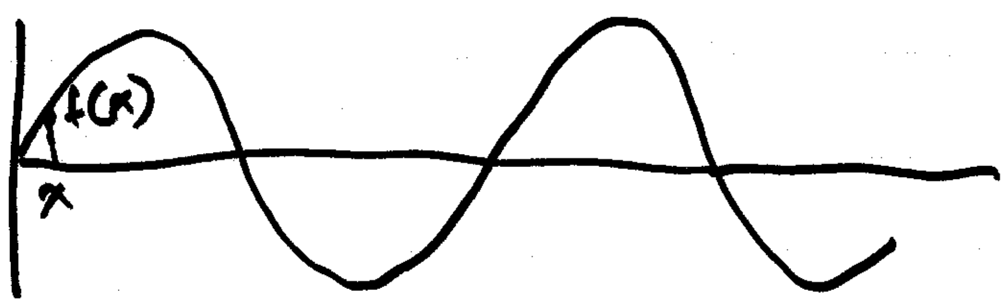
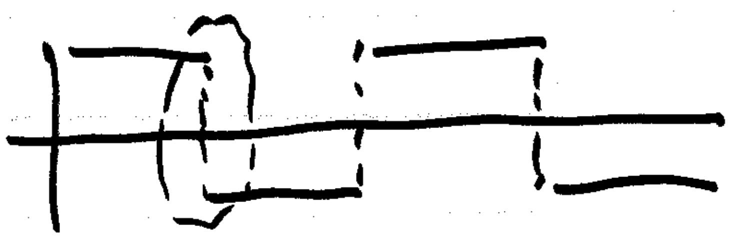
Webpage: engineering.purdue.edu/ME513

Single-valued function

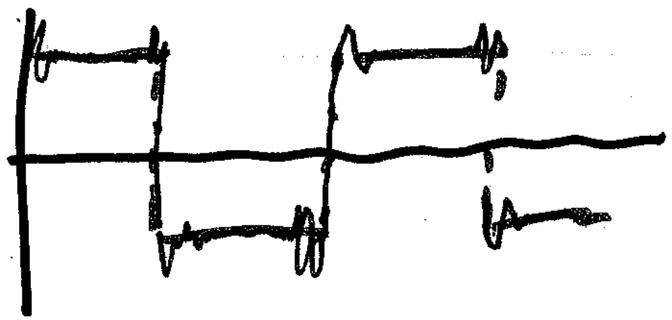
One value of function for each value of independent variable





Gibb's Phenomenon

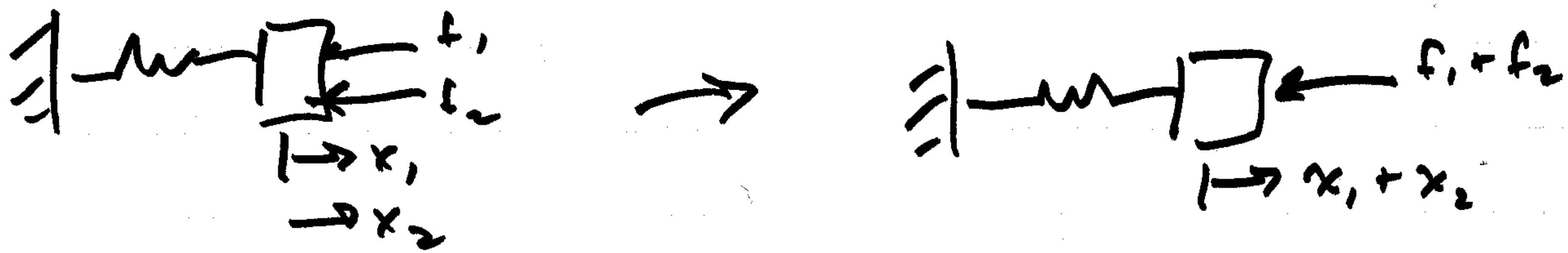
effect of discontinuity on Fourier series fit



Superposition

"Linear Acoustics"
- small oscillations

- Add functions and get results
  - sum of individual functions



- allows for frequency-dependent analysis

Fourier Analysis

Decompose periodic functions into sinusoidal components

$$f(t) = \tfrac{1}{2}A_0 + A_1\cos w_1 t + A_2 \cos w_2 t + \dots$$
$$+ B_1 \sin w_1 t + B_2 \sin w_2 t + \dots$$

$$w_1 = \frac{2\pi}{T} \qquad w_2 = 2w_1$$

$$A_n = \frac{2}{T}\int_0^T f(t)\cos w_n t \, dt \qquad w_n = \frac{2\pi}{T} n$$
$$B_n = \quad '' \quad '' \quad \sin \quad '' \quad ''$$

Summary

- physical model

represent in mathematical form

- To oscillate
  - mass
  - stiffness

Approach to problem solving

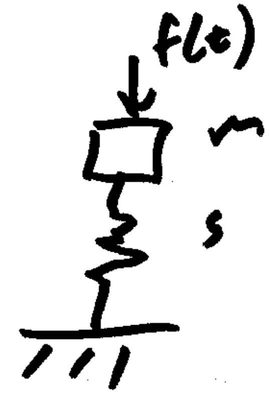
1. Governing equations
   - force
   - motion $(f = ma)$

2. Combine $\rightarrow$ 2nd ODE

3. Identify possible solutions in a convenient form

$$x = A e^{j\omega t}$$

4 Find the solutions that satisfy boundary conditions



Free response
- respond at natural frequency

$$w_0 = \sqrt{\frac{s}{m}}$$

Forced response
$f(t) = Fe^{jwt}$ - responds at driving frequency

Resonance

- system is driven at a natural frequency

$$\mathrm{Im}\{\tilde{Z}_m\} = 0$$

- mechanical impedance

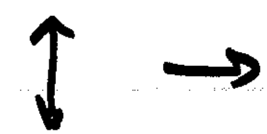
$$\tilde{Z}_m = \frac{\hat{f}_e}{\tilde{u}}$$

$$\tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

- mass-like
- stiffness-like
- resistive

2. Vibrating String

- vibration of extended systems
- derive a wave equation



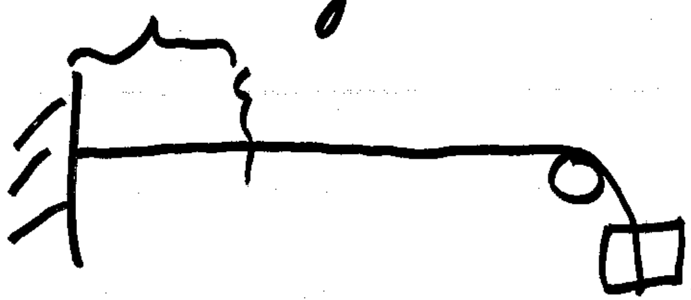
- transverse wave propagation
  - normal to direction of propagation

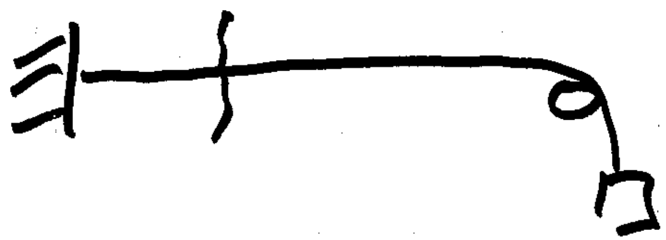
- phase speed
  + particle velocity

- wave number

- application of various BC's
- characteristic impedance
- standing waves & propagating waves

2.1 Derivation of a wave equation governing transverse vibration of tensioned string



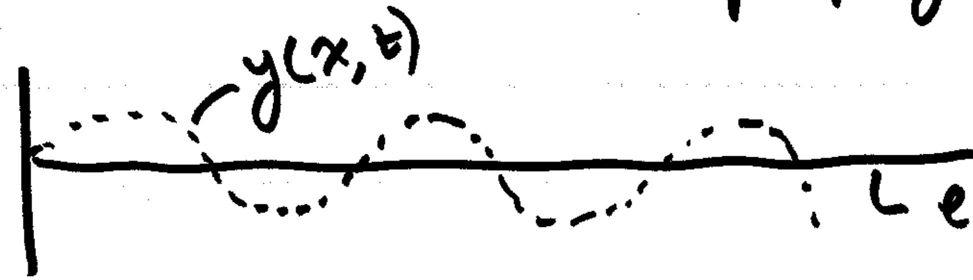


P, T

- restoring force
- dynamic equation

Transverse vibration

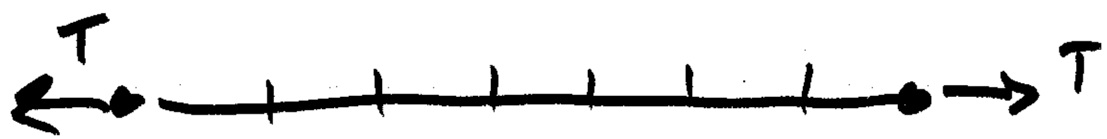
- displacement $\perp$ to the equilibrium axis + the direction of propagation



$y(x,t)$ - instantaneous transverse displacement

Assumptions:

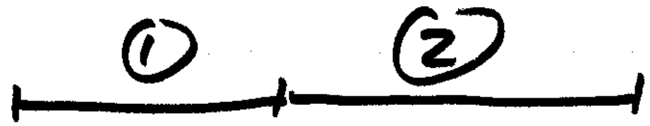
1. Uniform static tension



2. Small amplitude response
   - linear approximation

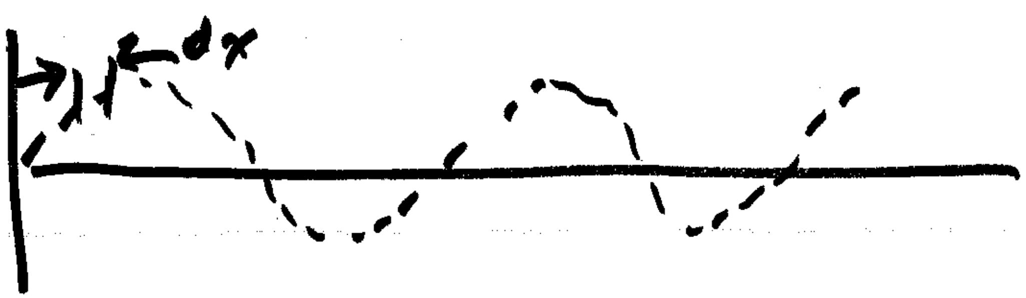
3. No losses in any segment

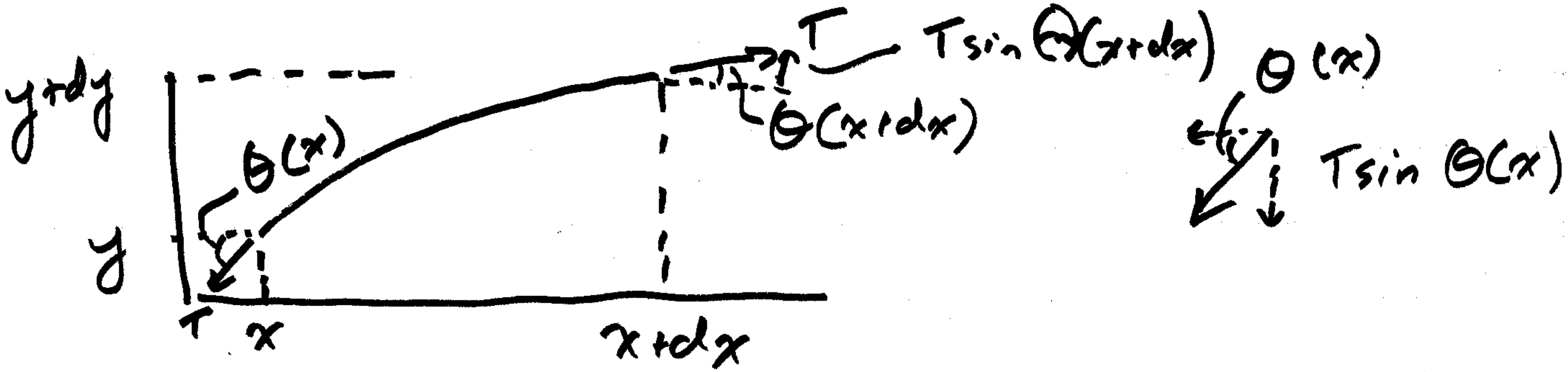
4. Uniform mass/length



5. Gravity force acting on the string is negligible

2.1.1 Restoring force equation





Net vertical force (in the y-direction)

$$df_y = T\sin\theta(x+dx) - T\sin\theta(x)$$

Linear sin(x) via Taylor's series expansion

$$f(x+dx) = f(x) + \left(\frac{df}{\partial x}\right) dx + \cancel{\frac{1}{2}\left(\frac{d^2f}{\partial x^2}\right)(dx)^2} + \ldots$$

small amplitude → linear approximation
so series can be truncated

$$\sin\theta(x+dx) \simeq \sin\theta(x) + \frac{d[\sin\theta(x)]}{\partial x}\, dx$$

$$df_y = \cancel{T\sin\theta(x)} + T\,\frac{d[\sin\theta(x)]}{\partial x}\, dx$$

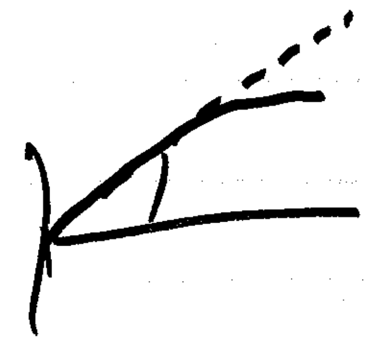
$$- \cancel{T\sin\theta(x)}$$

$$d f_y = T \frac{d[\sin\theta(x)]}{dx} dx$$

small response $\rightarrow$ θ is alway small

when θ is small

$$\sin\theta \simeq \tan\theta \simeq \frac{dy(x)}{\partial x} \quad \text{slope}$$



$$d f_y \simeq T \frac{d\left[\frac{dy}{\partial x}\right]}{\cancel{\partial x}\, dx} dx$$

$$\boxed{d f_y = T \underbrace{\frac{\partial^2 y}{\partial x^2}}_{\text{curvature of the string}} dx} \quad (1)$$

$y(x,t)$

Curvature