Sept 11 Due date for <u>HW 1</u>

Vibration of strings

Extended System

$$y(x,t) = y_1(\underbrace{ct - x}) + y_2(\underbrace{ct + x})$$

waves propagate without changing shape at a speed c

$$y(x,t) = \underbrace{Y(x)}_{}e^{j\omega t}$$

$$\frac{d^2Y}{dx^2} + k^2 Y = 0$$

$$k = \frac{\omega}{c}$$

$$Y(x) = Ae^{\pm jkx}$$

$$y(x,t) = A_1 e^{+jkx} e^{j\omega t} + A_2 e^{-ikx} e^{j\omega t}$$

$$= A_1 e^{i(kx+\omega t)} + A_2 e^{+j(-kx+\omega t)} \qquad k = \frac{\omega}{c}$$

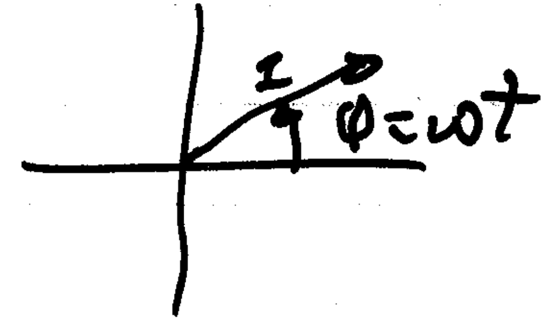
$$= \underbrace{A_1 e^{jk(x+ct)}}_{y_2(ct+x)} + \underbrace{A_2 e^{+jk(-x+ct)}}_{y_1(ct-x)}$$

– ve going wave

+ ve going wave

What is k ?

Recall $e^{j\omega t} \rightarrow e^{j\phi}$ $\qquad \underline{\phi = \omega t}$



$$\frac{d\phi}{dt} = \omega$$

ω: rate of change of phase with time

$\omega = \frac{2\pi}{T}$ (rad/s) $\qquad f = \frac{\omega}{2\pi}$ (Hz)

↑

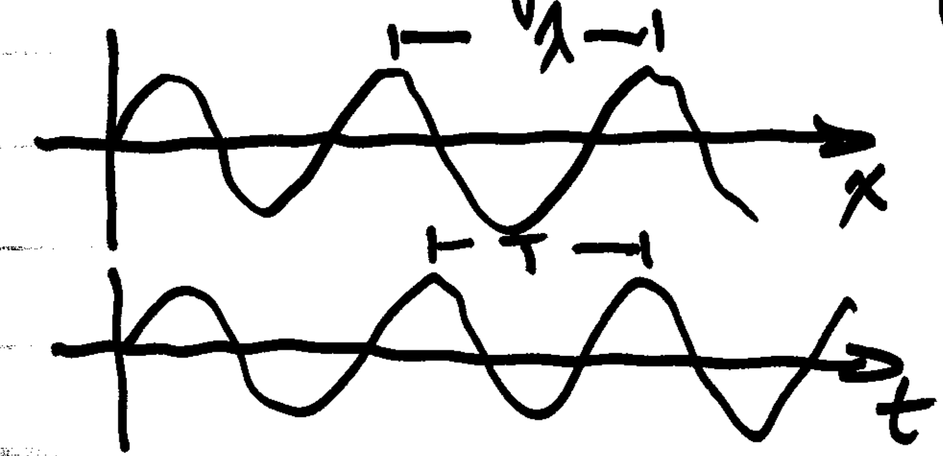
Temporal frequency

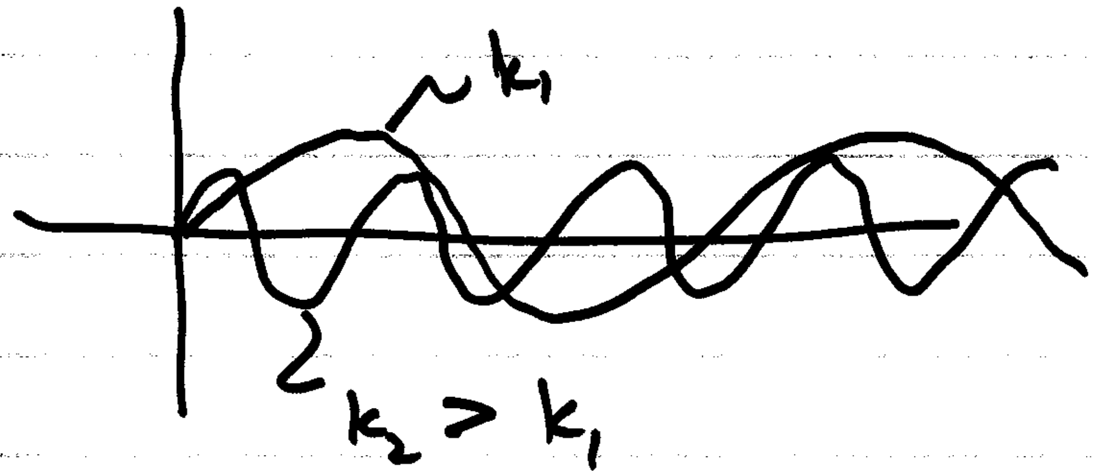
$$e^{jkx} \rightarrow e^{j\phi} \qquad \phi = kx$$

$$\frac{d\phi}{dx} = k$$

k: rate of change of phase with position

$k \rightarrow$ spatial frequency





$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \qquad \omega = \frac{2\pi}{T} \qquad \left(f = \frac{1}{T}\right)$$

$$c = f\lambda$$

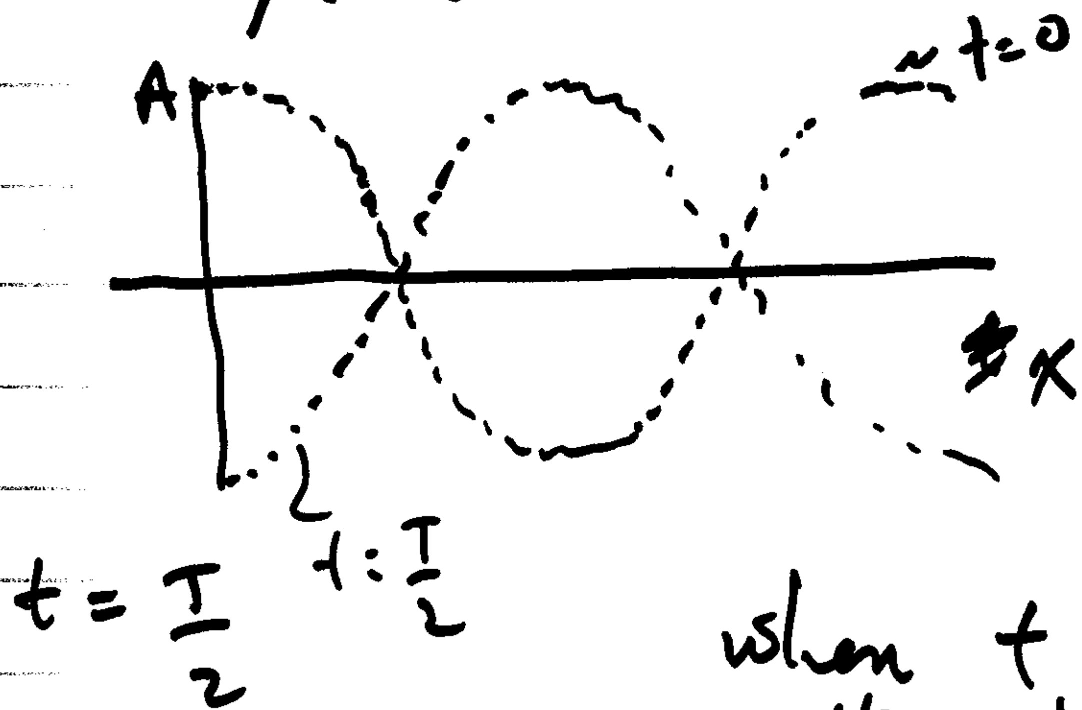
$$\lambda = \frac{c}{f}$$

$$k = \boxed{\frac{2\pi}{\lambda} = \frac{\omega}{c}}$$

exactly equivalent

plot of $y = A e^{j(\omega t - kx)}$ $\qquad \cos(\omega t - kx) - j\sin(\omega t - kx)$

$Re\{y\}$ at $t = 0$ when A = real



when $t = T$ wave pattern has advanced by λ

wave propagation speed $= c = \frac{\lambda}{T}$

$cT = \lambda$

$$\boxed{c = f\lambda}$$

$f = \frac{1}{T}$

ω [rad/s]

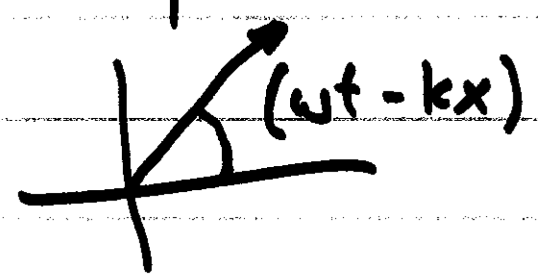
$$\frac{\omega}{c} = \frac{2\pi}{\lambda} = k$$

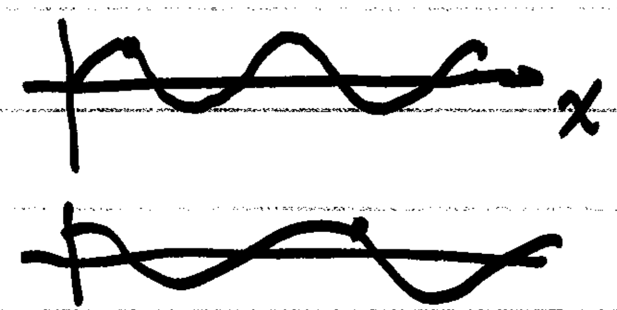
k [rad/m]

	Time	Space
frequency	ω	k
period	T	λ

General Solution

$$y(x,t) = A_1 e^{i(\omega t - kx)} + A_2 e^{i(\omega t + kx)}$$





transverse velocity (harmonic)

$$\frac{\partial y}{\partial t}(x,t) = u(x,t) = j\omega\, y(x,t)$$

transverse acceleration

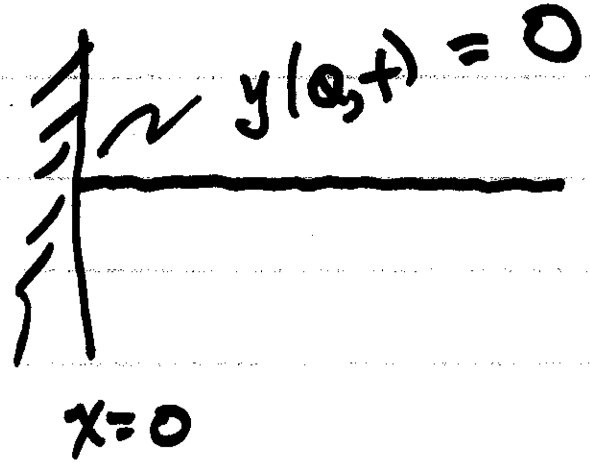
$$\frac{\partial^2 y}{\partial t^2}(x,t) = a(x,t) = \frac{\partial u}{\partial t} = (j\omega)(j\omega)\, y(x,t) = -\omega^2 y(x,t)$$

2.3 Boundary Conditions

General sol'n: $y(x,t) = y_1(w_1) + y_2(w_2)$

$w_1 = ct - x \quad w_2 = ct + x$

2.3.1 Fixed



$$y(0,t) = y_1\left(w_1\big|_{x=0}\right) + y_2\left(w_2\big|_{x=0}\right) = 0$$

$$w_1 = ct - x \qquad w_2 = ct + x$$

$$w_1\big|_{x=0} = ct \qquad w_2\big|_{x=0} = ct$$

$$\left. y_2(ct) = -y_1(ct) \right]$$

at $x=0$ y_1 + y_2 are equal and opposite

Cancel to create zero displacement

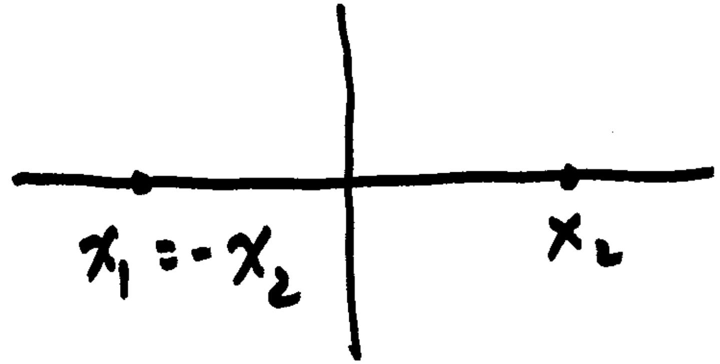
more generally as a result of the b.c.

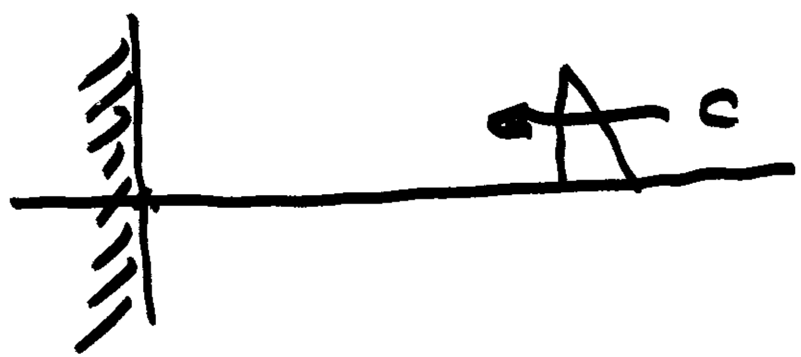
y_1 & y_2 are equal & opposite whenever $w_1 = w_2$

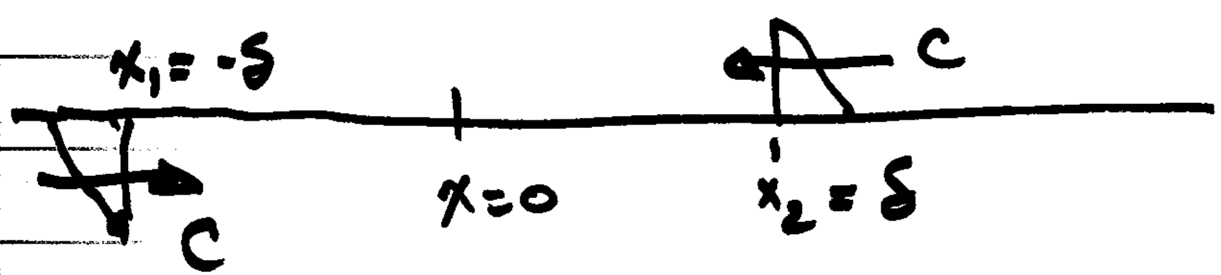
and at a particular $t = t_1$

$$\begin{array}{ccc} w_1 & & w_2 \\ ct_1 - x_1 & = & ct_1 + x_2 \end{array}$$

$$x_1 = -x_2$$

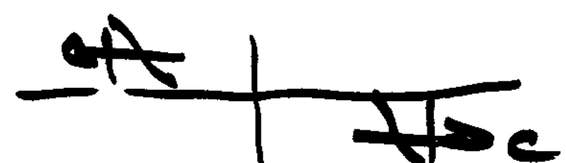




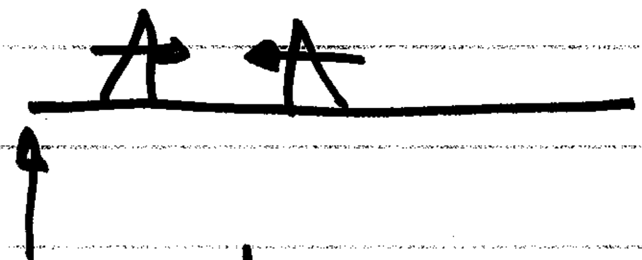


hard boundary causes a reflection that is upside down & backwards

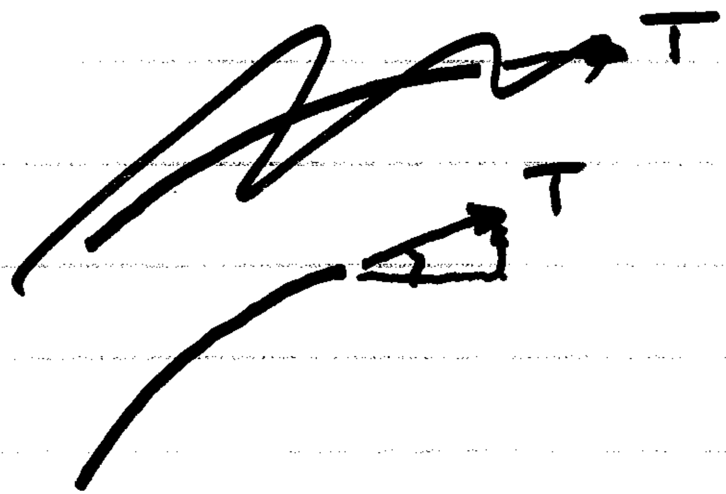
→ real world



free end



zero transverse force at the free end of a string

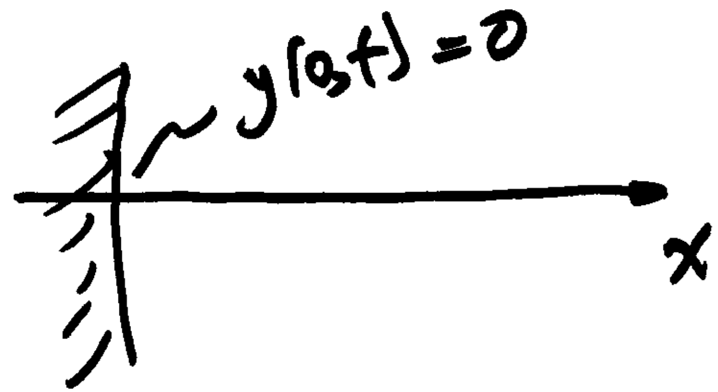


$$\frac{\partial y}{\partial x} = 0 \text{ at a free end}$$

frequency domain

$$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)} \qquad k = \frac{\omega}{c}$$

two b.c.'s to solve for A + B



$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$y(0,t) = Ae^{j\omega t} + Be^{j\omega t} = 0$$

$$B = -A$$

$$y(x,t) = Ae^{j\omega t}\boxed{\left(e^{-jkx} - e^{+jkx}\right)} \qquad -2j\sin kx$$

$$y(x,t) = -2jAe^{j\omega t}\sin kx$$