10 10.2.4

(a) From Eqn. (10.2.4)

$$\frac{Z_{mo}}{\rho_0 cS} = \frac{(Z_{mL}/\rho_0 cS) + j\tan kL}{1 + j(Z_{mL}/\rho_0 cS)\tan kL}$$

From Eqn (10.2.10)

$Z_{mL}/\rho_0 cS = \frac{1}{2}(ka)^2 + j(8/3\pi)ka$ (flanged)

$a = 0.01$ m, $L = 1$ m, air, $\rho_0 c = 415$ pa.s/m

(b) $kL_1 = 3.115$ $\quad k = \frac{\omega}{c} = \frac{2\pi f}{c}$, $f = \frac{kc}{2\pi}$

$kL_2 = 6.230$

$kL_3 = 9.346$

$f_1 = 170.0$ Hz
$f_2 = 340.1$ Hz
$f_3 = 510.2$ Hz
→ resonance frequencies from plot.

$f_1 = \frac{c}{2L} = \frac{343}{2} = 171.5$ Hz

$f_2 = 2\frac{c}{2L} = 2\frac{343}{2} = 343$ Hz

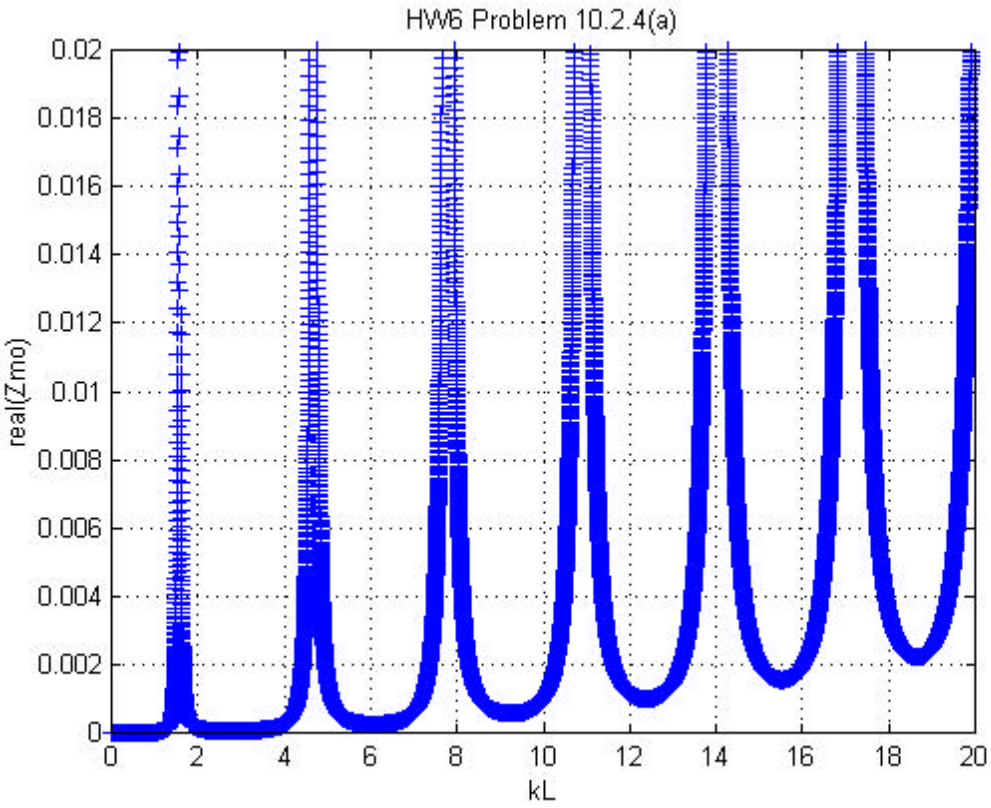
$f_3 = 3\frac{c}{2L} = 3\frac{343}{2} = 514.5$ Hz

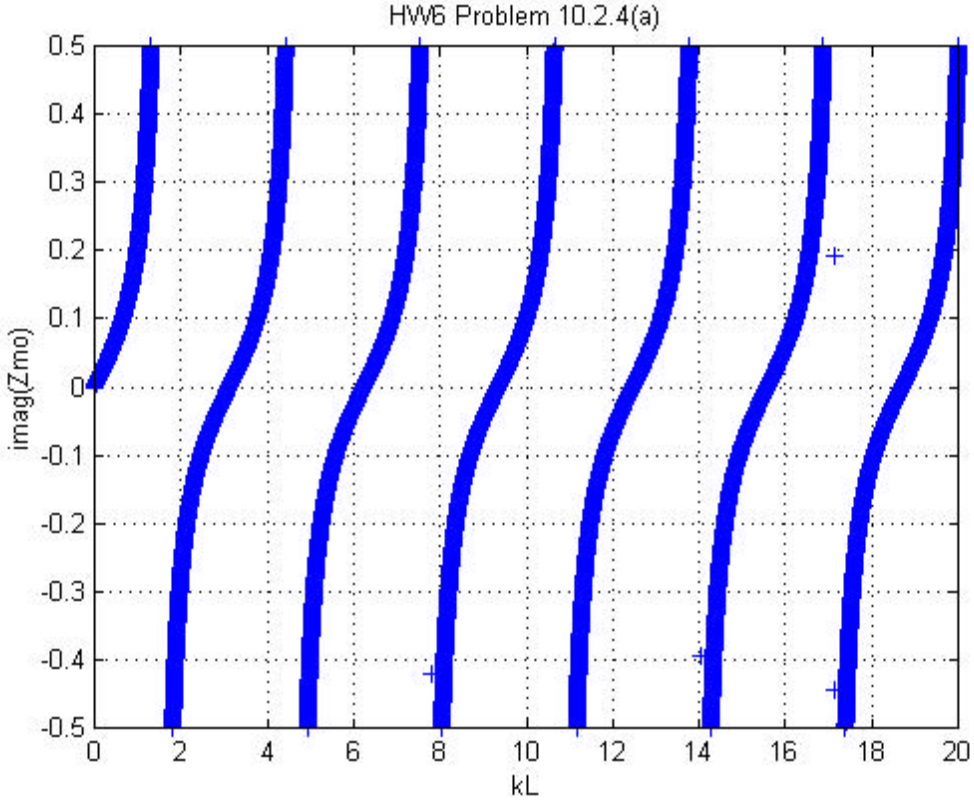
→ resonance frequencies from integral number of half-waves in pipe

Error in $f_1 = \frac{171.5 - 170}{170} = 0.0088$, 0.88%

Error in $f_2 = \frac{343 - 340.1}{340.1} = 0.0085$, 0.85%

Error in $f_3 = \frac{514.5 - 510.2}{510.2} = 0.0084$, 0.84%


HW6 Problem 10.2.4(a)
real(Zmo)
kL
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
0
2
4
6
8
10
12
14
16
18
20


HW6 Problem 10.2.4(a)
imag(Zmo)
kL
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0
2
4
6
8
10
12
14
16
18
20

10.3.3C

(a) From Eqn's (10.3.2), (10.3.1), (10,2,10)

power transmission coefficient (flanged open end)

$$T_\pi = 1 - |B/A|^2$$

$$= 1 - \left| \frac{Z_{mL}/\rho_0 cS - 1}{Z_{mL}/\rho_0 cS + 1} \right|^2$$

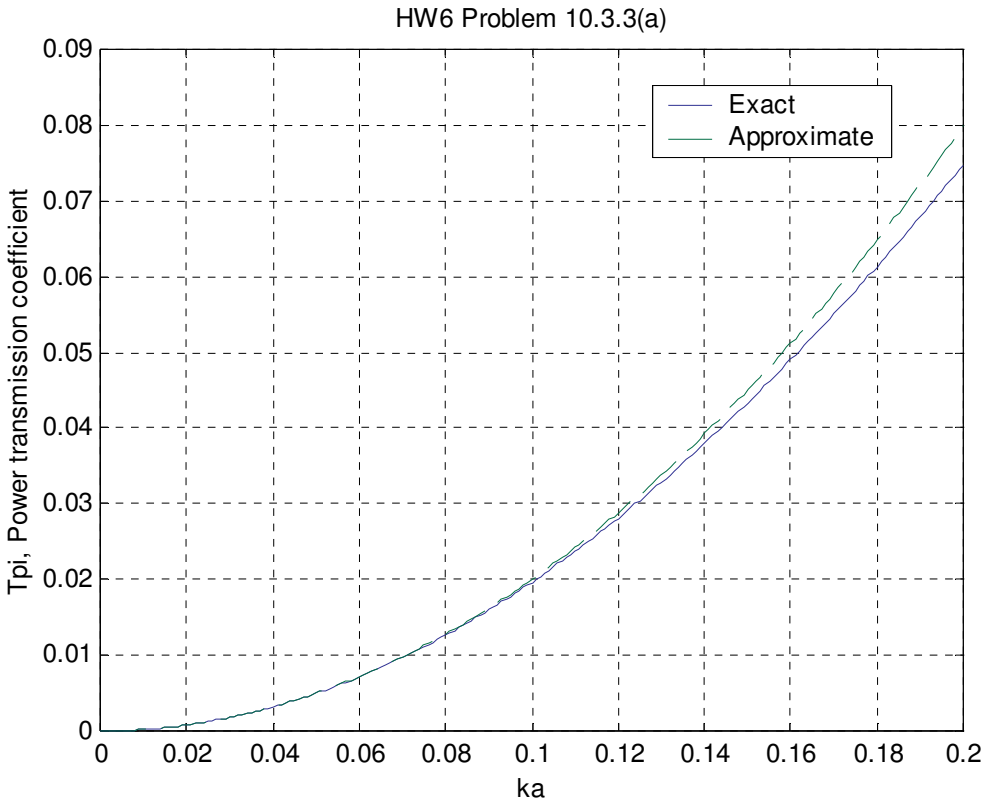
$$= 1 - \left| \frac{\frac{1}{2}(ka)^2 + j(8/3\pi)ka - 1}{\frac{1}{2}(ka)^2 + j(8/3\pi)ka + 1} \right|^2$$

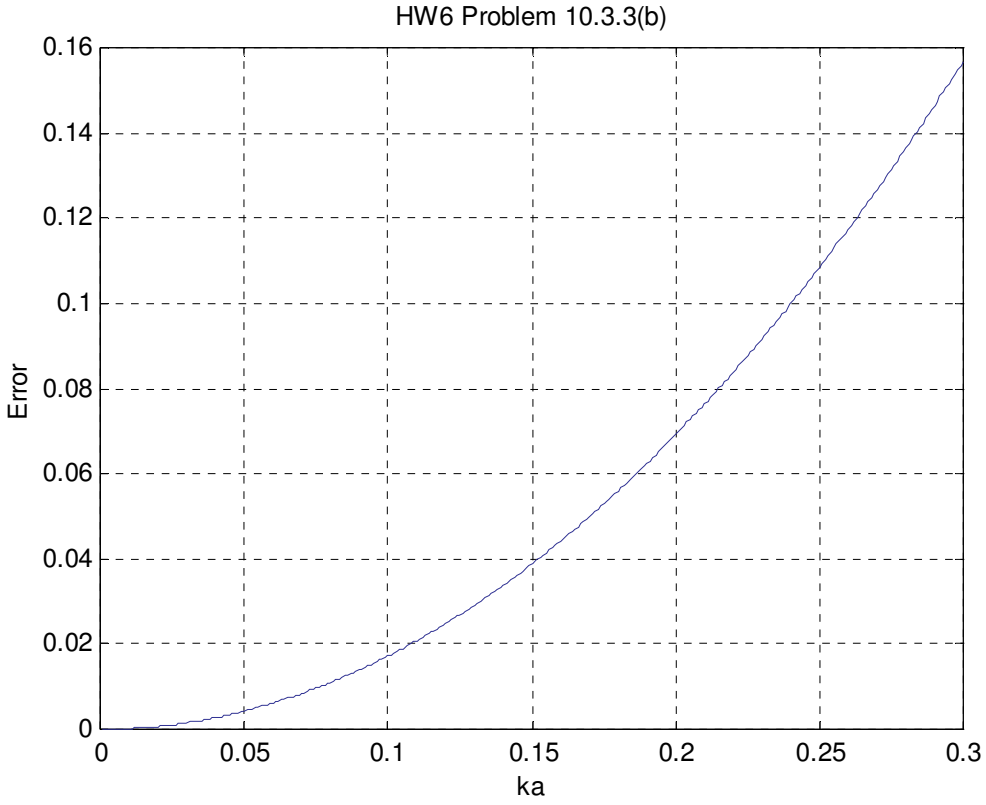
If $ka \ll 1$, from Eqn. (10.3.4)

$$T_\pi \approx 2(ka)^2$$

(b) $$\text{Error} = \left| \frac{T_{\pi,\text{exact}} - T_{\pi,\text{approx.}}}{T_{\pi,\text{exact}}} \right| = 0.1$$

$$ka = 0.240$$


HW6 Problem 10.3.3(a)
Exact
Approximate
Tpi, Power transmission coefficient
ka
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2


HW6 Problem 10.3.3(b)
Error
ka
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.05
0.1
0.15
0.2
0.25
0.3

10.4.3 200 Hz, air, $k = \frac{\omega}{c} = \frac{200(2\pi)}{343} = 3.664$ rad/m

Specific acoustic impedance: $1000 - j2000$ Pa·s/m

(a) From Eqn. (10.4.2)

$$\frac{Z_{mL}}{\rho_0 cS} = \frac{1 + (B/A)e^{j\theta}}{1 - (B/A)e^{j\theta}}$$

$$\frac{Z_{mL}}{\rho_0 cS} - \frac{Z_{mL}}{\rho_0 cS}(B/A)e^{j\theta} = 1 + (B/A)e^{j\theta}$$

$$(B/A)e^{j\theta} = \frac{\frac{Z_{mL}}{\rho_0 cS} - 1}{\frac{Z_{mL}}{\rho_0 cS} + 1}$$

$$= \frac{\frac{1000 - j2000}{415} - 1}{\frac{1000 - j2000}{415} + 1} = \frac{585 - j2000}{1415 - j2000}$$

$$|B/A| = \frac{\sqrt{585^2 + 2000^2}}{\sqrt{1415^2 + 2000^2}} = 0.8506$$

From Eqn. (10.4.4)

$$SWR = \frac{A+B}{A-B} = \frac{1 + B/A}{1 - B/A} = \frac{1.8506}{1 - 0.8506} = 12.39$$

(b) $$(B/A)e^{j\theta} = \frac{(585 - j2000)(1415 + j2000)}{(1415 - j2000)(1415 + j2000)}$$

$$\theta = \tan^{-1}\left(\frac{(585 - 1415)2000}{585 \cdot 1415 + 2000^2}\right) = -18.98°, \text{ or } -0.3312 \text{ rad}$$

Eqn. top of P.211

$$k(L - x_n) - \theta/2 = (n - \tfrac{1}{2})\pi$$

$$L - x_n = \frac{(n - \frac{1}{2})\pi + \theta/2}{k}$$

$$L - x_1 = \frac{(1 - \frac{1}{2})\pi - 0.3312/2}{3.664} = 0.3835 \text{ m}$$

$$L - x_2 = \frac{(2 - \frac{1}{2})\pi - 0.3312/2}{3.664} = 1.2409 \text{ m}$$

5

10.4.5 200 Hz, air, SWR = 10, $L - X_1 = 50\text{cm}$

$$k = \frac{\omega}{c} = \frac{200(2\pi)}{343} = 3.664 \text{ rad/m}$$

From Eqn. (10.4.6)

$$\theta = 2k(L - X_1) - \pi$$

$$= 2(3.664)(0.5) - \pi = 0.5224$$

From Eqn. (10.4.5)

$$\frac{B}{A} = \frac{SWR - 1}{SWR + 1} = \frac{9}{11}$$

From Eqn. (10.4.2)

$$\frac{Z_{mL}}{\rho_0 cS} = \frac{1 + (B/A)e^{j\theta}}{1 - (B/A)e^{j\theta}}$$

$$= \frac{1 + \frac{9}{11}e^{j0.5224}}{1 - \frac{9}{11}e^{j0.5224}}$$

$$\frac{Z_{mL}}{S} = \rho_0 c \frac{1 + \frac{9}{11}e^{j0.5224}}{1 - \frac{9}{11}e^{j0.5224}} = 415(1.3154 + j3.2489)$$

$$= 545.90 + j1348.30 \quad \text{Pa} \cdot \text{s/m}$$