ME 513 Fall 2017 – Homework No. 3 – Due Oct. 16, and off-site students, e-mailed before midnight Oct. 16

1. **A 0.01 m^2 piston oscillates at one end of a 1 m long tube that is terminated at the other end by a partially absorbing surface. At a single frequency, ω, the spatial dependence of the plane wave within the tube has the form**

$$p(x) = e^{-jkx} + 0.6e^{jkx}$$

where *x*=0 is at the surface of the absorbing material, and the positive *x*-direction is *into* the absorbing surface.

(i) **Calculate an expression for the mean square pressure and plot it as a function of position in the tube at 1000 Hz. Show that the minima in the standing wave pattern at a half wavelength apart.**

(ii) **Derive by using the linearized Euler equation an expression for the spatial dependence of the particle velocity field within the tube. Plot the mean square particle velocity as a function of position at 1000 Hz and plot on the same graph as the mean square pressure – and comment on the results. Also, calculate the velocity of the piston: i.e., the velocity in the sound field at x=-1.0 m.**

(iii) **The time-averaged acoustic intensity at any point within the tube can be calculated using the expression *I*(*x*) = (1/2)Re[*p*(*x*)*u**(*x*)] where Re denotes the real part, *u*(*x*) is the acoustic particle velocity and superscript * denotes the complex conjugate. Use this expression to show that the acoustic intensity does not depend on position within the tube, and then to determine the sound power delivered to the tube by the piston.**

(iv) **What is the specific acoustic impedance at the piston surface and at the absorbing surface?**

2. **A unit amplitude plane wave strikes a surface at *y* = 0 (with the positive *y*-axis pointing into the surface) at an angle θ from the normal. A reflected plane wave, having a complex amplitude *R* then propagates away from the surface at an angle of reflection θ.**

(i) **Write an expression for the sound pressure in the region *y* < 0 (defining quantities as necessary).**

(ii) **By using the linearized Euler equation, derive an expression for the *y*-component of the total particle velocity and also of the incident particle velocity.**

(iii) **Based solely on the field in the region $y < 0$, derive an expression for the time-averaged sound power per unit area flowing into the surface at $y = 0$ and also give an expression for the sound power per unit area being delivered to the surface by the incident wave. Calculate the absorption coefficient (i.e., the ratio of the power absorbed at the surface to the power incident on the surface) at an angle of incidence of 45 degrees when $R = 0.5$.**

3. **A sound field has the form**

$$p(x,t) = Ae^{-\alpha x}e^{-j\beta x}e^{j\omega t}$$

where A is complex and α and β are real.

(i) **Sketch the spatial variation of the sound field. What is the significance of α and β? What is the wavelength of the sound field in terms of the parameters of the sound field?**

(ii) **Derive an expression for the acoustic particle velocity.**

(iii) **Evaluate the acoustic intensity, and show that it is a function of position in this case. Derive an expression for the rate of decay of the sound intensity level in dB/m.**

4. **A two-dimensional sound field in the farfield of a cylindrical source can be expressed in cylindrical coordinates as:**

$$p(r,\theta) = \frac{A}{r^{1/2}} \sin\theta e^{-jkr}$$

(i) **By using the linearized Euler equation in cylindrical coordinates, find the vector particle velocity field associated with this pressure field. Note that the vector particle velocity field has two components, in this case.**
(ii) **Find the radial component of the acoustic intensity.**
(iii) **Find the farfield specific acoustic impedance based on the radial particle velocity – does it limit to the plane wave impedance in the farfield?**

From Kinsler, Frey, Coppens and Sanders: 5.12.3