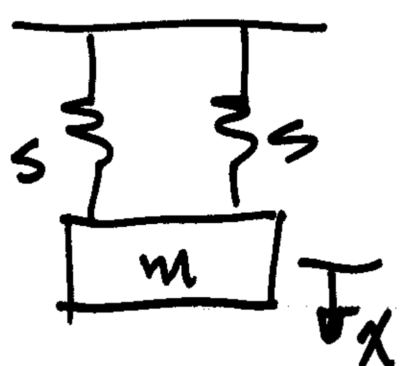
HW # 1 Sept 11

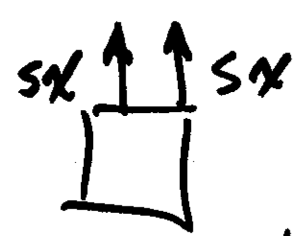
Homework Hints

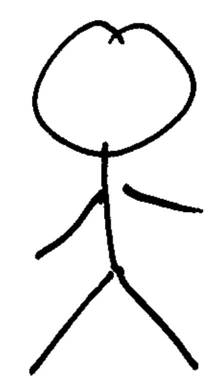
1.2.1 FBD, EOM] to create

$$\frac{d^2x}{dt^2} + \underbrace{\bigcirc}_{\omega_0^2}\, x = 0$$

(a)

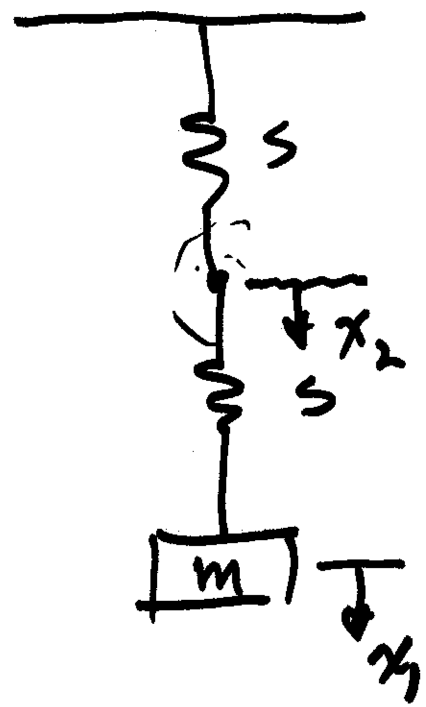


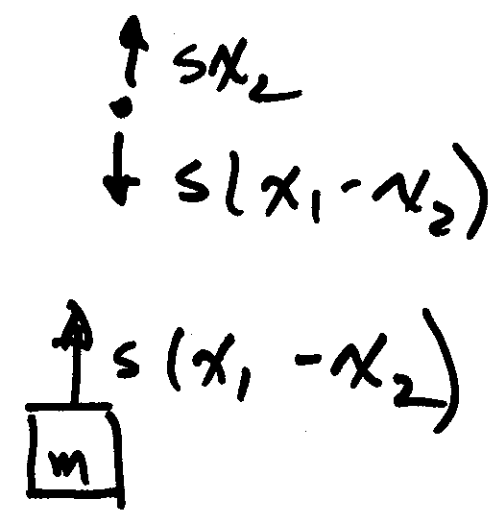




$$m\frac{d^2x}{dt^2} = -2sx \qquad \frac{d^2x}{dt^2} + \frac{2s}{m}x = 0$$

(c)





$\Sigma f = 0$

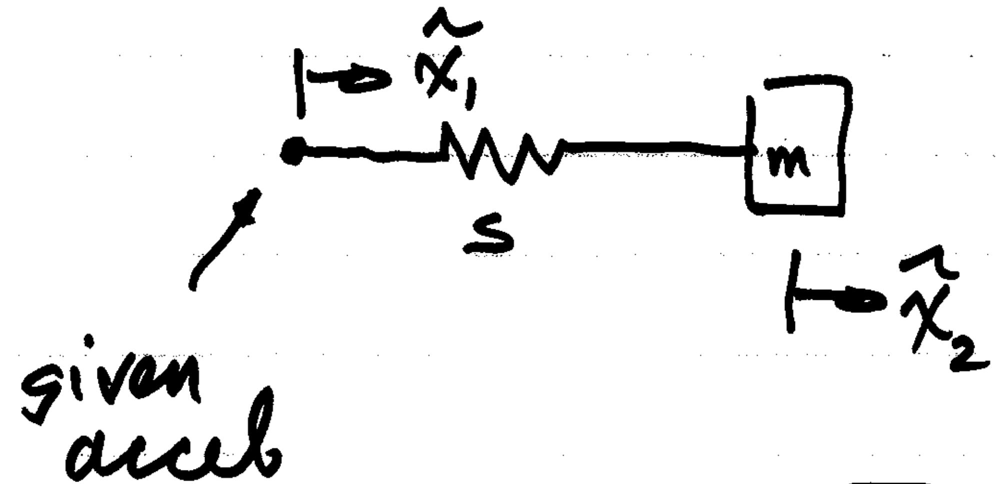
$$m \frac{d^2 x_1}{dt^2} = -s(x_1 - x_2)$$

1.3.2 $$x = x_0 \cos\omega_0 t + \frac{u_0}{\omega_0} \sin\omega_0 t$$

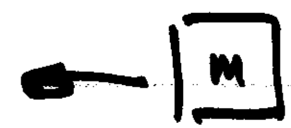
1.5.1 $$\hat{x} = \tilde{A} e^{j\omega t} \qquad \tilde{A} = A e^{j\phi} \qquad A \doteq |\tilde{A}|$$

$$\phi = \tan^{-1} \frac{I_m\{\hat{A}\}}{Re\{\tilde{A}\}}$$

1.7.5



$$\frac{1}{1 \cancel{s} - \left(\frac{\omega}{\omega_0}\right)^2}$$



$s(\tilde{x}_2 - \hat{x}_1)$

$$m \frac{d^2 x_2}{dt^2} = -s(\tilde{x}_2 - \hat{x}_1)$$

$$\frac{d^2 \hat{x}_1}{dt^2} = Ae^{j\omega t} \qquad \hat{a}_1 = j\omega \hat{u}_1 = (j\omega)(j\omega)\hat{x}_1$$

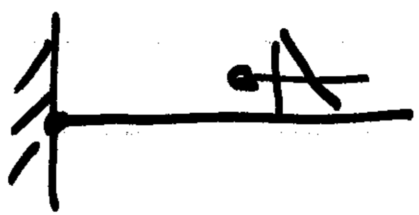
$$\hat{x}_2 = Be^{j\omega t}$$

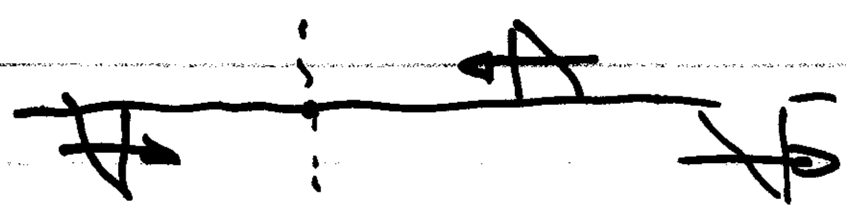
wave number

$$y(x,t) = A e^{j(\omega t - kx)}$$

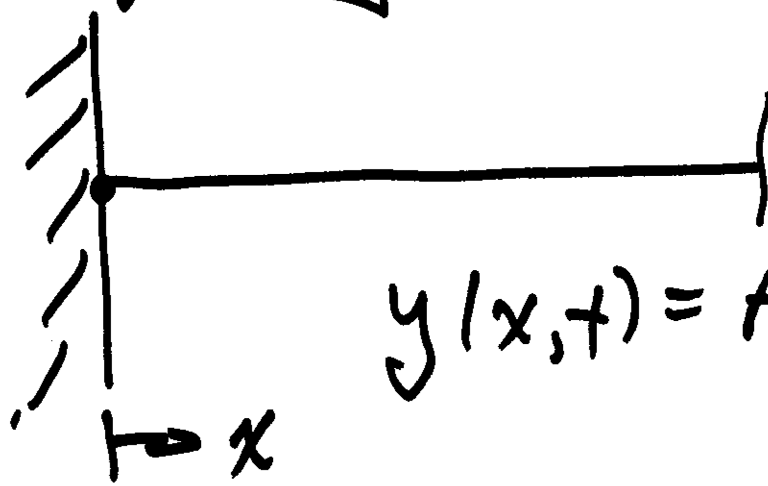
$$\left(\frac{\omega}{c}\right) = \left(\frac{2\pi}{\lambda}\right)$$

Boundary Conditions





Frequency Domain



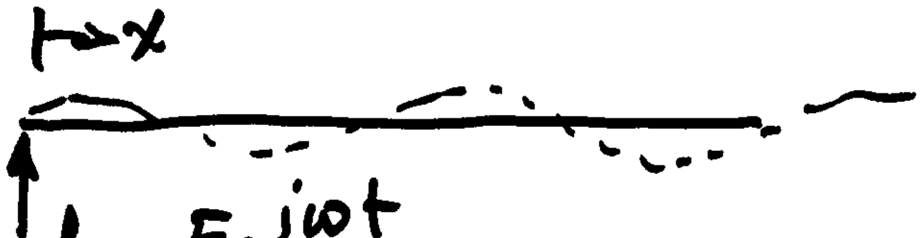
$$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)}$$

$y(0,t) = 0$ $\qquad$ $B = -A$

$$y(x,t) = A e^{j\omega t}\left(e^{-jkx} \cancel{B} - e^{+jkx}\right)$$

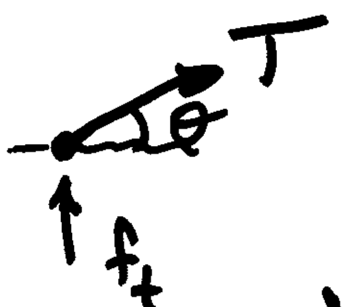
$\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2j}$ $\qquad$ $y(x,t) = -2jAe^{j\omega t}\sin kx$

2.3.2 Force b.c. at $x=0$



$$f_t = Fe^{i\omega t}$$

FBD

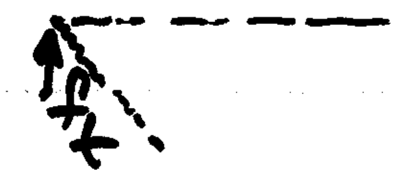


$$\Sigma f_y = 0$$

(since $m=0$)

$$f_t + T\overbrace{\sin\theta}^{dy/dx}\Big|_{x=0} = 0$$

$$f_t + T \frac{dy}{dx}\bigg|_{x=0} = 0$$



$$\frac{dy}{dx}\bigg|_{x=0} = -\frac{f_t}{T}$$

special case
if $f_t = 0$
no lateral constraint $\rightarrow$ $\frac{dy}{dx} = 0$

In the harmonic case

$$f_t = Fe^{j\omega t}$$

$$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{f_t}{T} \qquad y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

$$\frac{dy}{dx} = -jkAe^{j(\omega t - kx)} + jkBe^{j(\omega t + kx)}$$

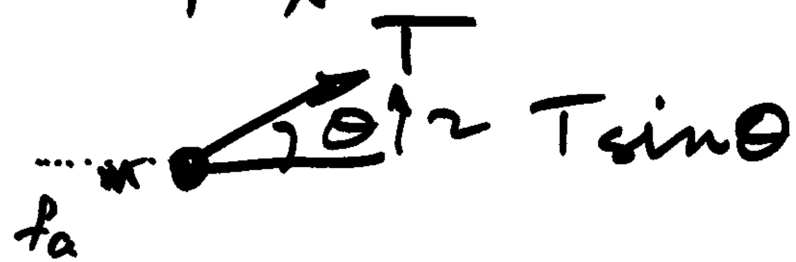
$$\left.\frac{dy}{dx}\right|_{x=0} = -jkA\cancel{e^{j\omega t}} + jkB\cancel{e^{j\omega t}} = -\frac{F\cancel{e^{j\omega t}}}{T}$$

$$B = -\frac{F}{jkT} + A$$

note correction

2.3.3 Mass at $x=0$

FBD

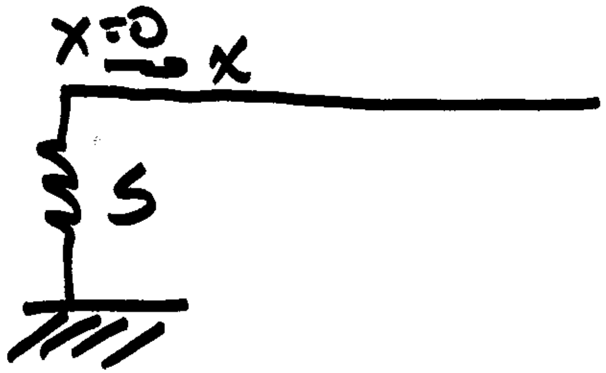


$$\sum f_y = ma$$

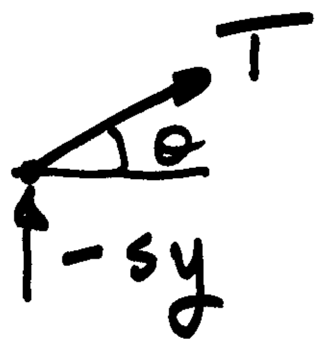
$$\underbrace{T\sin\theta}_{\partial y/\partial x}\Big|_{x=0} = m\,\frac{\partial^2 y}{\partial t^2}\Big|_{x=0}$$

$$\frac{\partial^2 y}{\partial t^2}\Big|_{x=0} = \frac{T}{m}\,\frac{\partial y}{\partial x}\Big|_{x=0}$$

2.3.4 stiffness b.c.



FBD



$\sum f_y = 0$

$$T\sin\theta\big|_{x=0} - sy\big|_{x=0} = 0$$

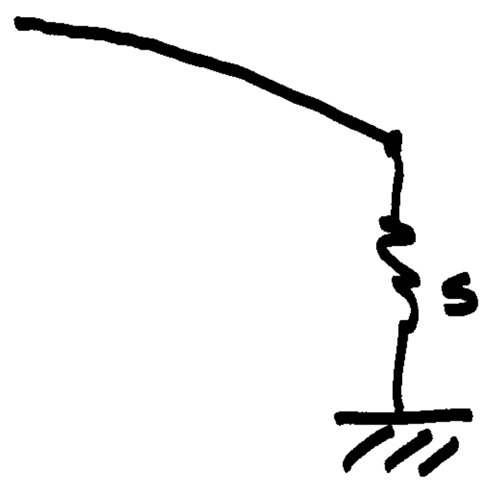
$$T\frac{\partial y}{\partial x}\bigg|_{x=0} - sy\big|_{x=0} = 0 \qquad \frac{\partial y}{\partial x}\bigg|_{x=0} = \frac{s}{T}\,y\big|_{x=0}$$

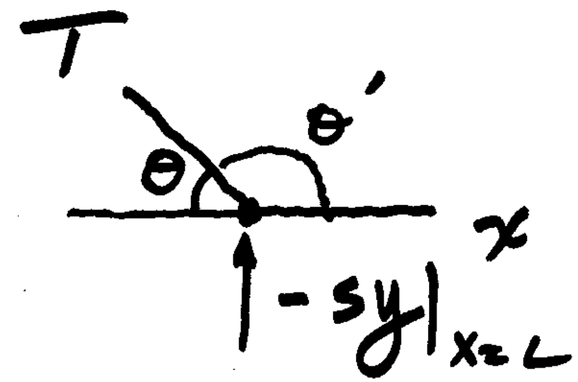
- fixed
- mass
- stiffness
- transverse force

$x = 0$

2.3.5 b.c. applied at the +ve x-end of the string

e.g., stiffness





$$T\sin\theta'\big|_{x=L} - sy\big|_{x=L} = 0$$

$$\sin\theta' = -\partial y/\partial x$$

$$-T\frac{\partial y}{\partial x}\bigg|_{x=L} = sy\big|_{x=L}$$

$$\boxed{\frac{\partial y}{\partial x}\bigg|_{x=L} = -\frac{s}{T}y\bigg|_{x=L}}$$

sign of The b.c. depends on position

$$y(x,t) = Ae^{j(\omega t - kx)} + \cancel{Be^{j(\omega t + kx)}} \rightarrow 0$$

nothing returns from infinity

$$y(x,t) = Ae^{j(\omega t - kx)}$$

$$\frac{\partial y}{\partial x} = -jkAe^{j(\omega t - kx)}$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = -jkA\cancel{e^{j\omega t}} = -\frac{F}{T}\cancel{e^{j\omega t}}$$

$$A = \frac{F}{jkT}$$

$$\boxed{y(x,t) = \frac{F}{jkT} e^{j(\omega t - kx)}}$$

physical solution

$Re\{y(x,t)\}$