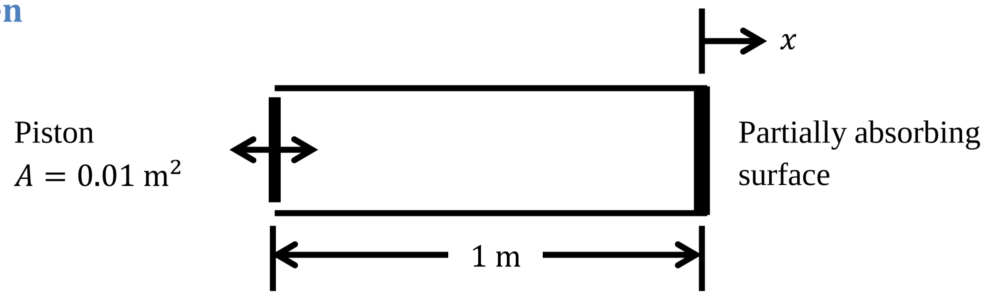
ME 513
Fall 2017
Daniel J. Carr

Homework 3 Solutions

80 points

Problem 1 (20 points)

Given



$\tilde{p}(x) = e^{-jkx} + 0.6e^{jkx}$ (at single frequency ω)

Find

a) p_{rms}; plot as function of x at 1000 Hz; show that minima in standing-wave pattern are $\lambda/2$ apart
b) $\tilde{u}(x)$ (using linearized Euler). Plot u_{rms} as a function of position at 1000 Hz (on pressure plot). Comment on results. Find velocity of piston, i.e. $\tilde{u}(x = -0.1)$.
c) Show that intensity $I(x) = \frac{1}{2}\text{Re}\{\tilde{p}(x)\tilde{u}^*(x)\}$ is not position-dependent; determine sound power from piston
d) $\tilde{z}$, both at piston and at absorbing surface

Assume air at 20°C: $c = 343\ \text{m/s}$, $\rho_0 c = 415\ \text{Pa}\cdot\text{s/m}$ [KFCS p. 528]

Solution

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi(1000\ \text{Hz})}{343\ \text{m/s}} = 18.3\ \text{m}^{-1}$$

$$\lambda = \frac{c}{f} = \frac{343\ \text{m/s}}{1000\ \text{s}^{-1}} = 0.343\ \text{m}$$

a) $$p_{rms}^2 = \frac{\tilde{p}(x)\tilde{p}^*(x)}{2} = \frac{|\tilde{p}(x)|^2}{2} = \frac{1}{2}\left(e^{-jkx} + 0.6e^{jkx}\right)\left(e^{jkx} + 0.6e^{-jkx}\right)$$
$$= \frac{1}{2}\left[1 + 0.6\left(e^{2jkx} + e^{-2jkx}\right) + 0.36\right] = \frac{1}{2}\left[1.36 + 1.2\cos(2kx)\right]$$
$$= 0.68 + 0.6\cos(2kx)$$
$$\therefore \boxed{p_{rms} = \sqrt{0.68 + 0.6\cos(2kx)}}$$ (see graph below)

b) $$\tilde{u}(x) = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial x} = -\frac{1}{j\omega\rho_0}\frac{\partial}{\partial x}\left(e^{-jkx} + 0.6e^{jkx}\right) = -\frac{1}{j\omega\rho_0}\left(-jke^{-jkx} + 0.6jke^{jkx}\right)$$
$$= -\frac{1}{j\omega\rho_0}(-jk)\left(e^{-jkx} - 0.6e^{jkx}\right) = \frac{1}{\omega\rho_0}\left(\frac{\omega}{c}\right)\left(e^{-jkx} - 0.6e^{jkx}\right)$$

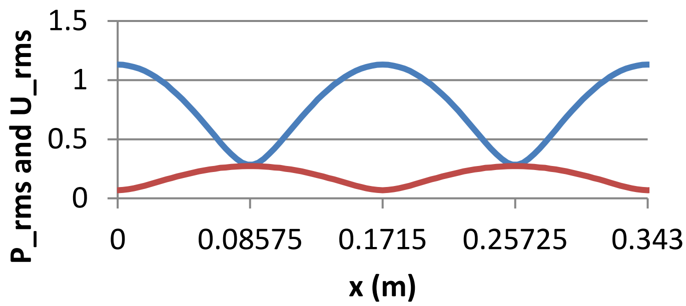
$$\boxed{\tilde{u}(x) = \frac{1}{\rho_0 c}\left(e^{-jkx} - 0.6e^{jkx}\right)}$$

$$u_{rms}^2 = \frac{\tilde{u}(x)\tilde{u}^*(x)}{2} = \frac{|\tilde{u}(x)|^2}{2} = \frac{1}{2}\left[\frac{1}{\rho_0 c}\left(e^{-jkx} - 0.6e^{jkx}\right)\right]\left[\frac{1}{\rho_0 c}\left(e^{jkx} - 0.6e^{-jkx}\right)\right]$$

$$= \frac{1}{2(\rho_0 c)^2}\left(e^{-jkx} - 0.6e^{jkx}\right)\left(e^{jkx} - 0.6e^{-jkx}\right)$$

$$= \frac{1}{2(\rho_0 c)^2}\left[1 - 0.6\left(e^{2jkx} + e^{-2jkx}\right) + 0.36\right] = \frac{1}{2(\rho_0 c)^2}\left[1.36 - 1.2\cos(2kx)\right]$$

$$\therefore \quad \boxed{u_{rms} = \frac{1}{\rho_0 c}\sqrt{0.68 - 0.6\cos(2kx)}}$$



p_{rms} is at maximum when u_{rms} is at minimum, and vice versa. u_{rms} is plotted at 100 scale.

Minima in standing-wave pressure pattern are 0.1715 m apart, which is $\lambda/2$.

$$u_p = \tilde{u}(x = -1) = \frac{1}{\rho_0 c}\left(e^{-jk(-1)} - 0.6e^{jk(-1)}\right) = \frac{1}{\rho_0 c}\left(e^{jk} - 0.6e^{-jk}\right)$$

$$\boxed{u_p = \frac{1}{\rho_0 c}\left[0.4\ \cos k + 1.6j\sin k\right]}$$

c) $$I_x(x) = \frac{1}{2}\mathrm{Re}\{\tilde{p}(x)\tilde{u}^*(x)\} = \frac{1}{2}\mathrm{Re}\left\{\left(e^{-jkx} + 0.6e^{jkx}\right)\left[\frac{1}{\rho_0 c}\left(e^{-jkx} - 0.6e^{jkx}\right)\right]^*\right\}$$

$$= \frac{1}{2\rho_0 c}\mathrm{Re}\left\{\left(e^{-jkx} + 0.6e^{jkx}\right)\left(e^{jkx} - 0.6e^{-jkx}\right)\right\}$$

$$= \frac{1}{2\rho_0 c}\mathrm{Re}\left\{1 + 0.6e^{2jkx} - 0.6e^{-2jkx} - 0.36\right\} = \frac{1}{2\rho_0 c}\mathrm{Re}\left\{0.64 + 0.6\left(e^{2jkx} - e^{-2jkx}\right)\right\}$$

$$= \frac{1}{2\rho_0 c}\mathrm{Re}\{0.64 + 1.2j\sin 2kx\} = \frac{0.32}{\rho_0 c} = \frac{0.32}{415\ \mathrm{Pa{\cdot}s/m}} = 7.71 \cdot 10^{-4}\ \frac{\mathrm{W}}{\mathrm{m}^2}$$

$$\boxed{I_x(x) = \frac{0.32}{\rho_0 c} = 7.71 \cdot 10^{-4}\ \frac{\mathrm{W}}{\mathrm{m}^2} \quad \forall\ x}$$

$$W = \int_S I\ ds = I\int_S ds = IA = \frac{0.32}{\rho_0 c}(0.01\ \mathrm{m}^2) = \frac{3.2{\cdot}10^{-3}\ \mathrm{m}^2}{\rho_0 c} = \frac{3.2{\cdot}10^{-3}\ \mathrm{m}^2}{415\ \ \mathrm{Pa{\cdot}s/m}} = 7.71 \cdot 10^{-6}\ \mathrm{W}$$

$$\boxed{W = \frac{3.2{\cdot}10^{-3}}{\rho_0 c} = 7.71 \cdot 10^{-6}\ \mathrm{W}}$$

d) $$\tilde{z}_p = \tilde{z}(-1) = \frac{\tilde{p}(-1)}{u(-1)} = \frac{e^{-jk(-1)} + 0.6e^{jk(-1)}}{\frac{1}{\rho_0 c}\left(e^{-jk(-1)} - 0.6e^{jk(-1)}\right)} = \boxed{\rho_0 c\frac{e^{jk} + 0.6e^{-jk}}{e^{jk} - 0.6e^{-jk}}}$$ (plug in k and $\rho_0 c$)

$$= \rho_0 c\left.\frac{1.6\cos k + 0.4j\sin k}{0.4\cos k + 1.6j\sin k}\right|_{k = 18.3\ \mathrm{m}^{-1}} = \left(415\frac{\mathrm{Pa{\cdot}s}}{\mathrm{m}}\right)(0.825 + 1.351j) = \boxed{342 + 561j\ \frac{\mathrm{Pa{\cdot}s}}{\mathrm{m}}}$$

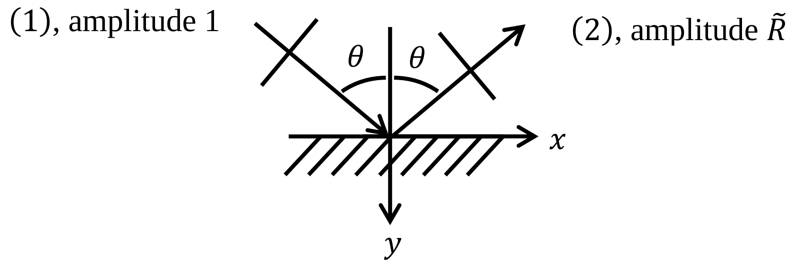
$$\tilde{z}_a = \tilde{z}(0) = \frac{\tilde{p}(0)}{\tilde{u}(0)} = \frac{e^{-jk(0)} + 0.6e^{jk(0)}}{\frac{1}{\rho_0 c}\left(e^{-jk(0)} - 0.6e^{jk(0)}\right)} = \rho_0 c\ \frac{1 + 0.6}{1 - 0.6} = \boxed{4\rho_0 c} = 4\left(415\frac{\mathrm{Pa{\cdot}s}}{\mathrm{m}}\right)$$

$$= \boxed{1660\ \frac{\mathrm{Pa{\cdot}s}}{\mathrm{m}}}$$

Problem 2 (20 points)

Given

Unit amplitude plane wave
Strikes and reflects from surface $y = 0$



Find

a) $p(x, y)$ for $y < 0$ (define quantities as necessary)
b) $\tilde{u}_y$ for $y < 0$ (using linearized Euler equation)
c) Sound power per unit area flowing into the surface at $y = 0$; sound power delivered to surface by incident wave; absorption coefficient $\left(\alpha = \frac{\text{power absorbed}}{\text{power incident}}\right)$ for $\theta = 45°$, $R = 0.5$

Solution

a) Wave 1: travels in $+x$, $+y$ direction, amplitude $A = 1$

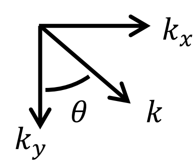
$$\tilde{p}_1(x, y) = Ae^{j(-k_x x - k_y y)} = e^{-jk_x x}e^{-jk_y y}$$

Wave 2: travels in $+x$, $-y$ direction, amplitude = R

$$\tilde{p}_2(x, y) = \tilde{R}e^{j(-k_x x + k_y y)} = \tilde{R}e^{-jk_x x}e^{jk_y y}$$

Whole pressure expression: superimpose

$$\tilde{p}(x, y) = \tilde{p}_1(x, y) + \tilde{p}_2(x, y) = e^{-jk_x x}e^{-jk_y y} + \tilde{R}e^{-jk_x x}e^{jk_y y}$$
$$= e^{-jk_x x}\left(e^{-jk_y y} + \tilde{R}e^{jk_y y}\right)$$



$$k_x = k\sin\theta$$
$$k_y = k\cos\theta$$

$$\boxed{\tilde{p}(x, y) = e^{-jkx\sin\theta}\left(e^{-jky\cos\theta} + \tilde{R}e^{jky\cos\theta}\right)}$$

b) $$\tilde{u}_y = -\frac{1}{j\omega\rho_0}\frac{\partial\tilde{p}}{\partial y} = -\frac{1}{j\omega\rho_0}e^{-jkx\sin\theta}\frac{\partial}{\partial y}\left(e^{-jky\cos\theta} + \tilde{R}e^{jky\cos\theta}\right)$$
$$= -\frac{1}{j\omega\rho_0}e^{-jkx\sin\theta}\left(-jk\cos\theta\, e^{-jky\cos\theta} + jk\tilde{R}\cos\theta\, e^{jky\cos\theta}\right)$$
$$= -\frac{1}{j\omega\rho_0}e^{-jkx\sin\theta}(-jk\cos\theta)\left(e^{-jky\cos\theta} - \tilde{R}e^{jky\cos\theta}\right)$$
$$= -\frac{1}{j\omega\rho_0}e^{-jkx\sin\theta}\left(-\frac{j\omega\cos\theta}{c}\right)\left(e^{-jky\cos\theta} - \tilde{R}e^{jky\cos\theta}\right)$$
$$\boxed{\tilde{u}_y = \frac{\cos\theta}{\rho_0 c}e^{-jkx\sin\theta}\left(e^{-jky\cos\theta} - \tilde{R}e^{jky\cos\theta}\right)}$$

c) $\tilde{u}_i = \frac{\cos\theta}{\rho_0 c} e^{-jkx\sin\theta} e^{-jky\cos\theta}$

$I_i = \frac{1}{2}\text{Re}\{\tilde{p}_1(x,y)\tilde{u}_{1y}^*\} = \frac{1}{2}\text{Re}\left\{\left(e^{-jkx\sin\theta}e^{-jky\cos\theta}\right)\left[\frac{\cos\theta}{\rho_0 c}e^{-jkx\sin\theta}e^{-jky\cos\theta}\right]^*\right\}$

$= \frac{\cos\theta}{2\rho_0 c}\text{Re}\{\left(e^{-jkx\sin\theta}e^{-jky\cos\theta}\right)\left[e^{jkx\sin\theta}e^{jky\cos\theta}\right]\} = \frac{\cos\theta}{2\rho_0 c}$

Note: this expression is not dependent on y, so it applies at $y = 0$ without any further specification

$I_y = \frac{1}{2}\ \text{Re}\{\tilde{p}(x,y)\tilde{u}_y^*\}$

$= \frac{1}{2}\text{Re}\left\{\left[e^{-jkx\sin\theta}\left(e^{-jky\cos\theta} + \tilde{R}e^{jky\cos\theta}\right)\right]\left[\frac{\cos\theta}{\rho_0 c}e^{-jkx\sin\theta}\left(e^{-jky\cos\theta} - \tilde{R}e^{jky\cos\theta}\right)\right]^*\right\}$

$= \frac{\cos\theta}{2\rho_0 c}\text{Re}\{\left[e^{-jkx\sin\theta}\left(e^{-jky\cos\theta} + \tilde{R}e^{jky\cos\theta}\right)\right]\left[e^{jkx\sin\theta}\left(e^{jky\cos\theta} - \tilde{R}^*e^{-jky\cos\theta}\right)\right]\}$

$= \frac{\cos\theta}{2\rho_0 c}\text{Re}\{\left(e^{-jky\cos\theta} + \tilde{R}e^{jky\cos\theta}\right)\left(e^{jky\cos\theta} - \tilde{R}^*e^{-jky\cos\theta}\right)\}$

$= \frac{\cos\theta}{2\rho_0 c}\text{Re}\left\{1 + \tilde{R}e^{2jky\cos\theta} - \tilde{R}^*e^{-2jky\cos\theta} - \left|\tilde{R}\right|^2\right\}$

A note about conjugation: $\tilde{a}^*\tilde{b}^* = \left(\tilde{a}\tilde{b}\right)^*$

$= \frac{\cos\theta}{2\rho_0 c}\text{Re}\left\{1 + \tilde{R}e^{2jky\cos\theta} - \left[\tilde{R}e^{2jky\cos\theta}\right]^* - \left|\tilde{R}\right|^2\right\}$

$= \frac{\cos\theta}{2\rho_0 c}\text{Re}\left\{1 + 2j\ \text{Im}\{\tilde{R}e^{2jky\cos\theta}\} - \left|\tilde{R}\right|^2\right\}$

$= \frac{\cos\theta}{2\rho_0 c}\left(1 - \left|\tilde{R}\right|^2\right)$

Note: this expression is not dependent on y, so it applies at $y = 0$ without any further specification

Sound power per unit area: $W = \int_S I\ ds = I\int_S ds = IA = I(1) = I$

$$\boxed{\begin{gathered} W = I_y = \frac{\cos\theta}{2\rho_0 c}\left(1 - \left|\tilde{R}\right|^2\right) \\ W_i = I_i = \frac{\cos\theta}{2\rho_0 c} \end{gathered}}\ [\text{units: W}]$$

$\alpha = \frac{W}{W_i} = \frac{\frac{\cos\theta}{2\rho_0 c}\left(1 - |\tilde{R}|^2\right)}{\frac{\cos\theta}{2\rho_0 c}} = 1 - \left|\tilde{R}\right|^2 = 1 - |0.5|^2 = \boxed{0.75}$

Problem 3 (15 points)

Given

$p(x,t) = Ae^{-\alpha x}e^{-j\beta x}e^{j\omega t}$

A is complex, α and β are real

Find

a) Sketch spatial variation of sound field; explain significance of α and β; find wavelength

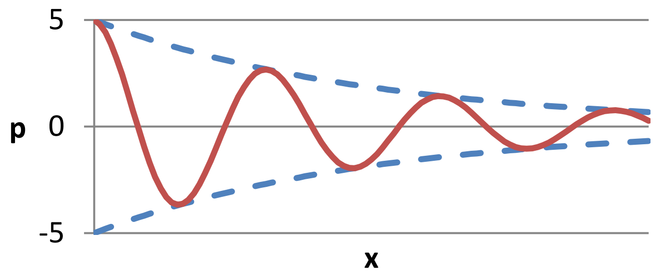
b) u_x

c) I_x; show that it is a function of position; derive expression for rate of decay of sound intensity level in dB/m

Solution

a) α is a decay term (real negative exponential); β is an oscillatory term

$$\beta = \frac{2\pi}{\lambda_x} \quad \rightarrow \quad \boxed{\lambda_x = \frac{2\pi}{\beta}}$$



b) $$u_x = -\frac{1}{j\omega\rho_0}\frac{\partial p_x}{\partial x} = -\frac{1}{j\omega\rho_0}\frac{\partial}{\partial x}\left(Ae^{-\alpha x}e^{-j\beta x}\right) = -\frac{A}{j\omega\rho_0}\frac{\partial}{\partial x}\left(e^{-(\alpha+j\beta)x}\right)$$

$$= \frac{A}{j\omega\rho_0}(\alpha + j\beta)e^{-(\alpha+j\beta)x}$$

$$\boxed{u_x = \frac{A(\beta - j\alpha)}{\omega\rho_0}e^{-(\alpha+j\beta)x}}$$

c) $$I_x = \frac{1}{2}\text{Re}\{\tilde{p}_x\tilde{u}_x^*\} = \frac{1}{2}\text{Re}\left\{Ae^{-(\alpha+j\beta)x}\left[\frac{A(\beta-j\alpha)}{\omega\rho_0}e^{-(\alpha+j\beta)x}\right]^*\right\}$$

$$= \frac{|A|^2}{2\rho_0\omega}\text{Re}\left\{e^{-(\alpha+j\beta)x}\left[(\beta + j\alpha)e^{(j\beta-\alpha)x}\right]\right\} = \frac{|A|^2}{2\rho_0\omega}\text{Re}\{(\beta + j\alpha)e^{-2\alpha x}\} = \frac{|A|^2\beta}{2\rho_0\omega}e^{-2\alpha x}$$

$$\boxed{I_x = \frac{|A|^2\beta}{2\rho_0\omega}e^{-2\alpha x}}$$

$$I_L = 10\log\left(\frac{I_x}{I_{ref}}\right) = 10\log\left(\frac{\frac{|A|^2\beta}{2\rho_0\omega}e^{-2\alpha x}}{I_{ref}}\right) = 10\log\left(\frac{|A|^2\beta}{2\rho_0\omega I_{ref}}\right) - 20\alpha x\log(e)\ \text{dB}$$

$$\boxed{\frac{d}{dx}I_L = -20\alpha\log(e)\ \frac{\text{dB}}{\text{m}}}$$

Problem 4 (15 points)

Given

2D cylindrical sound field (far-field)

$p(r,\theta) = \frac{A}{\sqrt{r}} \sin\theta \, e^{-jkr}$

Find

a) $\vec{u}(r,\theta)$ (use linearized Euler)
b) I_r
c) z_r; find if it limits to plane wave impedance in the far field

Solution

a) $\vec{u}(r,\theta) = -\frac{1}{j\omega\rho_0}\nabla p = -\frac{1}{j\omega\rho_0}\left[\hat{r}\frac{\partial p}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial p}{\partial \theta} + \hat{z}\frac{\partial p}{\partial z}\right]$

$$= -\frac{1}{j\omega\rho_0}\left[\hat{r}\, A\sin\theta\frac{\partial}{\partial r}\left(\frac{e^{-jkr}}{\sqrt{r}}\right) + \hat{\theta}\frac{1}{r}\left(\frac{A}{\sqrt{r}}e^{-jkr}\right)\frac{\partial}{\partial\theta}(\sin\theta)\right]$$

$$= -\frac{A}{j\omega\rho_0}\left[\hat{r}\sin\theta\left(-\frac{jke^{-jkr}}{\sqrt{r}} - \frac{e^{-jkr}}{2r\sqrt{r}}\right) + \hat{\theta}\frac{e^{-jkr}}{r\sqrt{r}}\cos\theta\right]$$

$$= \frac{Ae^{-jkr}}{j\omega\rho_0}\left[\hat{r}\sin\theta\left(\frac{jk}{\sqrt{r}} + \frac{1}{2r\sqrt{r}}\right) - \hat{\theta}\cos\theta\frac{1}{r\sqrt{r}}\right]$$

$$= \frac{Ae^{-jkr}}{j\omega\rho_0}\left(\frac{jk}{\sqrt{r}}\right)\left[\hat{r}\sin\theta\left(1 + \frac{1}{2jkr}\right) - \hat{\theta}\cos\theta\frac{1}{jkr}\right]$$

$$= \frac{Ae^{-jkr}}{j\omega\rho_0}\left[\frac{j(\omega/c)}{\sqrt{r}}\right]\left[\hat{r}\sin\theta\left(1 + \frac{1}{2jkr}\right) - \hat{\theta}\cos\theta\frac{1}{jkr}\right]$$

$$\boxed{\vec{u}(r,\theta) = \frac{1}{\rho_0 c}\frac{A}{\sqrt{r}}e^{-jkr}\left[\left(1 + \frac{1}{2jkr}\right)\sin\theta\,\hat{r} - \frac{1}{jkr}\cos\theta\,\hat{\theta}\right]}$$

In far-field, the $\frac{1}{jkr}$ terms are close to zero. In this case, $\boxed{\vec{u}(r,\theta) \approx \frac{1}{\rho_0 c}\frac{A}{\sqrt{r}}e^{-jkr}\sin\theta\,\hat{r}}$

b) $I_r = \frac{1}{2}\mathrm{Re}\{\tilde{p}(r,\theta)\,\tilde{u}_r^*(r,\theta)\} = \frac{1}{2}\mathrm{Re}\left\{\left[\frac{A}{\sqrt{r}}\sin\theta\, e^{-jkr}\right]\left[\frac{1}{\rho_0 c}\frac{A}{\sqrt{r}}e^{-jkr}\left(1 + \frac{1}{2jkr}\right)\sin\theta\right]^*\right\}$

$$= \frac{|A|^2\sin^2\theta}{2r\rho_0 c}\mathrm{Re}\left\{\left[e^{-jkr}\right]\left[e^{-jkr}\left(1 + \frac{1}{2jkr}\right)\right]^*\right\}$$

A note about conjugation: $\tilde{a}^*\tilde{b}^* = \left(\tilde{a}\tilde{b}\right)^*$

$$= \frac{|A|^2\sin^2\theta}{2r\rho_0 c}\mathrm{Re}\left\{\left[e^{-jkr}\right]\left[e^{jkr}\left(1 - \frac{1}{2jkr}\right)\right]\right\} = \frac{|A|^2\sin^2\theta}{2r\rho_0 c}\mathrm{Re}\left\{1 - \frac{1}{2jkr}\right\} = \frac{|A|^2\sin^2\theta}{2r\rho_0 c}$$

$$\boxed{I_r = \frac{|A|^2\sin^2\theta}{2r\rho_0 c}}$$

c) $z_r = \frac{p}{u_r} = \frac{\frac{A}{\sqrt{r}}\sin\theta\, e^{-jkr}}{\frac{1}{\rho_0 c}\frac{A}{\sqrt{r}}e^{-jkr}\left(1+\frac{1}{2jkr}\right)\sin\theta} = \boxed{\rho_0 c\frac{1}{\left(1+\frac{1}{2jkr}\right)}}$

Far-field simplification: $\frac{1}{jkr}$ terms are close to zero. In this case, $\boxed{z_r \approx \rho_0 c}$

Problem 5.12.3 (10 points)

Given

Plane sound wave in air

$f = 100\ \text{Hz}$

$p_{peak} = 2\ \text{Pa}$

Find

a) I, IL
b) Amplitude of y_{peak}
c) Amplitude of u_{peak}
d) p_{rms}
e) SPL ref 20 μPa

Solution

From KFSC, p. 528:

$$\rho_0 = 1.21\ \frac{\text{kg}}{\text{m}^3} \quad @ \quad 20°\text{C}$$

$$c = 343\ \frac{\text{m}}{\text{s}} \quad @ \quad 20°\text{C}$$

$$\rho_0 c = 415\ \frac{\text{Pa·s}}{\text{m}}$$

a) $I = \frac{p_e^2}{\rho_0 c} = \frac{p_{peak}^2}{2\rho_0 c} = \frac{(2\ \text{Pa})^2}{2(415\ \text{Pa·s/m})} = \boxed{4.82 \cdot 10^{-3}\ \text{W/m}^2}$

From KFSC, table 5.12.1: $I_{ref} = 10^{-12}\ \frac{\text{W}}{\text{m}^2}$

$IL = 10 \log\left(\frac{I}{I_{ref}}\right) = 10 \log\left(\frac{4.82 \cdot 10^{-3}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = \boxed{96.8\ \text{dB}}$

b) For a plane wave, $z = \rho_0 c$

$u_{peak} = \frac{p_{peak}}{z} = \frac{p_{peak}}{\rho_0 c}$

$y = \int u\, dt = \frac{u}{j\omega}$

$y_{peak} = \left|\frac{u_{peak}}{j\omega}\right| = \frac{u_{peak}}{\omega} = \frac{p_{peak}}{\omega \rho_0 c} = \frac{p_{peak}}{2\pi f \rho_0 c} = \frac{2\ \text{Pa}}{2\pi(100\ \text{s}^{-1})(415\ \text{Pa·s/m})} = \boxed{7.67 \cdot 10^{-6}\ \text{m}}$

c) From b): $u_{peak} = \frac{p_{peak}}{\rho_0 c} = \frac{2\ \text{Pa}}{415\ \text{Pa·s/m}} = \boxed{4.82 \cdot 10^{-3}\ \text{m/s}}$

d) $p_{rms} = p_e = \frac{p_{peak}}{\sqrt{2}} = \frac{2\ \text{Pa}}{\sqrt{2}} = \boxed{\sqrt{2}\ \text{Pa} = 1.41\ \text{Pa}}$

e) $SPL = 20 \log\left(\frac{p_e}{p_{ref}}\right) = 20 \log\left(\frac{\sqrt{2}\ \text{Pa}}{20 \cdot 10^{-6}\ \text{Pa}}\right) = \boxed{97.0\ \text{dB ref } 20\ \mu\text{Pa}}$