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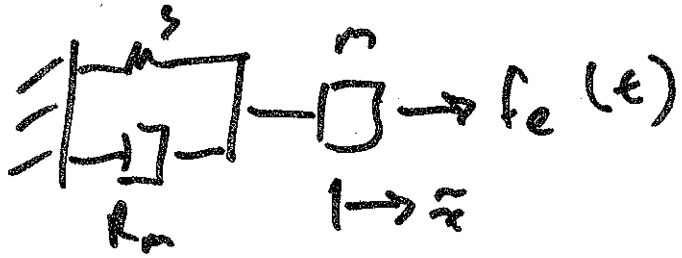
1.3 Forced Oscillations



$f_e(t) = Fe^{j\omega t}$ $\tilde{x} = \tilde{A}e^{j\omega t}$

$$\tilde{x} = \frac{Fe^{j\omega t}}{j\omega\left(R_m + j\left(\omega m - \frac{s}{\omega}\right)\right)}$$

Mechanical Impedance

Property of the system

$$\tilde{Z}_m = \frac{\tilde{f}_e}{\tilde{u}} = \frac{\tilde{f}_e}{j\omega \tilde{x}} = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

$$\tilde{u} = \frac{\tilde{f}_e}{\tilde{Z}_m}$$

Recall: $\tilde{x} = x_{trans} + \underbrace{x_{ss}}_{\text{Impedance applies to SS solution}}$

1.3.4 Mechanical Resonance

Defn: Occurs when the imaginary component of mechanical impedance = 0

$$\mathrm{Im}\{\tilde{Z}_m\} = 0$$

$$\tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right) \qquad \tilde{X}_m = \mathrm{Im}\{\tilde{Z}_m\} = \omega m - \frac{s}{\omega}$$

$$X_m = 0 \qquad \omega = \omega_0 = \sqrt{\frac{s}{m}}$$

undamped natural frequency

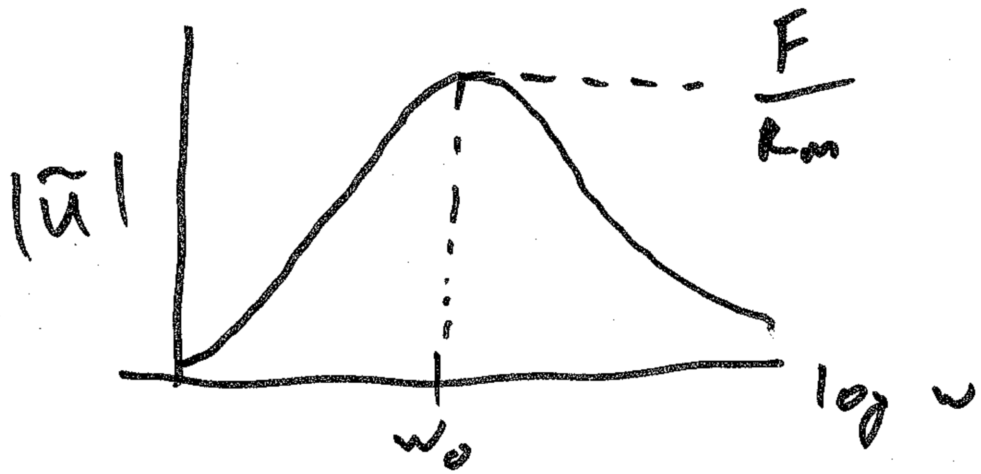
resonance occurs when the system is driven at its undamped natural frequency

$$\tilde{u} = \frac{F e^{j\omega t}}{R_m + j\underbrace{\left(\omega m - \frac{s}{\omega}\right)}_{\to 0}}$$

at resonance $\omega = \omega_0$

$\hat{Z}_m$ = purely real

- velocity reaches a maximum



3 frequency ranges of interest

$w < w_0$

$w \simeq w_0$

$w > w_0$

b

(i) $\omega < \omega_0$ $\qquad$ $\tilde{Z}_m = R_m + j(\omega m - \frac{s}{\omega})$

Stiffness controlled region

$$\tilde{Z}_m \simeq -j\frac{s}{\omega}$$

$$\tilde{x} \simeq \frac{F}{s} e^{j\omega t}$$

$|\tilde{x}|$ is independent of frequency

(ii) near resonance $\omega \approx \omega_0$

$\tilde{Z}_m \approx R_m$ $\qquad$ $\tilde{Z}_m = R_m + j\cancel{(\omega m - \frac{s}{\omega})}$

$$\tilde{x} = \frac{F}{j\omega R_m} e^{j\omega t} \qquad j\omega\tilde{x} = \frac{F}{R_m} e^{j\omega t} = \tilde{u}$$

$|\tilde{u}| \simeq \frac{F}{R_m}$ independent of frequency

"damping controlled region"

(iii) $\omega > \omega_0$ mass controlled region

$$\tilde{Z} \simeq j\omega m$$

$$\tilde{a} \simeq \frac{F}{m} e^{j\omega t}$$

$|\tilde{q}|$ is independent of frequency

Transducer are designed to operate in one of these ranges

$\tilde{Z}_m \simeq -j\frac{s}{\omega}$ stiffness-controlled region

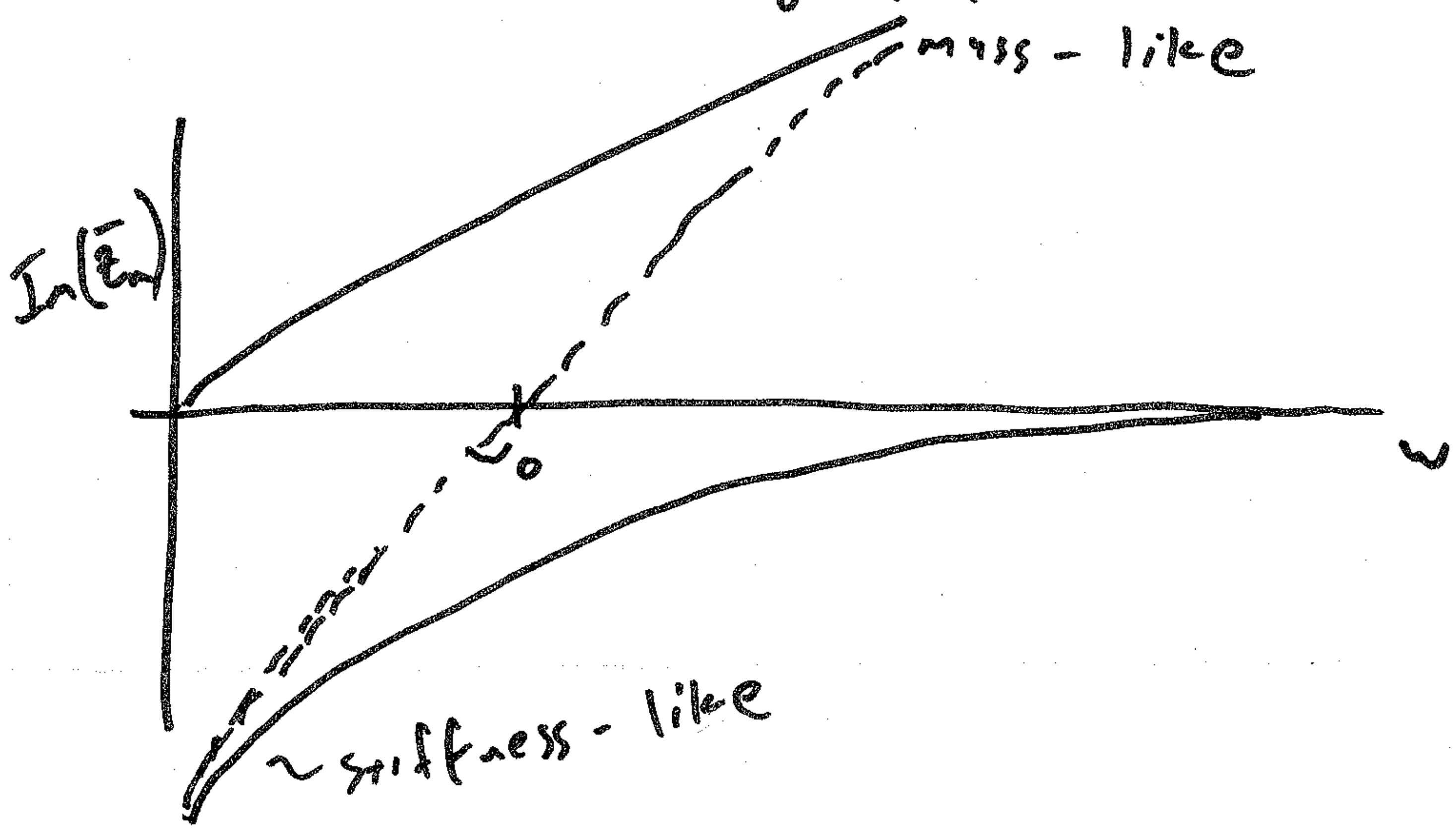
" -like impedance

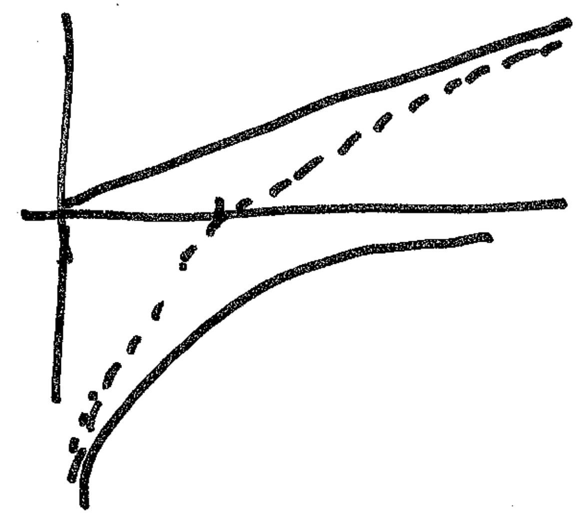
$\tilde{Z}_m$ is:

- imaginary
- negative
- inversely proportional to frequency

$\tilde{Z}_m \approx j\omega m$ mass-like impedance

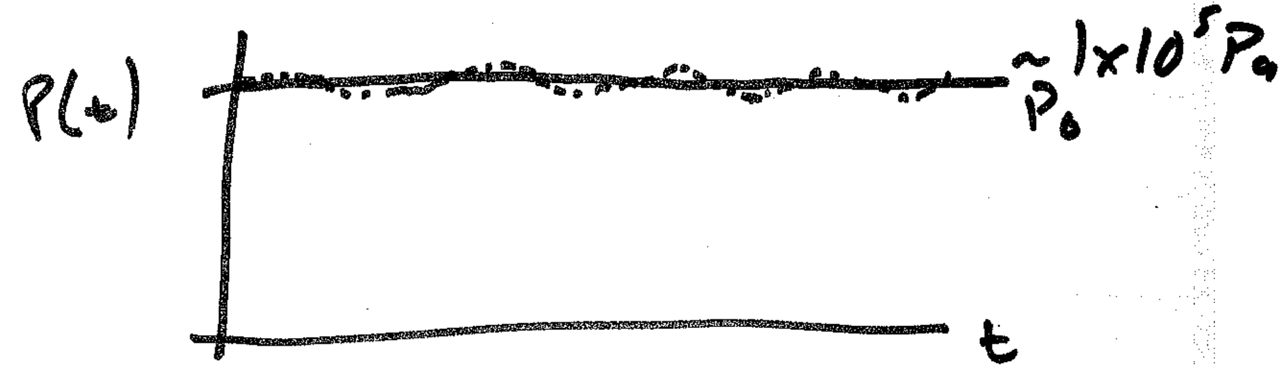
- imaginary
- positive
- linearly proportional to frequency





1.4 Superposition

"linear" acoustics
"linear" vibration



loud sound
1 Pa

small amplitude fluctuations in sound pressure or vibration

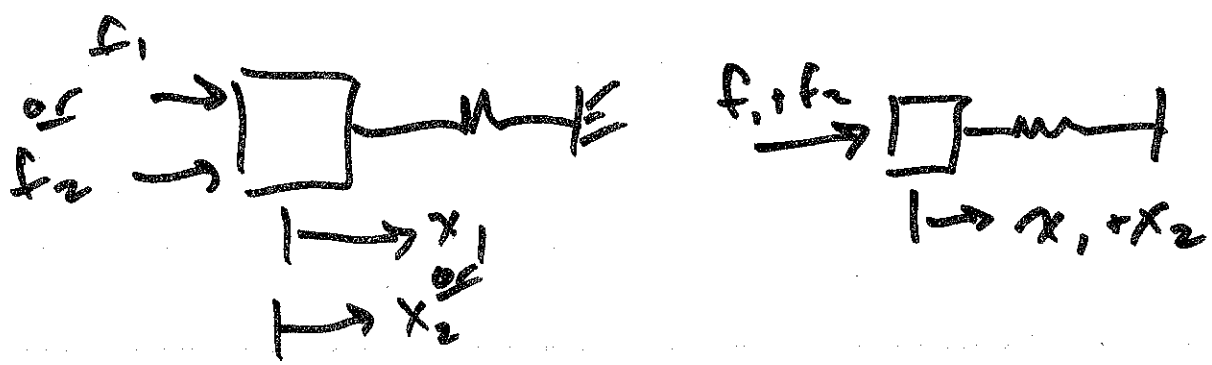
linear, small amplitude response

- output of a linear system is linearly proportional to the input
  double F, double $\bar{x}$

- linear system respond at the forcing frequency

$$F e^{j\omega t} \rightarrow \tilde{A} e^{j\omega t}$$

- response to two or more inputs is the linear sum of the individual ~~inputs~~ responses

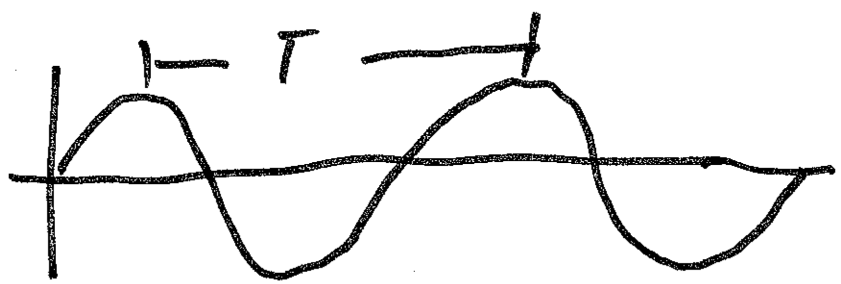


- ~~conver~~
- convenient to break input forces into frequency components (frequency analysis)
- find the response to each frequency component
- add up individual results
  → total response

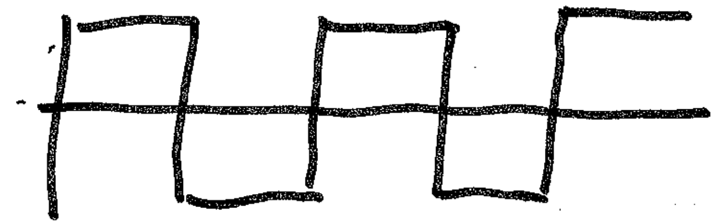
Frequency Domain Analysis

1.5 Fourier Analysis

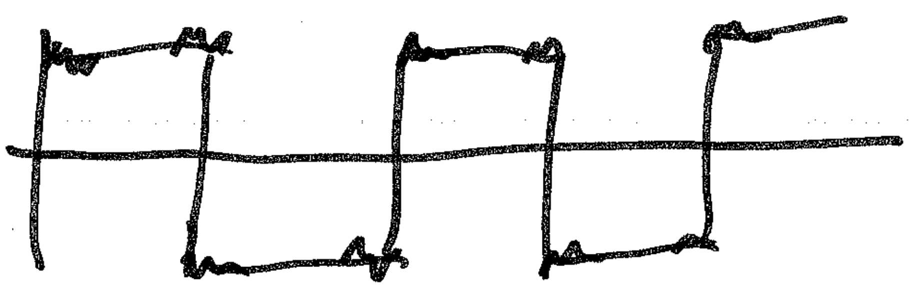
periodic signal - repeats itself in T



sine wave



square wave



— any single-valued periodic signal
can be represented as a sum of
sinusoidals periodic in T

$f(t)$ periodic

$$f(t) = \frac{1}{2}A_0 + A_1 \cos w_1 t + A_2 \cos w_2 t + \cdots + B_1 \sin w_1 t + B_2 \sin w_2 t + \ldots$$

$$w_1 = \frac{2\pi}{T}, \quad w_2 = 2w_1$$

periodic function is represented
by contributions at discrete,
harmonically related frequencies

$w_1 = \frac{2\pi}{T}$ = fundamental = first harmonic

$w_2 = 2 w_1$ 2nd harmonic

$w_3 = 3 w_1$ 3rd harmonic

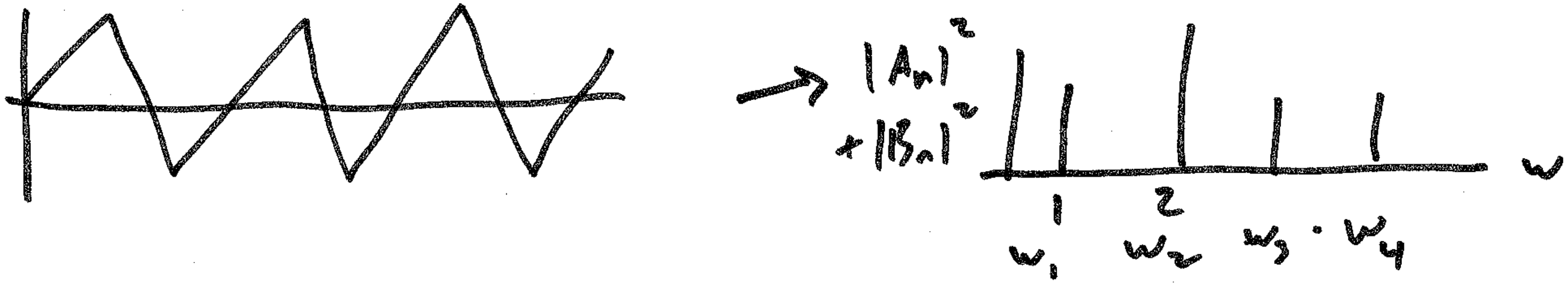
$\vdots$

$w_n = \frac{n 2\pi}{T}$

$$A_n = \frac{2}{T} \int_0^T f(t) \cos(w_n t)\, dt$$

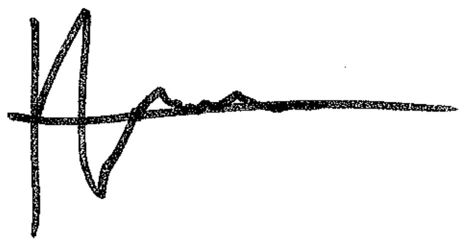
$n = 1, 2, 3, \dots$

$$B_n = \frac{2}{T} \int_0^T f(t) \sin(w_n t)\, dt$$



non-periodic signals

transient



random noise (white)