MR 513 FALL 2013
PS #5 solution

10 7.1.3

(a) From Eqn. (7.1.5)

$$p(r,t) = \rho_0 c U_0 (a/r) \cos\theta_a e^{j[\omega t - k(r-a) + \theta_a]}$$

where $\cot\theta_a = ka$

$$Z(a) = \rho_0 c \cos\theta_a e^{j\theta_a} \qquad (7.1.2)$$

↑ specific acoustic impedance

$ka = 1, \quad \cot\theta_a = 1, \quad \theta_a = \frac{\pi}{4}$

$(\rho_0 c)_{water} = 1.48 \times 10^6 \ (Pa \cdot s/m)$

$$Z(a) = 1.48 \times 10^6 \cos(\tfrac{\pi}{4}) e^{j\frac{\pi}{4}}$$
$$= 0.74 \times 10^6 (1+j) \quad (Pa \cdot s/m)$$

Acoustic intensity

$$I = \tfrac{1}{2}\rho_0 c U_0^2 (a/r)^2 \cos^2\theta_a \qquad (7.1.6)$$

For $ka \ll 1$

$$I = \tfrac{1}{2}\rho_0 c U_0^2 (a/r)^2 (ka)^2 \qquad (7.1.8)$$

$$\text{Error} = \left| \frac{I_{actual} - I_{approx.}}{I_{actual}} \right|$$
$$= \left| \frac{\cos^2\theta_a - (ka)^2}{\cos^2\theta_a} \right| = \left| \frac{\frac{1}{2} - 1}{\frac{1}{2}} \right| = 1, (100\%)$$

(b) From Eqn. (7.18), for $ka \ll 1$

$$I = \frac{1}{2} \rho_0 c U_0^2 (a/r)^2 (ka)^2$$

Radiated sound power,

$$\Pi = \int_s I \, ds$$

$$= \frac{1}{2} \rho_0 c U_0^2 (ka)^2 4\pi a^2$$

$$= 2\pi \rho_0 c U_0^2 k^2 a^4 , \quad k = \frac{\omega}{c}$$

$$= 2\pi \rho_0 c U_0^2 \frac{\omega^2}{c^2} a^4$$

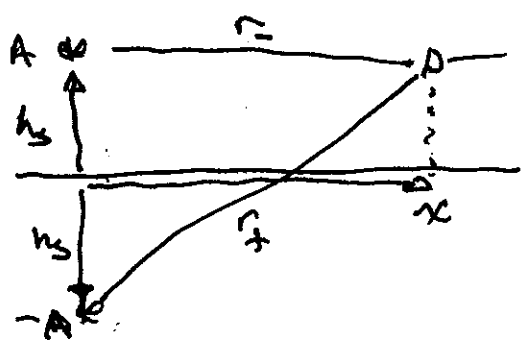
$$= 2\pi \frac{\rho_0 U_0^2}{c} \omega^2 a^4 \quad \left(\frac{\rho Q^2 \omega^2}{8\pi c} , \; Q = 4\pi a^2 U_0 \right)$$

If source is operated with constant acceleration amplitude,

$$U_0 \omega = \text{const.} = A$$

$$\Pi = 2\pi \frac{\rho_0 A^2 a^4}{c}$$: constant for different freguencies

6.8.3



(a) for pressure release surface

$$p = p_i + p_r = A\left(\frac{1}{r_-} e^{-jkr_-} - \frac{1}{r_+} e^{jkr_+}\right)$$

where $r_- = r - \Delta r$

$r_+ = r + \Delta r$

$\Delta r \doteq d \sin\theta$ and $d = h_s$

for receiver points on the surface

$$r_+ = \sqrt{h_s^2 + x^2} = r_- = r$$

so that

$$p_{surface} = A\left(\frac{1}{r} e^{-ikr} - \frac{1}{r} e^{-jkr}\right) e^{jwt}$$

$$= 0$$

(b) $$p(r,\theta) = \frac{A}{r} e^{j(wt-kr)}\left(\frac{e^{ik\Delta r}}{1-\frac{\Delta r}{r}} - \frac{e^{-ik\Delta r}}{1+\frac{\Delta r}{r}}\right)$$

$$p(r,\theta) = \frac{A}{r} e^{j(\omega t - kr)} \left[\frac{e^{jk\Delta r}\left(1+\frac{\Delta r}{r}\right) - e^{-jk\Delta r}\left(1-\frac{\Delta r}{r}\right)}{1 + \cancel{\frac{\Delta r}{r}} - \cancel{\frac{\Delta r}{r}} - \left(\frac{\Delta r}{r}\right)^2} \right]$$

$$= \frac{A}{r} e^{j(\omega t - kr)} \left[\frac{e^{jk\Delta r} - e^{-jk\Delta r} + \left(\frac{\Delta r}{r}\right)\left(e^{jk\Delta r} + e^{-jk\Delta r}\right)}{1 - \left(\frac{\Delta r}{r}\right)^2} \right]$$

Then when $\left(\frac{\Delta r}{r}\right) << 1$

$$p(r,\theta) \simeq \frac{A}{r} e^{j(\omega t - kr)} \left(e^{jk\Delta r} - e^{-jk\Delta r}\right)$$

$$= 2j \frac{A}{r} e^{j(\omega t - kr)} \sin k\Delta r$$

(c) $p = 0$ when $\sin k\Delta r = 0$

i.e., when $\sin(kd\sin\theta) = 0$

$\therefore\ kd\sin\theta = n\pi \qquad n = 0, 1, 2, 3 \ldots$

$$\sin\theta = \frac{n\pi}{kd}$$

$$\theta = \sin^{-1}\left(\frac{n\pi}{kd}\right)$$

7.2.3.

Eqn. (6.8.6).

$$p \simeq \frac{2A}{r}\cos(kd\sin\theta)e^{j(\omega t-kr)}$$

In the limit $kd \ll 1$.

$$kd\sin\theta \ll 1 \Rightarrow \cos(kd\sin\theta) \simeq 1.$$

$$\therefore p \simeq \frac{2A}{r}e^{j(\omega t-kr)} = 2\left[\frac{A}{r}e^{j(\omega t-kr)}\right].$$

Therefore, in the limit $kd \ll 1$, when a simple source is very close to a rigid boundary, the sound pressure is doubled.

This is consistent with Eqn. (7.2.14) and (7.2.17), in which:

$p = \frac{1}{2}\rho_0 c Q/\lambda r$ for a simple source. (7.2.14)

& $p = \rho_0 c Q/\lambda r$ for baffled simple source. (7.2.17).

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7.4.1.

(a). Eqn (7.4.17).

$$p(r,\theta,t)=\frac{j}{2}\rho_0 c U_0 \frac{a}{r} ka\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right] \cdot e^{j(\omega t-kr)}$$

Set the angular dependent term to zero:

$$\Rightarrow J_1(ka\sin\theta)=0.$$

For the smallest angle θ_1:

$$ka\sin\theta_1 = j_{11} = 3.83.$$

$$\therefore \sin\theta_1 = \frac{3.83}{ka}.$$

(b). From Eqn. (7.4.4).

$$p(r,0,t)=\rho_0 c U_0\{1-\exp[-jk(\sqrt{r^2+a^2}-r)]\} \cdot e^{j(\omega t-kr)}$$

For pressure to be zero:

$$1-\exp[-jk(\sqrt{r^2+a^2}-r)]=0$$

$$\Rightarrow k(\sqrt{r^2+a^2}-r)=2n\pi. \quad (n=1,2,\cdots).$$

For the greatest finite nodal distance:

$$k(\sqrt{r_1^2+a^2}-r_1)=2\pi$$

$$ka\left(\sqrt{(r_1/a)^2+1} - r_1/a\right) = 2\pi.$$

$$\sqrt{(r_1/a)^2+1} - r_1/a = 2\pi/ka.$$

$$\Rightarrow r_1/a = (ka/4\pi)\left[1-(2\pi/ka)^2\right].$$

(c). To have a small angular width of the main lobe ($\theta_1 \ll 1$) requires the wavelength is much smaller than the piston radius: $ka \gg 1$. Then from eqn. (7.4.10). $r_1/a = a/\lambda - \lambda/4a$. must ~~be~~ be very large: $r_1/a \gg 1$. So. it's impossible to have $\theta_1 \ll 1$ & $r_1/a \ll 1$ simultaneously.

In other words. $r_1/a \ll 1$ requires the piston can be modeled as a simple source. but ~~than~~ the main lobe for a simple source is very ~~large~~ broad. or $\theta_1 \gg 1$.

$\Rightarrow$ impossible.

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7.4.6C

(a) For a circular piston, on axis pressure amplitude, (7.4.5)

$$P(r,0) = 2\rho_0 c U_0 \left| \sin\left\{\frac{1}{2} kr\left[\sqrt{1+(a/r)^2}-1\right]\right\}\right|$$

$$= 2\rho_0 c U_0 \left| \sin\left\{\frac{1}{2} ka \frac{r}{a}\left[\sqrt{1+(a/r)^2}-1\right]\right\}\right|$$

(b) The pressure amplitude on the axis has asymptotic form,

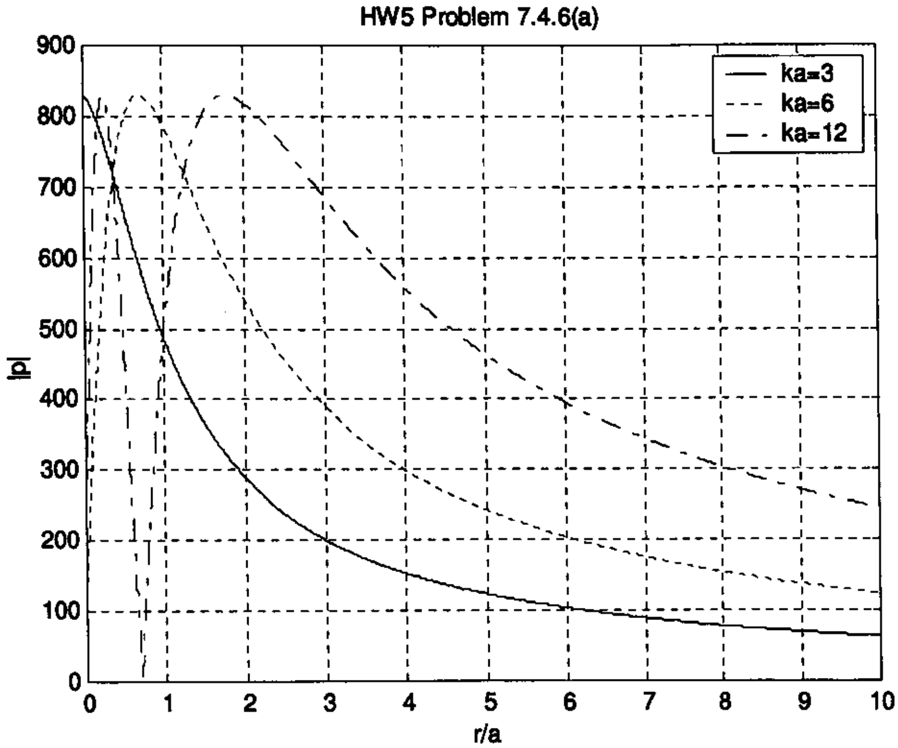
$$P_{ax}(r) = \frac{1}{2}\rho_0 c U_0 (a/r) ka \qquad (7.4.7)$$

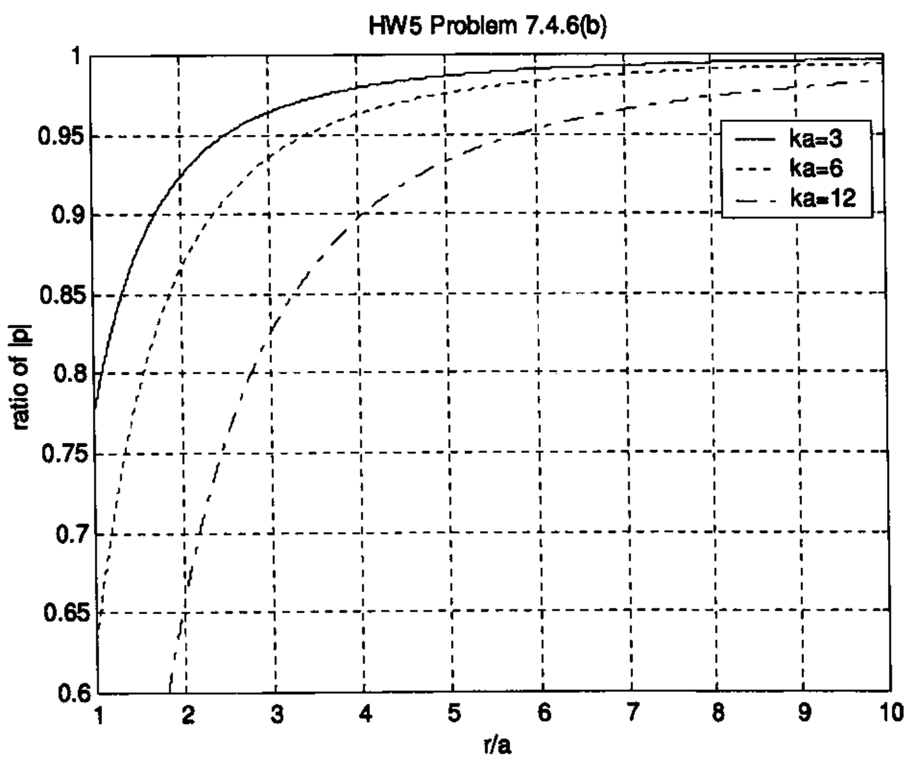
$ka = 3$, $r/a = 1.697$, $\frac{r/a}{ka} = 0.566$
$ka = 6$, $r/a = 2.353$, $\frac{r/a}{ka} = 0.392$
$ka = 12$, $r/a = 4.049$, $\frac{r/a}{ka} = 0.337$

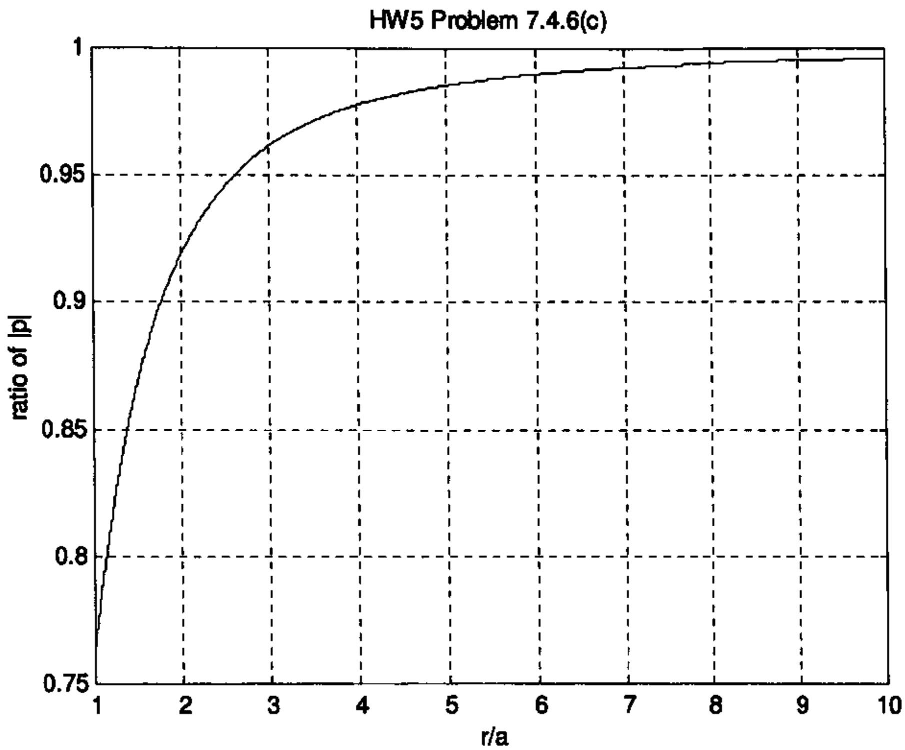
(c) $a = 0.2\,m$, $f = 4000\,Hz$, $c = 1481\,m/s$

$ka = 3.394$, $r/a = 1.767$, $\frac{r/a}{ka} = 0.521$

$r = 0.353\,m$


HW5 Problem 7.4.6(a)
|p|
r/a
ka=3
ka=6
ka=12
900
800
700
600
500
400
300
200
100
0
0
1
2
3
4
5
6
7
8
9
10


HW5 Problem 7.4.6(b)
ratio of |p|
r/a
ka=3
ka=6
ka=12
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
1
2
3
4
5
6
7
8
9
10


HW5 Problem 7.4.6(c)
ratio of |p|
r/a
1
0.95
0.9
0.85
0.8
0.75
1
2
3
4
5
6
7
8
9
10

7.5.1

(a). In the limit $ka \gg 1$. the radiation impedance is

$$Z_r \to R_r \simeq S \rho_0 c.$$

The resonance is reached when

$$\mathrm{Imag}\{Z_m + Z_r\} = 0$$

$$\Rightarrow \omega_0 m - s/\omega_0 = 0$$

$$\Rightarrow f_0 = \frac{1}{2\pi}\sqrt{s/m}.$$

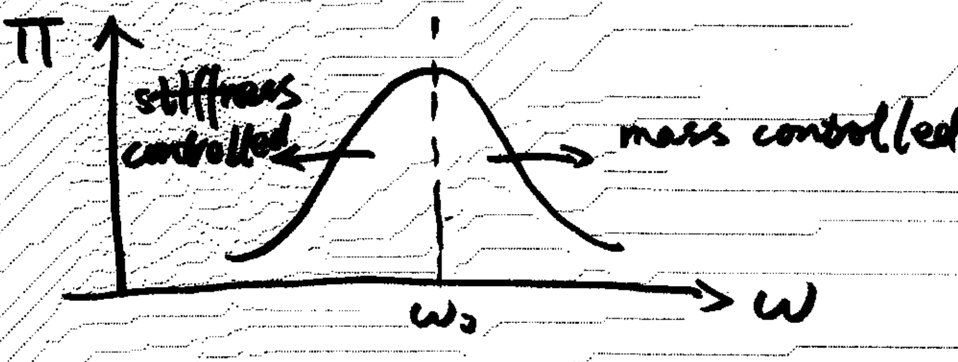
(b). With $ka \gg 2$.

From Eqn. (7.5.16)

$$\Pi \simeq \frac{1}{2}\rho_0 c S U_0^2$$

$$|U_0| = \left|\frac{f}{Z_m + Z_r}\right| = \frac{F}{\sqrt{(R_m + S\rho_0 c)^2 + (\omega m - s/\omega)^2}}$$

$$\therefore \Pi \simeq \frac{1}{2}\rho_0 c S \frac{F^2}{(R_m + S\rho_0 c)^2 + (\omega m - s/\omega)^2}$$



Stiffness controlled when

$$S/\omega \gg \omega m$$

$$\& \quad S/\omega \gg R_m + S\rho_0 c.$$

Mass controlled when:

$$\omega m \gg S/\omega$$

$$\& \quad \omega m \gg R_m + S\rho_0 c.$$