MR 513

PS # 4 solution

6.2.2.

From Eqn. 6.2.8.

10 $$R = \frac{r_2 - r_1}{r_2 + r_1} = \frac{r_2/r_1 - 1}{r_2/r_1 + 1}$$

$$20\log_{10}|R| = 20 \Rightarrow R = \pm 0.1$$

$$\therefore \frac{r_2/r_1 - 1}{r_2/r_1 + 1} = \pm 0.1$$

$$\therefore \frac{r_2}{r_1} = \frac{11}{9}, \quad \frac{r_2}{r_1} = \frac{9}{11}.$$

With $r_1 = (\rho c)_{water}$ = 1.54x10^6 (Pa.S/m)

The specific acoustic impedance of the fluid bottom material is:

$$\frac{11}{9}\rho c \text{ or } \frac{9}{11}\rho c.$$

= 1.88x10^6 or 1.26x10^6 (Pa.S/m)

6.2.6.C.

10 $R = \dfrac{\gamma_2/\gamma_1 - 1}{\gamma_2/\gamma_1 + 1}$. $T = \dfrac{2\gamma_2/\gamma_1}{\gamma_2/\gamma_1 + 1}$

$R_I = \left(\dfrac{\gamma_2/\gamma_1 - 1}{\gamma_2/\gamma_1 + 1}\right)^2$. $T_I = \dfrac{4\gamma_2/\gamma_1}{(\gamma_2/\gamma_1 + 1)^2}$.

R, T, R_I, T_I as a function of γ_1/γ_2 are plotted on next page.

For $\gamma_1/\gamma_2 = 0$.

$R = 1$. $T = 2$. $R_I = 1$. $T_I = 0$.

For $\gamma_1/\gamma_2 = 1$.

$R = 0$. $T = 1$. $R_I = 0$. $T_I = 1$.

For $\gamma_1/\gamma_2 \to \infty$

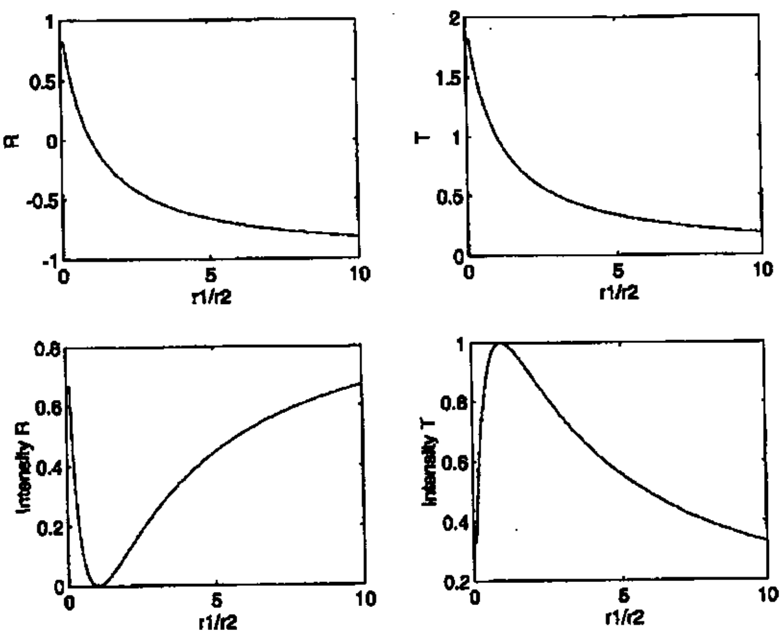
$R = -1$. $T = 0$. $R_I = 1$. $T_I = 0$.

Comments: when a wave in a small impedance media incident into a high impedance media, the wave is totally reflected and the reflected wave is in phase with the incident wave at the boundary.

On the other limit, when $r_1/r_2 \to \infty$, the wave energy is also totally reflected while the reflected wave is out of phase this time with the incident wave at the boundary.

When the two media have the same impedance, the wave propagates as if in a one media. the wave is totally transmitted.

6.2.6C



6.3.4.

10 Eqn. (6.3.8).

$$T_I = \frac{4}{2 + (r_3/r_1 + r_1/r_3)\cos^2 k_2 L + (r_2^2/r_1 r_3 + \frac{r_1 r_3}{r_2^2})\sin^2 k_2 L}$$

(a). with r_2 the only variable. the optimum transmission is reached when:

$$r_2 = \sqrt{r_1 r_3}.$$

(b). with $r_2^2 = r_1 r_3$. (see page 155)

when $k_2 L = (n - \frac{1}{2})\pi$, then

$$\cos^2 k_2 L = 0, \quad \sin^2 k_2 L = 1$$

$$\Rightarrow T_I = \frac{4}{2+2} = 1$$

$$\therefore c = \frac{2\pi f L}{(n-\frac{1}{2})\pi} = \frac{2\pi \cdot 20\times10^3 \times 1\times10^{-2}}{(n-\frac{1}{2})\cdot\pi}.$$

$$\rho = \frac{r_2}{c} = \frac{\sqrt{r_1 r_3}}{c}.$$

From page 527. & 526.

$r_1 = 1.48\times10^6$. $r_3 = 47.0\times10^6$.

$$\therefore c = \frac{400}{(n-\frac{1}{2})}. \quad \rho = 20851\times(n-\tfrac{1}{2}).$$

$n = 1, 2, \cdots$

6.4.1.

10 (a). $C_2 = C_1$, $\rho_2 > \rho_1$.

Since $C_2 = C_1$, $\theta_i = \theta_t$, the critical angle doesn't exist. R ~~is~~ doesn't change with the incident angle.

(b). $C_2 > C_1$, $\rho_2 = \rho_1$.

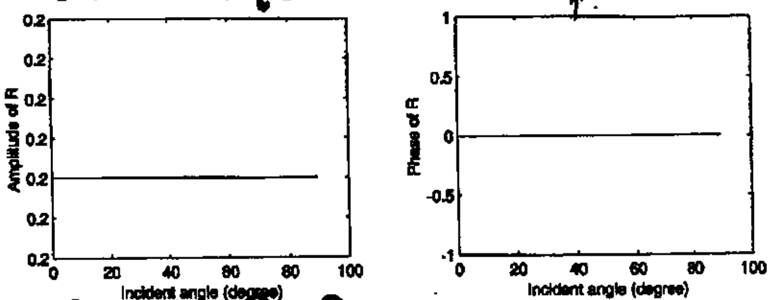
There exists an angle of total reflection.

$|R|$ increases as the incident angle increases; ~~at~~ when $\theta_i = 40°$, $|R| \to 1$, the wave is totally reflected. when $\theta_i > 40°$, $|R| = 1$, there's a phase difference between P_i and P_t at the boundary. As $\theta_i \to 90°$, P_i and P_t is 180° out of phase, $R = -1$.

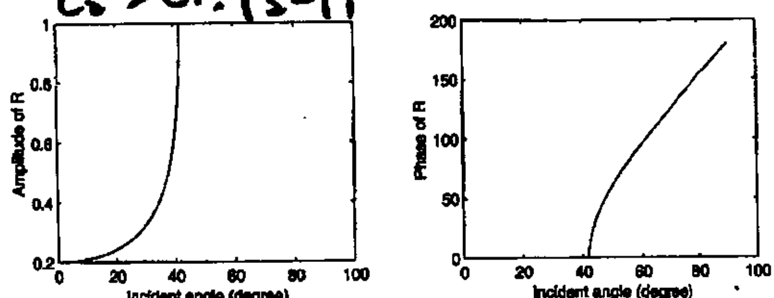
6.4.1

Comment: The figure 6.4.4 on the book is not correct. the phase angle should be greater than zero. if we choose $\cos\theta_t = -j[\;\;]^{1/2}$.

$C_2 = C_1,\ \rho_2 > \rho_1$



$C_2 > C_1,\ \rho_2 = \rho_1$



$$R = \frac{r_2/\cos\theta_t - r_1/\cos\theta_i}{r_2/\cos\theta_t + r_1/\cos\theta_i}$$

$$\cos\theta_t = \begin{cases} [1-(C_2/C_1)^2\sin^2\theta_i]^{1/2}; & \theta_i \le \theta_c \\ -j[(C_2/C_1)^2\sin^2\theta_i - 1]^{1/2}; & \theta_i > \theta_c \end{cases}$$

10 6.6.1.

Eqn. (6.6.5).

$$R = \frac{(r_n - r_1/\cos\theta_i) + jX_n}{(r_n + r_1/\cos\theta_i) + jX_n}. \qquad r_n = 900$$

$$X_n = 1200$$

$$R_I = \frac{(r_n - r_1/\cos\theta_i)^2 + X_n^2}{(r_n + r_1/\cos\theta_i)^2 + X_n^2}. \qquad r_1 = 415$$

(a). Set $\frac{dR_I}{d(r_1/\cos\theta_i)} = 0$.

$$\Rightarrow 4r_n(r_n^2 - r_1^2/\cos^2\theta_i + X_n^2) = 0.$$

$\cancel{\Rightarrow r_1^2}$

$\therefore$ when $r_1^2 = (r_n^2 + X_n^2)\cos^2\theta_i$ the power reflection coefficient is minimum.

$$\Rightarrow \theta_i = \arccos\left[\frac{r_1^2}{r_n^2 + X_n^2}\right]^{1/2}$$

$$= 73.9^\circ.$$

(b). $\theta_i = 80^\circ$.

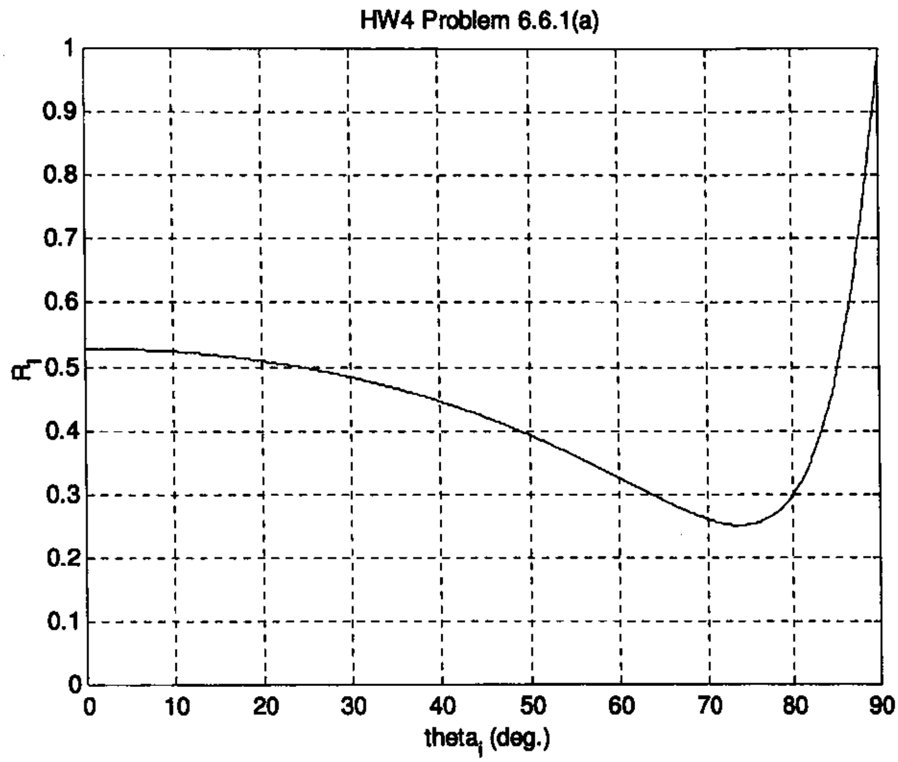
$$R_I = \frac{(900 - 415/\cos 80^\circ)^2 + 1200^2}{(900 + 415/\cos 80^\circ)^2 + 1200^2}$$

$= 0.30$.

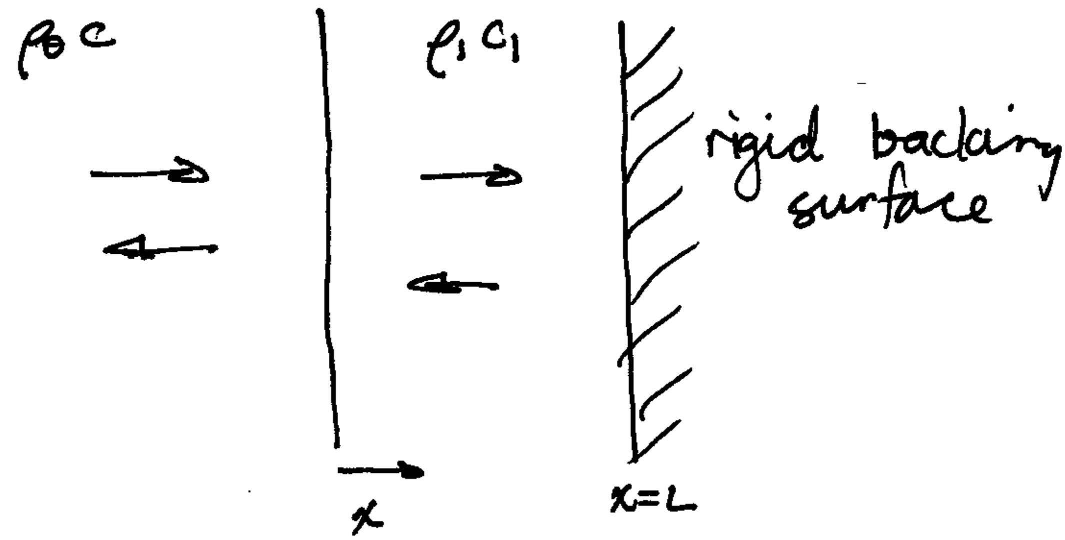
(c). $\theta_i = 0^\circ$.

$$R_I = \frac{(900-415)^2 + 1200^2}{(900+415)^2 + 1200^2}$$

$= 0.53$


HW4 Problem 6.6.1(a)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_i
0
10
20
30
40
50
60
70
80
90
$theta_i$ (deg.)

(10) <u>Additional Problem</u>



(i) $p_1 = A e^{-jk_1 x} + B e^{jk_1 x}$

$u_1 = \frac{1}{\rho_1 c_1}\left(A e^{-jk_1 x} - B e^{jk_1 x}\right)$

Apply rigid surface b.c. at $x=L$ (i.e., $u_1|_{x=L} = 0$)

$$\frac{1}{\rho_1 c_1}\left(A e^{-jk_1 L} - B e^{jk_1 L}\right) = 0$$

$$\therefore \; B = A e^{-2jk_1 L}$$

$$\therefore \; p_1 = A\left(e^{-jk_1 x} + e^{-2jk_1 L} e^{+jk_1 x}\right)$$

$$= A e^{-jk_1 L}\left(e^{+jk_1(L-x)} + e^{-jk_1(L-x)}\right)$$

$$= 2Ae^{-jk_1L}\cos k_1(L-x)$$

$$u_1 = \frac{1}{\rho_1 c_1} A\left(e^{-jk_1x} - e^{-2jk_1L}e^{+jk_1x}\right)$$

$$= \frac{Ae^{-jk_1L}}{\rho_1 c_1}\left(e^{+jk_1(L-x)} - e^{-jk_1(L-x)}\right)$$

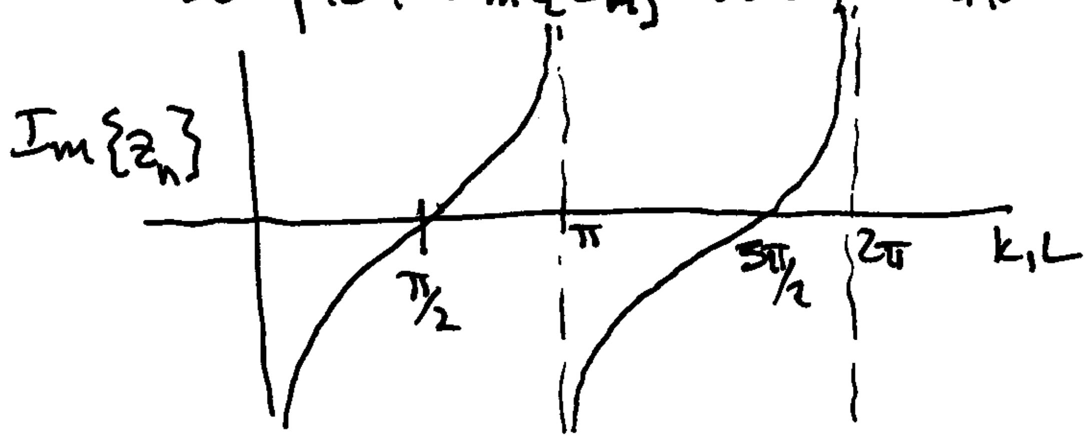
$$= 2j\frac{A}{\rho_1 c_1}e^{-jk_1L}\sin k_1(L-x)$$

Then

$$z_n = \left.\frac{p_1}{u_1}\right|_{x=0} = \frac{\cancel{2A e^{-jk_1L}}\cos k_1(L\cancel{-x})}{\cancel{2j\frac{A}{\rho_1 c_1}e^{-jk_1L}}\sin k_1(L\cancel{-x})}$$

$$= -j\rho_1 c_1 \cot k_1(L)$$

(ii) Impedance is purely imaginary, so plot $\mathrm{Im}\{z_n\}$ as function of k_1L



(iii) $$R = \frac{z_n - \rho_0 c}{z_n + \rho_0 c}$$

$$= \frac{-j\rho_1 c_1 \cot k_1 L - \rho_0 c}{-j\rho_1 c_1 \cot k_1 L + \rho_0 c}$$

$$= \left[(-1) \frac{(\rho_0 c + j\rho_1 c_1 \cot k_1 L)}{(\rho_0 c - j\rho_1 c_1 \cot k_1 L)} \right]$$

since numerator and denominator are complex conjugates, their magnitude is the same

$$\therefore |R| = 1$$