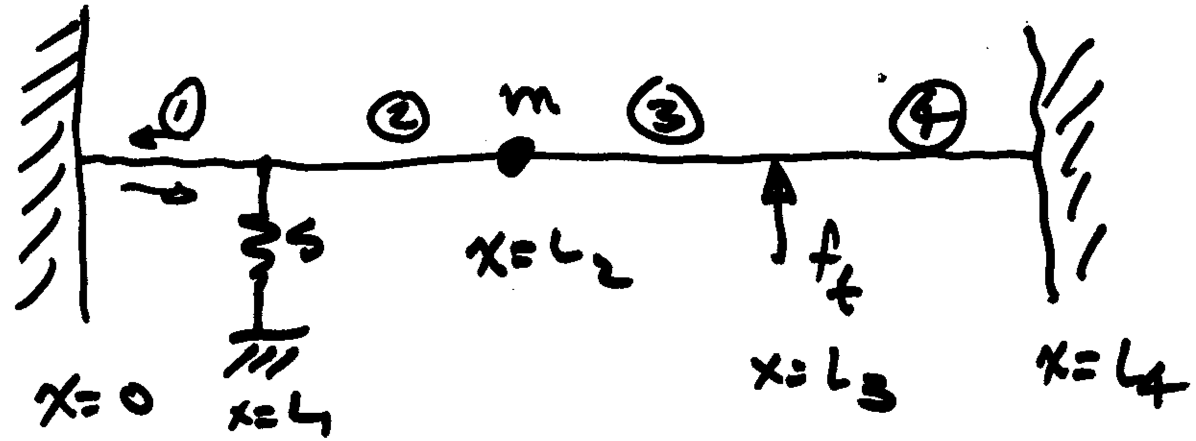


(i) solution for y_1 applies only in the region $x \le 0$

(ii) solution for y_2 applies only in the region $0 \le x \le L$

(iii) same approach even if the two segments have the same material properties

(iv) same approach can be extended to any number of segments



$$f_t = F e^{j\omega t}$$

$$y_1(x,t) = A e^{j(\omega t - k_1 x)} + B e^{j(\omega t + k_1 x)} \qquad k_1 = \omega/c_1 \quad c_1 = \sqrt{T/\rho_{L_1}}$$

$$0 \leq x \leq L_1$$

$$y_2(x,t) = C \sim\sim\sim + D \sim\sim\sim \qquad k_2$$

$$y_3(x,t) = E \sim\sim\sim + F \sim\sim\sim \qquad k_3$$

$$y_4(x,t) = G \sim\sim\sim + H \sim\sim\sim \qquad k_4$$

8 unknowns

8 b.c.'s

General Approach

- write general solution for each segment (8 unknowns)
- write down the b.c.'s (8)
- substitute the solutions into the b.c.'s & solve

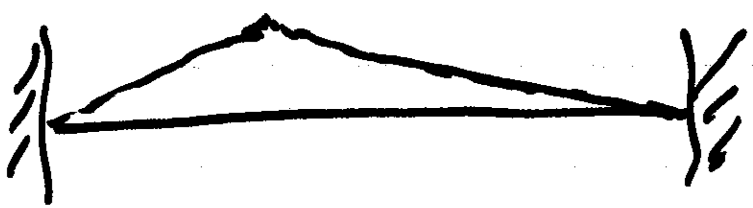
$$\begin{bmatrix} 8 \times 8 \\ \text{coefficient} \\ \text{matrix} \end{bmatrix} \begin{bmatrix} A \\ \vdots \\ \vdots \\ H \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

forcing vector

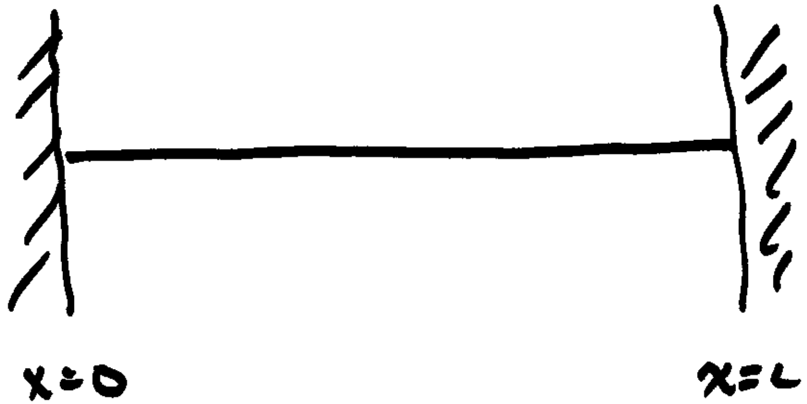
2.5 Normal Modes of Finite Strings

Free vibration

- when the string is "forced" into motion by initial conditions



2.5.1 Characteristic Equation



One segment

$$y(x,t) = A e^{j(\omega t - \underset{\sim}{k} x)} + B e^{j(\omega t + \underset{\sim}{k} x)}$$