ME 513
Fall 2017
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Homework 2 Solutions

93 points

Problem 2.4.1 (18 points, 3 per part)

Given

Wave equation: $\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$

Find

By direct substitution, show that each of the following is a solution of the wave equation:

a) $f_1(x - ct)$
b) $\ln[a(ct - x)]$
c) $a(ct - x)^2$
d) $\cos[a(ct - x)]$

Similarly, show that each of the following is not a solution of the wave equation:

e) $a(ct - x^2)$
f) $at(ct - x)$

Solution

Solution works if $\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$

a) $y = f_1(x - ct)$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2}{\partial x^2} f_1(x - ct) = f_1''(x - ct)$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2} f_1(x - ct) = \frac{\partial}{\partial t}[-c\, f_1'(x - ct)] = (-c)^2 f_1''(x - ct) = c^2 f_1''(x - ct)$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = f_1''(x - ct) - \frac{1}{c^2}[c^2 f_1''(x - ct)] = f_1''(x - ct) - f_1''(x - ct) = 0$$

$\boxed{\text{Solution checks}}$

b) $y = \ln[a(ct - x)]$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2}{\partial x^2}\ln[a(ct - x)] = \frac{\partial}{\partial x}\left[\frac{-a}{a(ct-x)}\right] = -\frac{\partial}{\partial x}(ct - x)^{-1} = -(-1)^2(ct - x)^{-2}$$
$$= -(ct - x)^{-2}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2}\ln[a(ct - x)] = \frac{\partial}{\partial t}\left[\frac{ac}{a(ct-x)}\right] = c\frac{\partial}{\partial t}(ct - x)^{-1} = (-c^2)(ct - x)^{-2}$$
$$= -c^2(ct - x)^{-2}$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = -(ct - x)^{-2} - \frac{1}{c^2}\{-c^2(ct - x)^{-2}\} = -(ct - x)^{-2} + (ct - x)^{-2} = 0$$

$\boxed{\text{Solution checks}}$

c) $y = a(ct-x)^2$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2}{\partial x^2}\ a(ct-x)^2 = a\frac{\partial}{\partial x}2(-1)(ct-x) = -2a\frac{\partial}{\partial x}(ct-x) = -2a\,(-1) = 2a$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2}\ a(ct-x)^2 = a\frac{\partial}{\partial t}2c(ct-x) = 2ac\frac{\partial}{\partial t}(ct-x) = 2ac(c) = 2ac^2$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 2a - \frac{1}{c^2}(2ac^2) = 2a - 2a = 0$$

Solution checks

d) $y = \cos[a(ct-x)]$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2}{\partial x^2}\cos[a(ct-x)] = -a\frac{\partial}{\partial x}\{-\sin[a(ct-x)]\} = (-a)^2\{-\cos[a(ct-x)]\}$$

$$= -a^2\cos[a(ct-x)]$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2}\cos[a(ct-x)] = ac\frac{\partial}{\partial x}\{-\sin[a(ct-x)]\} = (ac)^2\{-\cos[a(ct-x)]\}$$

$$= -(ac^2)\cos[a(ct-x)]$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = -a^2\cos[a(ct-x)] - \frac{1}{c^2}\{-(ac^2)\cos[a(ct-x)]\}$$

$$= -a^2\cos[a(ct-x)] + a^2\cos[a(ct-x)] = 0$$

Solution checks

e) $y = a(ct-x^2)$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2}{\partial x^2}a(ct-x^2) = a\frac{\partial}{\partial x}(-2x) = -2a$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2}a(ct-x^2) = a\frac{\partial}{\partial t}c = 0$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = -2a - \frac{1}{c^2}(0) = -2a \neq 0$$

Solution does not check

f) $y = at(ct-x)$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2}{\partial x^2}at(ct-x) = at\frac{\partial}{\partial x}(-1) = 0$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2}{\partial t^2}at(ct-x) = \frac{\partial}{\partial t}[a(ct-x) + at(c)] = ac + ac = 2ac$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0 - \frac{1}{c^2}(2ac) = -\frac{2a}{c} \neq 0$$

Solution does not check

Problem 2.8.1 (10 points)

Given

Waveform $y = 4\cos(3t - 2x)$
Propagates on string of linear density $\rho_L = 0.1\ \text{g/cm}$
Units: y and x in cm, t in seconds

Find

a) A, c, f, λ, k
b) $u(0,0)$

Solution

a) $y = A\cos(\omega t - kx) = 4\cos(3t - 2x)$

$$\boxed{\begin{array}{c} A = 4\ \text{cm} \\ k = 2\ \text{cm}^{-1} \end{array}}$$

$$f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}(3\ \text{rad/s}) = 0.477\ Hz \qquad \boxed{f = 0.477\ Hz}$$

$$k = \frac{\omega}{c} \rightarrow \quad c = \frac{\omega}{k} = \frac{3\text{rad/s}}{2\ \text{cm}^{-1}} = \frac{3}{2}\text{cm/s} \qquad \boxed{c = 1.5\ \ \text{cm/s}}$$

$$k = \frac{2\pi}{\lambda} \rightarrow \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{2\ \text{cm}^{-1}} = \pi\ \text{cm} \qquad \boxed{\lambda = \pi\ \text{cm}}$$

b) $u = \frac{\partial y}{\partial t} = \frac{\partial}{\partial t}A\cos(\omega t - kx) = -\omega A\sin(\omega t - kx)$

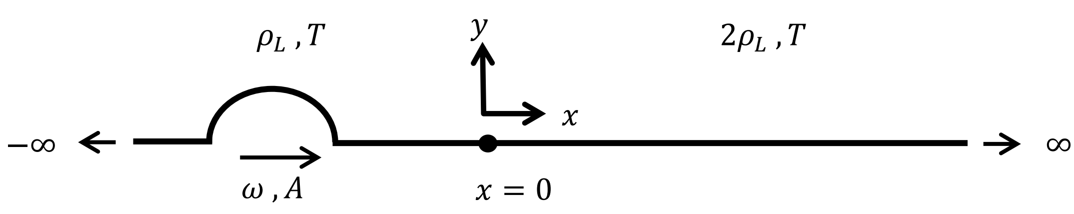
$$u(0,0) = -\omega A\sin\big(\omega(0) - k(0)\big) = -\omega A\sin(0) = 0$$

$$\boxed{u(0,0) = 0\ \ \text{cm/s}}$$

Problem 2.8.2 (15 points)

Given

Infinite string $(-\infty < x \le 0)$ with density ρ_L under tension T
Attached at $x = 0$ to another infinite string $(0 < x < \infty)$ with density $2\rho_L$ under tension T
Wave of angular frequency ω and amplitude A traveling in the $+x$ direction on the first string



Find

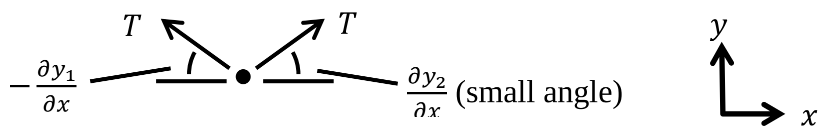
Amplitude of the wave traveling on the second string

Solution

Original wave: $y_1(x,t) = Ae^{j(\omega t - k_1 x)}$
Wave reflects at transfer point $x = 0$
Thus, $y_1(x,t) = Ae^{j(\omega t - k_1 x)} + Be^{j(\omega t + k_1 x)}$
A wave also transfers into the right string: $y_2(x,t) = Ce^{j(\omega t - k_2 x)}$

Displacement continuity BC: $y_1(0,t) = y_2(0,t)$
$Ae^{j\omega t} + Be^{j\omega t} = Ce^{j\omega t}$
$B = C - A$

Force balance BC:



$$\sum F_y = -T\frac{\partial y_1}{\partial x}\bigg|_{x=0} + T\frac{\partial y_2}{\partial x}\bigg|_{x=0} = 0$$
$$\frac{\partial y_1}{\partial x}\bigg|_{x=0} = \frac{\partial y_2}{\partial x}\bigg|_{x=0}$$

$$\frac{\partial y_1}{\partial x}\bigg|_{x=0} = -jk_1Ae^{j\omega t} + jk_1Be^{j\omega t} = -jk_1Ae^{j\omega t} + jk_1(C-A)e^{j\omega t} = jk_1(C-2A)e^{j\omega t}$$
$$\frac{\partial y_2}{\partial x}\bigg|_{x=0} = -jk_2Ce^{j\omega t}$$

Substituting: $jk_1(C-2A)e^{j\omega t} = -jk_2Ce^{j\omega t}$
$k_1(C-2A) = -k_2C$

Note that $k_1 = \frac{\omega}{c_1} = \omega\sqrt{\frac{\rho_{L1}}{T}} = \omega\sqrt{\frac{\rho_L}{T}}$

$k_2 = \frac{\omega}{c_2} = \omega\sqrt{\frac{\rho_{L2}}{T}} = \omega\sqrt{\frac{2\rho_L}{T}} = \sqrt{2}\left(\omega\sqrt{\frac{\rho_L}{T}}\right) = \sqrt{2}k_1$

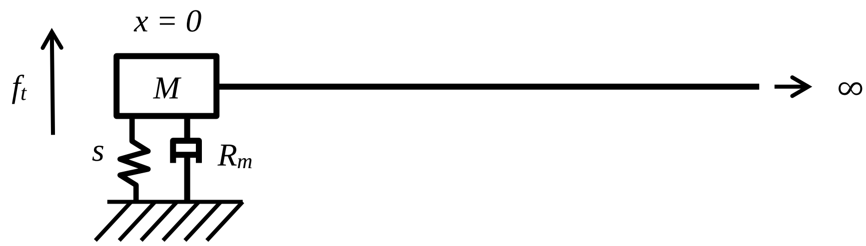
$k_1(C-2A) = -\sqrt{2}k_1C$
$(C-2A) = -\sqrt{2}C$
$C(1+\sqrt{2}) = 2A$
$C = \frac{2}{1+\sqrt{2}}A\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right) = \frac{2(\sqrt{2}-1)}{2-1}A = 2(\sqrt{2}-1)A$

$$\boxed{C = \frac{2}{1+\sqrt{2}}A = 2(\sqrt{2}-1)A}$$

Problem 2.9.2 (10 points)

Given

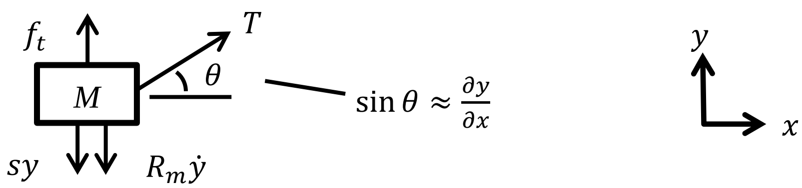
Simple harmonic oscillator with an infinite string extending transversely from the mass
Force applied to the oscillator



Find

z_{m0}
(assume $f_t = Fe^{j\omega t}$)

Solution



Force balance equation: $\sum F_y = f_t + T\frac{\partial y}{\partial x}\Big|_{x=0} - sy|_{x=0} - R_m\dot{y}|_{x=0} = m\ddot{y}|_{x=0}$

Wave equation: $y = Ae^{j(\omega t - kx)}$ (only travels in +x direction)

$$\frac{\partial y}{\partial x}\Big|_{x=0} = \frac{\partial}{\partial x}Ae^{j(\omega t-kx)}\Big|_{x=0} = -jkAe^{j(\omega t-kx)}\Big|_{x=0} = -jkAe^{j\omega t}$$

$$y(0,t) = Ae^{j(\omega t-kx)}\Big|_{x=0} = Ae^{j\omega t}$$

$$\dot{y}(0,t) = \frac{\partial}{\partial t}Ae^{j(\omega t-kx)}\Big|_{x=0} = j\omega Ae^{j(\omega t-kx)}\Big|_{x=0} = j\omega Ae^{j\omega t}$$

$$\ddot{y}(0,t) = \frac{\partial^2}{\partial t^2}Ae^{j(\omega t-kx)}\Big|_{x=0} = -\omega^2 Ae^{j(\omega t-kx)}\Big|_{x=0} = -\omega^2 Ae^{j\omega t}$$

Substitute into force balance equation:

$$Fe^{j\omega t} + T\left(-jkAe^{j\omega t}\right) - s\left(Ae^{j\omega t}\right) - R_m\left(j\omega Ae^{j\omega t}\right) = m\left(-\omega^2 Ae^{j\omega t}\right)$$

$$F - jkTA - sA - j\omega R_m A = -m\omega^2 A$$

$$F = jkTA + sA + j\omega R_m A - m\omega^2 A$$

Remember $k = \frac{\omega}{c}$, $T = \rho_L c^2$

$$F = j\left(\frac{\omega}{c}\right)(\rho_L c^2)A + sA + j\omega R_m A - m\omega^2 A = A[j\omega(\rho_L c + R_m) + (s - m\omega^2)]$$

$$A = \frac{F}{j\omega(\rho_L c + R_m) + (s - m\omega^2)}$$

$$z_{m0} = \frac{f_t}{\dot{y}(0,t)} = \frac{Fe^{j\omega t}}{j\omega A e^{j\omega t}} = \frac{F}{j\omega} A^{-1} = \frac{F}{j\omega}\left[\frac{j\omega(\rho_L c + R_m) + (s - m\omega^2)}{F}\right] = (\rho_L c + R_m) + \frac{1}{j}\left(\frac{s}{\omega} - \omega m\right)$$

$$= (\rho_L c + R_m) - j\left(\frac{s}{\omega} - \omega m\right) = (\rho_L c + R_m) + j\left(\omega m - \frac{s}{\omega}\right)$$

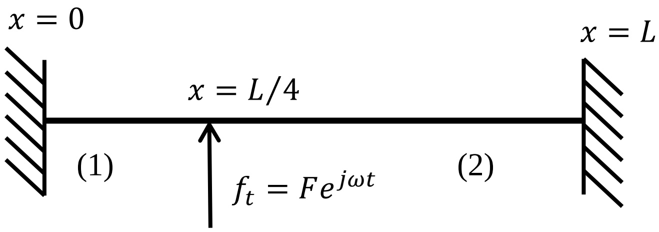
$$\boxed{z_{m0} = \rho_L c + R_m + j\left(\omega m - \frac{s}{\omega}\right)}$$

Problem 2.9.3a (edited) (20 points)

Given

Fixed-fixed string of length L

Driving force $Fe^{j\omega t}$ located at $x = L/4$



Find

Input mechanical impedance at $x = L/4$

(assume uniform T, ρ_L)

Solution

Sector 1: $y_1(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$

Sector 2: $y_2(x,t) = Ce^{j(\omega t - kx)} + De^{j(\omega t + kx)}$

Fixed-end boundary condition (BC) at $x = 0$:

$y_1(0,t) = Ae^{j\omega t} + Be^{j\omega t} = 0$

$B = -A$

Substituting, $y_1(x,t) = Ae^{j(\omega t - kx)} - Ae^{j(\omega t + kx)} = Ae^{j\omega t}\left(e^{-jkx} - e^{jkx}\right) = -2jA\sin(kx)\, e^{j\omega t}$

Fixed-end BC at $x = L$:

$y_2(L,t) = Ce^{j(\omega t - kL)} + De^{j(\omega t + kL)} = 0$

$Ce^{-jkL} = -De^{jkL}$

$D = -Ce^{-2jkL}$

Substituting: $y_2(x,t) = Ce^{j(\omega t - kx)} - \left(Ce^{-2jkL}\right)e^{j(\omega t + kx)}$

$= Ce^{j\omega t}\left(e^{-jkx} - e^{-2jkL}e^{jkx}\right)$

$= Ce^{j\omega t}e^{-jkL}\left(e^{jkL}e^{-jkx} - e^{-jkL}e^{jkx}\right)$

$= Ce^{j(\omega t - kL)}\left(e^{jk(L-x)} - e^{-jk(L-x)}\right)$

$= 2jCe^{j(\omega t - kL)} \sin k(L - x)$

Displacement continuity BC at $x = L/4$:

$y_1(L/4\,, t) = y_2(L/4\,, t)$

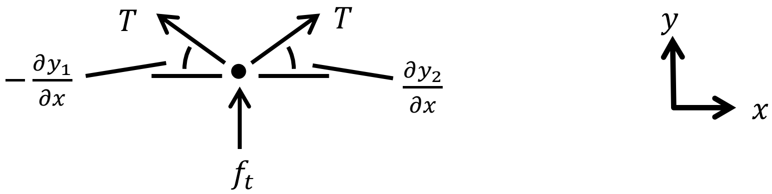
$-\cancel{2j}A \sin\left(\frac{kL}{4}\right) \cancel{e^{j\omega t}} = \cancel{2j}C\cancel{e^{j\omega t}}e^{-jkL} \sin k\left(L - \frac{L}{4}\right)$

$-A \sin\left(\frac{kL}{4}\right) = Ce^{-jkL} \sin\left(\frac{3kL}{4}\right)$

$Ce^{-jkL} = -A \sin\left(\frac{kL}{4}\right) \csc\left(\frac{3kL}{4}\right)$

Substituting into y_2: $y_2(x, t) = -2jA \sin\left(\frac{kL}{4}\right) \csc\left(\frac{3kL}{4}\right) \sin k(L - x)\, e^{j\omega t}$

Force balance BC at $x = L/4$:



$\sum F_y = f_t - T\left.\frac{\partial y_1}{\partial x}\right|_{x=L/4} + T\left.\frac{\partial y_2}{\partial x}\right|_{x=L/4} = 0$

$\left.\frac{\partial y_1}{\partial x}\right|_{x=L/4} = \frac{\partial}{\partial x}\left[-2jA \sin(kx)\, e^{j\omega t}\right]_{x=L/4} = -2jkA \cos(kx)\, e^{j\omega t}\Big|_{x=L/4} = -2jkA \cos\left(\frac{kL}{4}\right) e^{j\omega t}$

$\left.\frac{\partial y_2}{\partial x}\right|_{x=L/4} = \frac{\partial}{\partial x}\left[-2jA \sin\left(\frac{kL}{4}\right) \csc\left(\frac{3kL}{4}\right) \sin k(L - x)\, e^{j\omega t}\right]_{x=L/4}$

$= 2jkA \sin\left(\frac{kL}{4}\right) \csc\left(\frac{3kL}{4}\right) \cos k(L - x)\, e^{j\omega t}\Big|_{x=L/4}$

$= 2jkA \sin\left(\frac{kL}{4}\right) \csc\left(\frac{3kL}{4}\right) \cos k\left(L - \frac{L}{4}\right) e^{j\omega t}$

$= 2jkA \sin\left(\frac{kL}{4}\right) \cot\left(\frac{3kL}{4}\right) e^{j\omega t}$

$Fe^{j\omega t} - T\left[-2jkA \cos\left(\frac{kL}{4}\right) e^{j\omega t}\right] + T\left[2jkA \sin\left(\frac{kL}{4}\right) \cot\left(\frac{3kL}{4}\right) e^{j\omega t}\right] = 0$

$F + 2jkTA \cos\left(\frac{kL}{4}\right) + 2jkTA \sin\left(\frac{kL}{4}\right) \cot\left(\frac{3kL}{4}\right) = 0$

$\frac{F}{2jkTA} + \cos\left(\frac{kL}{4}\right) + \sin\left(\frac{kL}{4}\right) \cot\left(\frac{3kL}{4}\right) = 0$

$\frac{F}{2jkTA \sin(kL/4)} = -\cot\left(\frac{3kL}{4}\right) - \cot\left(\frac{kL}{4}\right)$

$A = -\frac{F}{2jkT \sin\left(\frac{kL}{4}\right)\left[\cot\left(\frac{kL}{4}\right) + \cot\left(\frac{3kL}{4}\right)\right]}$

Remember $k = \frac{\omega}{c}$, $T = \rho_L c^2$

$= -\frac{F}{2j\left(\frac{\omega}{c}\right)(\rho_L c^2) \sin\left(\frac{kL}{4}\right)\left[\cot\left(\frac{kL}{4}\right) + \cot\left(\frac{3kL}{4}\right)\right]} = -\frac{F}{2j\omega(\rho_L c) \sin\left(\frac{kL}{4}\right)\left[\cot\left(\frac{kL}{4}\right) + \cot\left(\frac{3kL}{4}\right)\right]}$

Solve for z_{m0} using either y_1 or y_2

From y_1: $z_{m0} = \frac{f_t}{u_1(L/4,t)} = \frac{f_t}{j\omega y_1(L/4,t)} = \frac{Fe^{j\omega t}}{j\omega\left[-2jA\sin(kx)e^{j\omega t}\right]_{x=L/4}} = \frac{F}{2\omega A\sin(kL/4)}$

$$= \frac{F}{2\omega\sin(kL/4)}\left\{-\frac{2j\omega(\rho_L c)\sin\left(\frac{kL}{4}\right)\left[\cot\left(\frac{kL}{4}\right)+\cot\left(\frac{3kL}{4}\right)\right]}{F}\right\} = -j\rho_L c\left[\cot\left(\frac{kL}{4}\right) + \cot\left(\frac{3kL}{4}\right)\right]$$

From y_2: $z_{m0} = \frac{f_t}{u_2(L/4,t)} = \frac{f_t}{j\omega y_2(L/4,t)} = \frac{Fe^{j\omega t}}{j\omega\left[-2jA\sin\left(\frac{kL}{4}\right)\csc\left(\frac{3kL}{4}\right)\sin k(L-x)e^{j\omega t}\right]_{x=L/4}}$

$= \frac{F}{2\omega A\sin\left(\frac{kL}{4}\right)\csc\left(\frac{3kL}{4}\right)\sin k\left(L-\frac{L}{4}\right)} = \frac{F}{2\omega A\sin(kL/4)} = \cdots$ (remaining steps are same as in y_1 case)

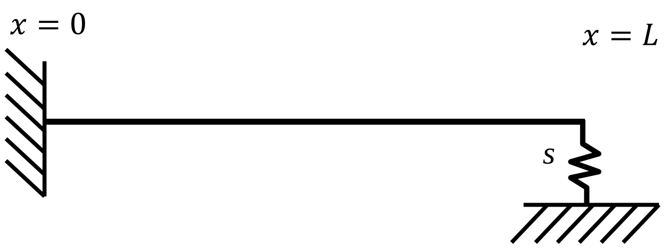
$$\boxed{z_{m0} = -j\rho_L c\left[\cot\left(\frac{kL}{4}\right) + \cot\left(\frac{3kL}{4}\right)\right]}$$

Problem 2.11.1 (10 points)

Given

Fixed, spring-loaded string (assume fixed at $x = 0$, spring-loaded at $x = L$)

$T = sL$



Find

Identify kL for the normal modes

Sketch the waveforms for the fundamental and for the first overtone

Solution

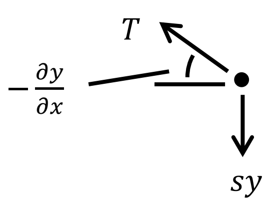
$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$

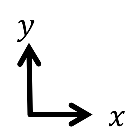
Fixed end BC: $y(0,t) = Ae^{j\omega t} + Be^{j\omega t} = 0$

$B = -A$

$y(x,t) = Ae^{j\omega t}\left(e^{-jkx} - e^{jkx}\right) = -2jA\sin(kx)\,e^{j\omega t}$

Spring-loaded end BC:





$$\sum F_y = -T \frac{\partial y}{\partial x}\Big|_{x=L} - sy(L,t) = 0$$

$$\frac{\partial y}{\partial x}\Big|_{x=L} = \frac{\partial}{\partial x}\left[-2jA\sin(kx)\,e^{j\omega t}\right]_{x=L} = -2jkA\cos(kx)\,e^{j\omega t}\Big|_{x=L} = -2jkA\cos(kL)\,e^{j\omega t}$$

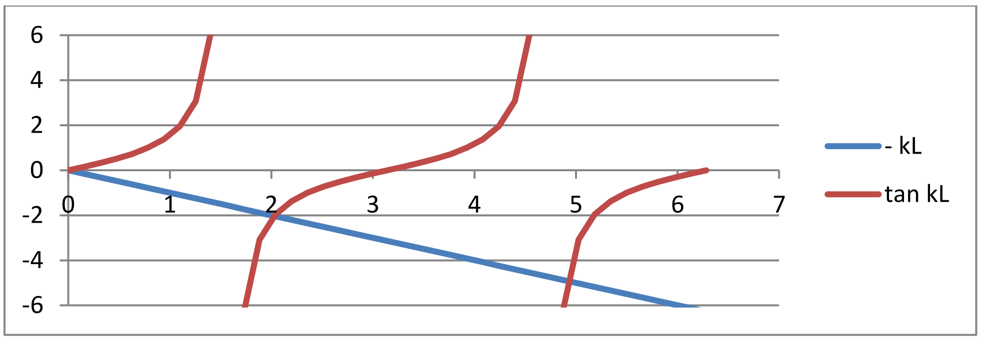
Thus, $(-T)\left[-2\cancel{j}\cancel{A}k\cos(kL)\,\cancel{e^{j\omega t}}\right] - s\left[-2\cancel{j}\cancel{A}s\sin(kL)\,\cancel{e^{j\omega t}}\right] = 0$

$$kT\cos(kL) + s\sin(kL) = 0$$

$$\tan(kL) = -\frac{kT}{s} = -\frac{k(sL)}{s} = -kL$$

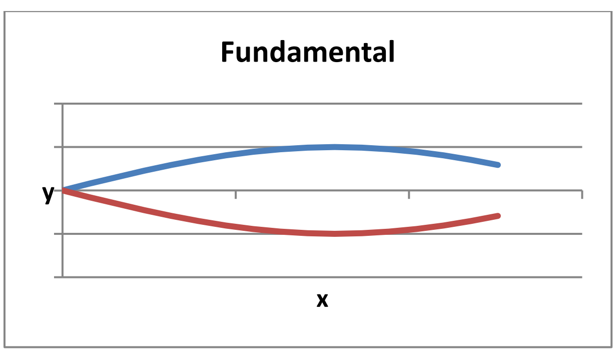
Characteristic equation: $\boxed{\tan(kL) = -kL}$

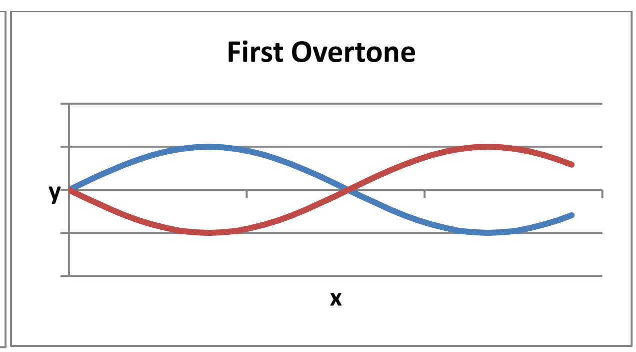
Solve for kL numerically. I will begin by plotting $-kL$ and $\tan(kL)$ in Excel.



I see from this plot that $k_1L \approx 2$ and $k_2L \approx 5$. To find the exact values, I set up two equations for $x = kL + \tan(kL)$ in Excel, make initial guesses of $kL = 2$ and 5, and run the Solver tool.

$$\boxed{\begin{array}{l} k_1L = 2.029 \\ k_2L = 4.913 \end{array}}$$





Problem 2.11.1 (addendum) (10 points)

Guitar string

$\rho_L = 8 \cdot 10^{-4}$ kg/m

$T = 50$ N

$L = 0.66$ m

Find

a) ω_1 (assume fixed-fixed)

b) Minimum stiffness of the transverse string **(misprint: spring?)** required to ensure that ω_1 is reduced by no more than 1% (assume $T = sL$ in this case, rather than 50 N)

Solution

$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$

Fixed end BC at $x = 0$: $y(0,t) = Ae^{j\omega t} + Be^{j\omega t} = 0$

$B = -A$

$y(x,t) = Ae^{j\omega t}\left(e^{-jkx} - e^{jkx}\right) = -2jA\sin(kx)\, e^{j\omega t}$

a) Fixed end BC at $x = L$: $y(L,t) = -2jA\sin(kL)\, e^{j\omega t} = 0$

Characteristic equation: $\sin(kL) = 0$

$k_1 L = \pi$

$k_1 L = \frac{\omega_1}{c} L = \pi$

$\omega_1 = \frac{\pi}{L} c = \frac{\pi}{L}\sqrt{\frac{T}{\rho_L}} = \frac{\pi}{0.66\text{ m}}\sqrt{\frac{50\text{ N}}{8 \cdot 10^{-4}\text{kg/m}}} = 1.19 \cdot 10^3 \text{ rad/s}$

$\boxed{\omega_1 = 1.19 \cdot 10^3 \text{ rad/s}}$

b) From Problem 2.11.1 above, for a fixed, spring-loaded string, $\tan(kL) = -\frac{kT}{s} = -\frac{T}{sL} kL$

$\omega_{1b} = \frac{(k_1 L)_b}{L}\sqrt{\frac{T}{\rho_L}} = \frac{(k_1 L)_b}{L} c = 0.99\omega_{1a}$

Remember $\omega_{1a} = \frac{\pi}{L} c$

$\frac{\omega_{1b}}{\omega_{1a}} = \frac{(k_1 L)_b \frac{c}{L}}{\frac{\pi}{L} c} = \frac{(k_1 L)_b}{\pi} = 0.99$

$(k_1 L)_b = 0.99\pi$

Substituting, $\tan(0.99\pi) = -\frac{T}{sL}(0.99\pi)$

$s = -\frac{0.99\pi T}{L\tan(0.99\pi)} = -\frac{0.99\pi(50\text{ N})}{(0.66\text{ m})\tan(0.99\pi)} = 7.50 \cdot 10^3 \text{ N/m}$

$\boxed{s = 7.50 \cdot 10^3 \text{ N/m}}$