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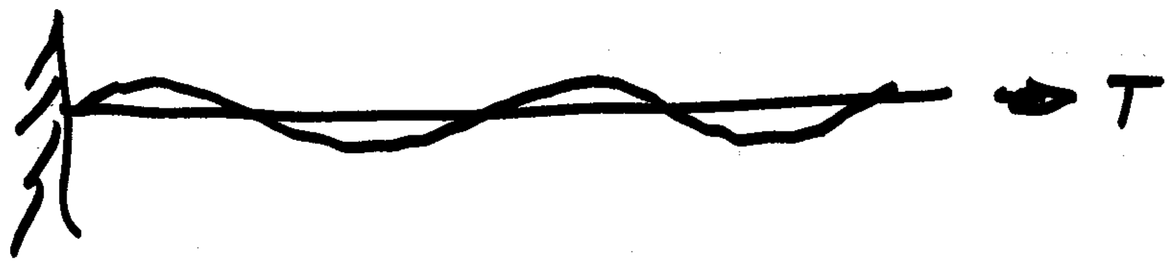
Harman RSVP by <u>Tuesday</u>

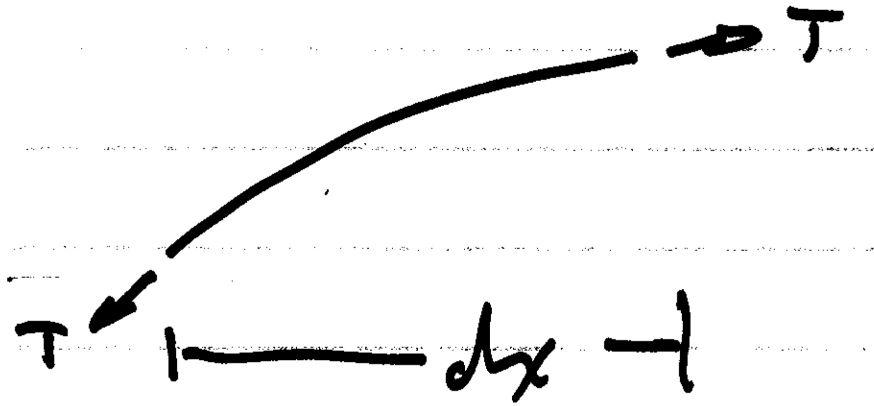
$$\tilde{z}_m = \frac{F}{\tilde{u}} \qquad \underline{\mathrm{Im}\{\tilde{z}_m\} = 0}$$

2.



Extended system

Two equations - restoring force eqn
- dynamic EOM

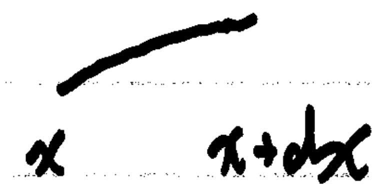


$$df_y = T \frac{\partial^2 y}{\partial x^2}$$

2.1.2 Equation of Motion

ρ_L = mass / unit length

$$f = ma$$



$$m = \rho_L \, dx \qquad a = \frac{\partial^2 y}{\partial t^2} \qquad f = df_y$$

linear density [kg/m]

$$\boxed{df_y = \rho_L \, dx \, \frac{\partial^2 y}{\partial t^2}} \qquad (2)$$

$$c = \sqrt{\frac{T}{\rho_L}}$$

2.1.3 Wave Equation

$$(1) = (2)$$

$$T \frac{\partial^2 y}{\partial x^2} \cancel{dx} = \rho_L \cancel{dx} \frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} - \frac{\rho_L}{T} \frac{\partial^2 y}{\partial t^2} = 0}$$

2nd Order PDE

$$\boxed{\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0} \quad (3)$$

Wave Equation governing small amplitude transverse displacement of a tensioned string

2.2 Solutions of the Wave Equation

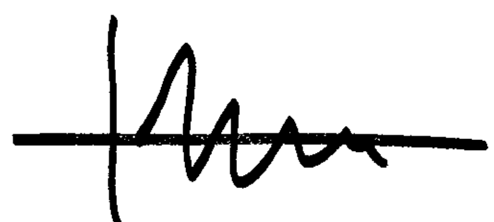
two independent variables $y(x,t)$

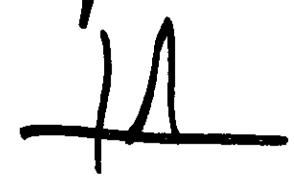
2.2.1 General solution

$$y(x,t) = y_1(\underline{ct-x}) + y_2(ct+x) = y_1(w_1) + y_2(w_2)$$

y_1 & y_2 are any functions of a single variable.

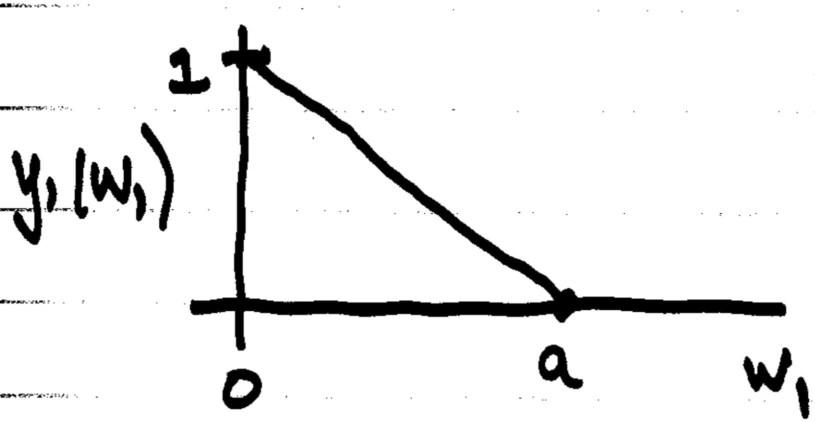
function can be a shape





$$y_1(\underbrace{ct - x}_{w_1}) \qquad y_2(\underbrace{ct + x}_{w_2})$$

Can prove by direct substitution that these are solutions



$y_1 = 1$ at $w_1 = 0$

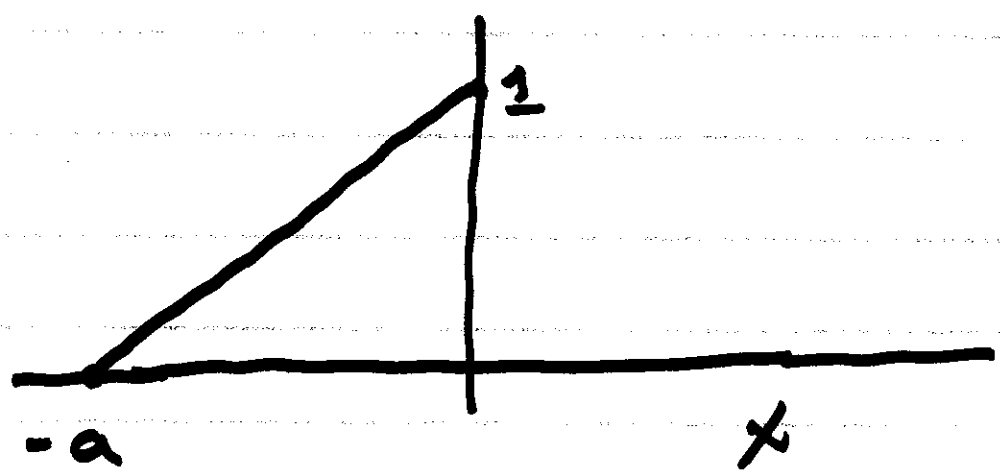
$y_1 = 0$ at $w_1 = a$

plot the function as a function of x at $t=0$

$w_1 = ct - x$

at $t=0$ $\quad w_1 = -x \qquad x = -w_1 \qquad w_1 = 0 \rightarrow x = 0$

$w_1 = a \rightarrow x = -a$



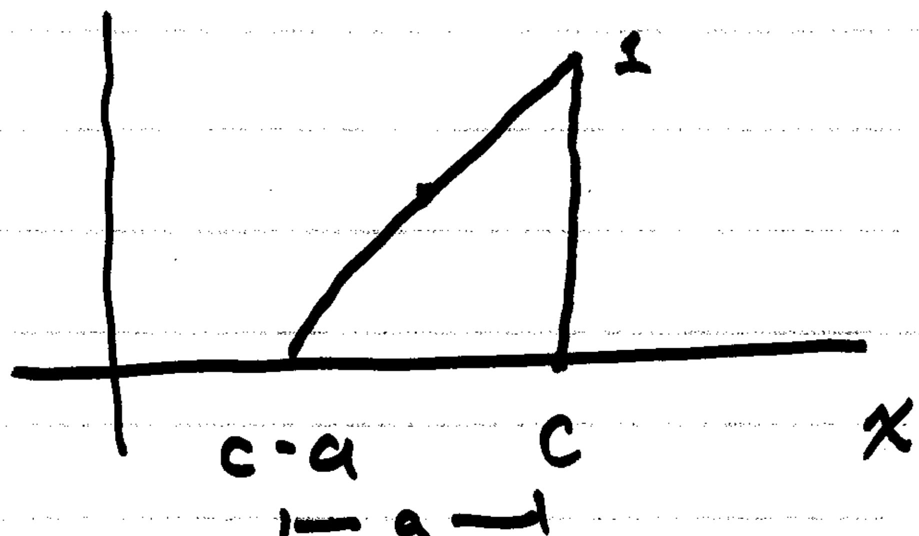
plot as a function of x at $t = 1\,s$

$w_1 = ct - x$ $\qquad t = 1 \qquad$ $w_1 = c - x$

$x = c - w_1$

$w_1 = 0 \rightarrow x = c$

$w_1 = a \rightarrow x = c - a$



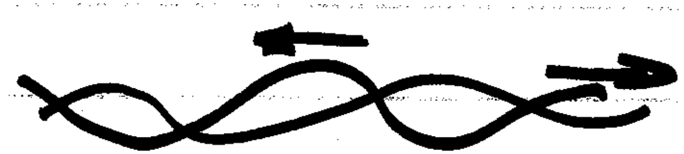
$$c = \sqrt{\frac{T}{\rho}}$$

$$\frac{\text{distance}}{\text{time}} = \frac{c}{1}$$ speed of wave propagation

$y_1(ct - x)$ disturbance that travels in the positive x-direction without changing shape as t increases

$y_2(ct + x)$ disturbance that travels in the negative x-direction. . . .

$y = y_1 + y_2$



what is the transverse velocity of the string?

$$v_t = \frac{\partial y_1}{\partial t} = \frac{\partial y_1(ct-x)}{\partial(ct-x)} \overset{c}{\cancel{\frac{\partial(ct-x)}{\partial t}}}$$

$$= c\,\frac{\partial y_1(w_1)}{\partial w_1} \neq c$$

value of v_t depends on the function

speed of wave propagation

$\neq$ transverse velocity of the string

$$y(x,t) = \underbrace{y_1(ct-x)}_{\text{+ve going wave}} + \underbrace{y_2(ct+x)}_{\text{-ve going wave}}$$

functions <u>propagate</u> without changing shape

non-dispersive system

2.2.2 Harmonic single-frequency solution

Assume a separable form of solution

$$y(x,t) = Y(x)e^{j\omega t}$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

$$\frac{d^2Y}{dx^2}\cancel{e^{j\omega t}} + \left(\frac{\omega^2}{c^2}\right)Y\cancel{e^{j\omega t}} = 0$$

define as k^2 $\quad \frac{\omega}{c} = k$ wave number

$$\boxed{\frac{d^2Y(x)}{dx^2} + k^2 Y(x) = 0} \qquad (4)$$

Scalar Helmholtz Eqn

SDOF $\quad \frac{d^2x}{dt^2} + \omega_0^2 x = 0 \qquad x = Ae^{\pm j\omega_0 t}$

$Y(x) = Ae^{\pm jkx}$ two solutions

solution for spatial dependence
of single frequency transverse vibration
of a string