ME 513 Homework #3 Solution

#1 $$\tilde{p}(x) = e^{-jkx} + 0.8e^{+jkx}$$

(i) linearized Euler Eqn - harmonic case

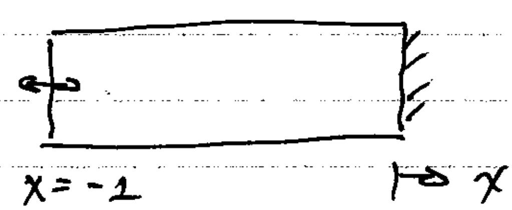
$$\tilde{u}(x) = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dx}$$

$$= -\frac{1}{j\omega\rho_0}\left(-jke^{-jkx} + jk0.8e^{+jkx}\right)$$

$$= \frac{k}{\omega\rho_0}\left(e^{-jkx} - 0.8e^{+jkx}\right)$$

$k = \frac{\omega}{c}$

$$= \frac{1}{\rho_0 c}\left(e^{-jkx} - 0.8e^{+jkx}\right)$$



velocity at piston - evaluate $\tilde{u}(-1)$

$$\tilde{u}(-1) = \frac{1}{\rho_0 c}\left(e^{+jk} - 0.8e^{-jk}\right)$$

(ii) $I = \frac{1}{2} Re\{\hat{p}(x)\, \hat{u}^*(x)\}$

$$= \frac{1}{2} Re\left\{\left(e^{-jkx} + 0.8e^{+jkx}\right)\left(\frac{e^{+jkx} - 0.8e^{-jkx}}{\rho_0 c}\right)\right\}$$

$$= \frac{1}{2\rho_0 c} Re\left\{1 - 0.8e^{-2jkx} + 0.8e^{+2jkx} - 0.64\right\}$$

$$= \frac{1}{2\rho_0 c} Re\left\{0.36 + 1.6j\sin 2kx\right\}$$

$$= \frac{0.18}{\rho_0 c} \simeq 0.43 \ W/m^2$$ not a function of x

The sound power delivered to the tube

$$W = IS = 0.43 \times 0.01 = 0.0043 \text{ watts}$$

a(iii) at absorbing surface

$$z\big|_{x=0} = \frac{\tilde{p}(0)}{\tilde{u}(0)} = \rho_0 c \, \frac{1+0.8}{1-0.8}$$

$$= 9.0(415)$$

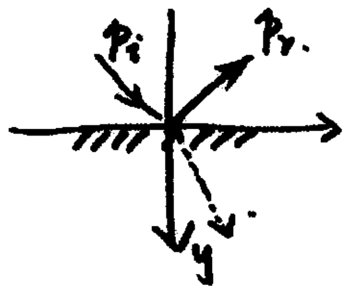
$$= 3735 \quad \text{Rayls}$$

at the piston

$$z\big|_{x=-1} = \frac{\tilde{p}(-1)}{\tilde{u}(-1)} = \rho_0 c \, \frac{\left(e^{+jk} + 0.8 e^{-jk}\right)}{\left(e^{+jk} - 0.8 e^{-jk}\right)}$$

#2.

(i).



$$p = p_i + p_r$$
$$= e^{-jk_x x - jk_y y} + R e^{-jk_x x + jk_y y}.$$

where: $k_x = k\sin\theta$, $k_y = k\cos\theta$.

$k = \frac{\omega}{c} = \frac{2\pi f}{c}$

f: frequency of the wave.

(ii). Euler's equation:

$$\underset{\frown}{u} = -\frac{1}{j\omega\rho_0}\nabla p.$$

$$\therefore u_y = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial y}$$
$$= -\frac{1}{j\omega\rho_0}\left[-jk_y e^{-jk_x x - jk_y y} + jk_y R e^{-jk_x x + jk_y y}\right].$$

$$= \frac{\cos\theta}{\rho_0 c}\left[e^{-jk_x x - jk_y y} - Re^{-jk_x x + jk_y y}\right].$$

(iii) Time-averaged sound power per unit area into the surface $y=0$ is:

$$\frac{1}{2}\mathrm{Re}[p \cdot u_y^*]\Big|_{y=0}$$

$$= \frac{1}{2}\mathrm{Re}\Big[(e^{-jk_x x - jk_y y} + Re^{-jk_x x + jk_y y})$$

$$\cdot \frac{\cos\theta}{\rho_0 c}(e^{jk_x x + jk_y y} - R^* e^{jk_x x - jk_y y})\Big]\Big|_{y=0}$$

$$= \frac{1}{2}\mathrm{Re}\Big[\frac{\cos\theta}{\rho_0 c}(1 - Re^{-2jk_y y} + Re^{2jk_y y}$$

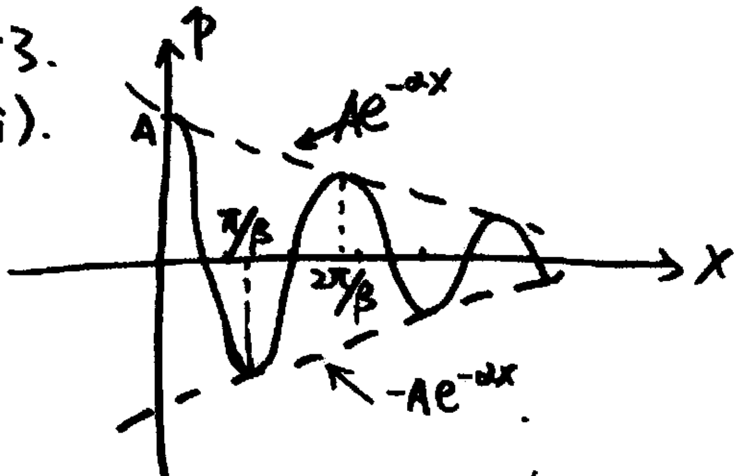
$$- |R|^2)\Big]\Big|_{y=0}$$

$$= \frac{\cos\theta}{2\rho_0 c}\mathrm{Re}\left[1 - |R|^2\right]$$

$$= \frac{\cos\theta}{2\rho_0 c}\left[1 - |R|^2\right].$$

#3.

(i).



α: attenuation factor

β: wavenumber

$\lambda = \frac{2\pi}{\beta}$.

(ii). $u_x = -\frac{1}{j\omega\rho_0}\frac{dp}{dx}$

$$= -\frac{1}{j\omega\rho_0}\left[Ae^{-\alpha x}(-j\beta)e^{-j\beta x} - \alpha A e^{-\alpha x}e^{-j\beta x}\right]e^{j\omega t}$$

$$= \frac{\beta A}{\rho_0 \omega} e^{-\alpha x}e^{-j\beta x}e^{j\omega t}\left[1+\frac{\alpha}{j\beta}\right].$$

(iii)

$$I = \frac{1}{2}\mathrm{Re}[p\, u_x^*]$$

$$= \frac{1}{2}\mathrm{Re}\left[A e^{-\alpha x} e^{-j\beta x} e^{j\omega t} \cdot \frac{\beta A^*}{\rho_0 \omega} e^{-\alpha x} e^{j\beta x} e^{-j\omega t} \cdot \left(1 - \frac{\alpha}{j\beta}\right)\right]$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{\beta |A|^2}{\rho_0 \omega} e^{-2\alpha x}\left(1 - \frac{\alpha}{j\beta}\right)\right]$$

$$= \frac{\beta |A|^2}{2\rho_0 \omega} e^{-2\alpha x}.$$

Intensity varies with position.

#4

(i). Euler's Equation:

$$\underset{\sim}{u} = -\frac{1}{j\omega\rho_0}\underset{\sim}{\nabla} p.$$

$$= -\frac{1}{j\omega\rho_0}\left[\frac{\partial p}{\partial r}\vec{r} + \frac{1}{r}\frac{\partial p}{\partial \theta}\hat{\theta} + \frac{\partial p}{\partial z}\hat{z}\right].$$

$$\frac{\partial p}{\partial r} = -\frac{A}{2}r^{-3/2}\sin\theta e^{-jkr} - jkr^{-1/2}\sin\theta e^{-jkr}A$$

$$= -\frac{A}{2}r^{-1/2}\sin\theta e^{-jkr}\left(\frac{1}{r} + jk\right)$$

$$\frac{\partial p}{\partial \theta} = \frac{A}{r^{1/2}}\cos\theta e^{-jkr}.$$

$$\frac{\partial p}{\partial z} = 0.$$

$$\therefore \underset{\sim}{u} = -\frac{1}{j\omega\rho_0}\left[-\frac{A}{2}\frac{1}{r^{1/2}}\sin\theta e^{-jkr}\left(\frac{1}{r} + 2jk\right)\vec{r} + \frac{A}{r^{3/2}}\cos\theta e^{-jkr}\vec{\theta}\right]$$

$$= \frac{1}{\rho_0 c}\frac{A}{r^{1/2}}\sin\theta e^{-jkr}\left(1 + \frac{1}{2jkr}\right)\vec{r} + \frac{j}{\omega\rho_0}\frac{A}{r^{3/2}}\cos\theta e^{-jkr}\vec{\theta}.$$

(ii)

$$I_r = \frac{1}{2} Re[p\, u_r^*]$$

$$= \frac{1}{2} Re\left[\frac{A}{r^{1/2}} \sin\theta\, e^{-jkr} \cdot \frac{A^*}{\rho_0 c} \frac{\sin\theta}{r^{1/2}} e^{jkr} \cdot \left(1 - \frac{1}{2jkr}\right)\right]$$

$$= \frac{|A|^2 \sin^2\theta}{2\rho_0 c r}$$

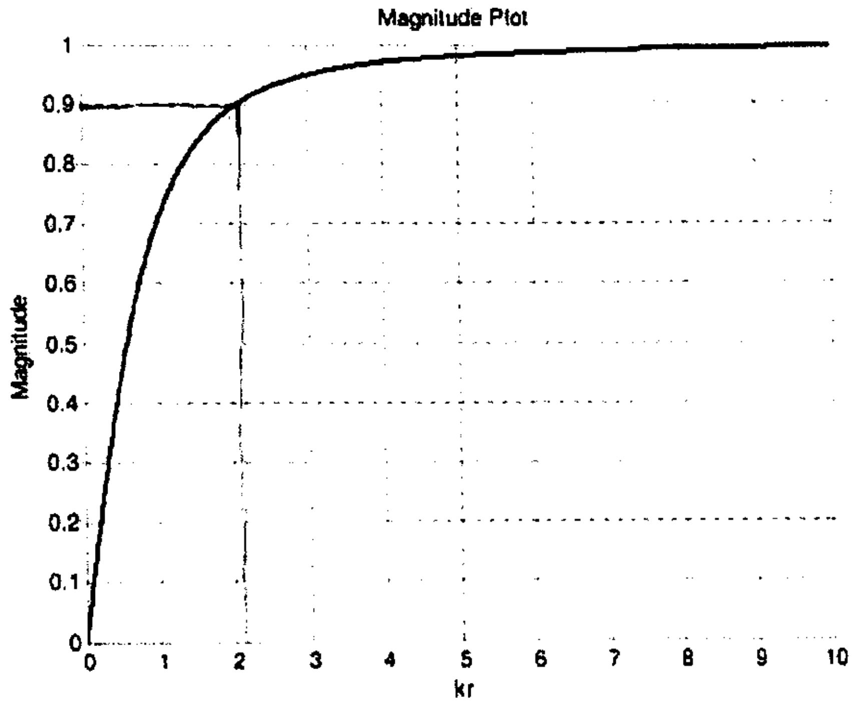
5.11.6C

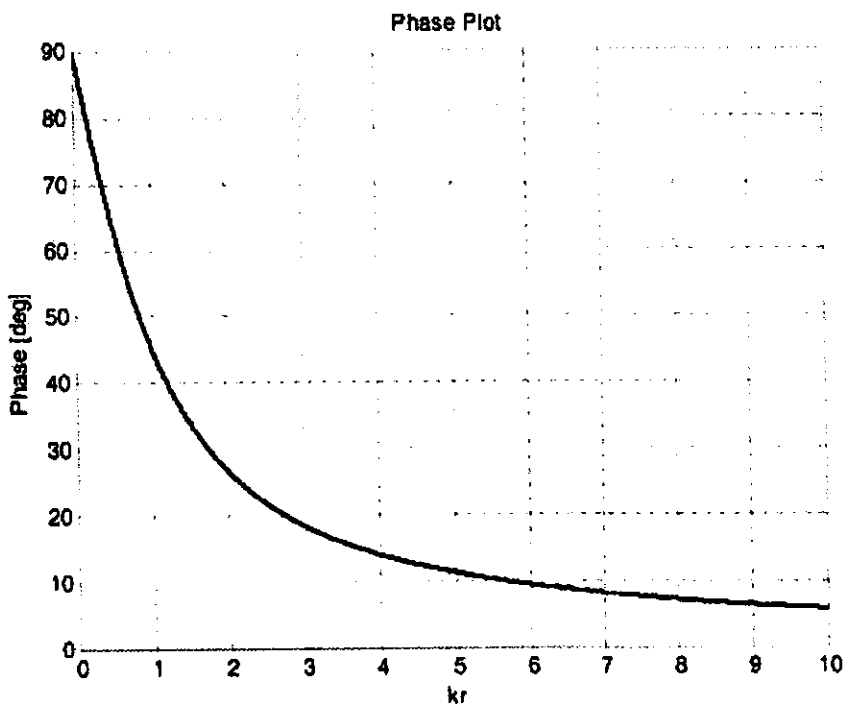
specific acoustic impedance of outward-going spherical wave

$$z = \rho_0 c \frac{kr}{[1+(kr)^2]^{1/2}} e^{j\theta}$$

$$|z| = \rho_0 c \frac{kr}{[1+(kr)^2]^{1/2}} \rightarrow \text{plot } \frac{|z|}{\rho_0 c} = \frac{kr}{[1+(kr)^2]^{1/2}}$$

$$\angle z = \theta = \cot^{-1}(kr)$$


Magnitude Plot
Magnitude
kr
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9
10


Phase Plot
Phase [deg]
kr
90
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
8
9
10

5.12.2

$$L_p = 10 \log \frac{p_{rms}^2}{p_{ref}^2} \qquad p_{ref} = 20\mu P$$

$L_p = 40$ dB re $20\mu Pa$

(a) $p_{rms}^2 = p_{ref}^2 \, 10^{40/10}$

$$= 4 \times 10^{-10} \, 10^4 = 4 \times 10^{-6}$$

$$p_{rms} = 2 \times 10^{-3} \text{ Pa}$$

$$|p| = 1.41 \times 2 \times 10^{-3}$$

$$= 2.82 \times 10^{-3} \text{ Pa}$$

(b) $I = \dfrac{p_{rms}^2}{\rho_0 c} = \dfrac{4 \times 10^{-6}}{4.15 \times 10^2} = 0.964 \times 10^{-8} \dfrac{W}{m^2}$

(c) $\dfrac{p}{u} = \rho_0 c \rightarrow |u| = \dfrac{|p|}{\rho_0 c}$

$$= \frac{2.82 \times 10^{-3}}{4.15 \times 10^2}$$

$$= 0.68 \times 10^{-5} \text{ m/s}$$

(d) $p = \beta s$ $\qquad \beta = \gamma P_0 = 1.4\times10^5$ Pa

$$|s| = \frac{|p|}{\beta} = \frac{2.82\times10^{-3}}{1.4\times10^5}$$

$$= \underline{\underline{2\times10^{-8}}}$$

Answer to (f)

$$p = \beta s$$

$$= \beta\left(\frac{\rho-\rho_0}{\rho_0}\right)$$

Acoustic density

$$= (\rho-\rho_0) = \frac{\rho_0 p}{\beta}$$

$$|\rho-\rho_0| = \frac{\rho_0 |p|}{\beta} = \frac{1.2\times2.82\times10^{-3}}{1.4\times10^5}$$

$$= 2.4\times10^{-8} \ \text{kg/m}^3$$

(e) $u = j\omega\xi$ $\quad\quad \xi$ = particle displacement

$$|\xi| = \frac{|u|}{\omega} = \frac{0.68 \times 10^{-5}}{2\pi(171)}$$

$$= \frac{6.8 \times 10^{-6}}{(2\pi)(1.71 \times 10^{2})}$$

$$= 0.63 \times 10^{-8} \text{ m}$$

5.12.3

(a) $I = \frac{|p|^2}{2\rho_0 c} = \frac{4}{2\rho_0 c} = 0.0048 \; \cancel{\text{watt}} \; W/m^2$

$L_I = 10 \log \frac{I}{I_{ref}} = \underline{96.81}$ dB re 1×10^{-12} W/m²

$I_{ref} = 1 \times 10^{-12}$ W/m²

(b) ~~m~~ for propagating plane wave

$$\frac{P}{u} = \rho_0 c \Rightarrow u = \frac{P}{\rho_0 c}$$

and $u = j\omega \xi$ where ξ = particle displacement

$$u = \frac{A e^{-jkx}}{\rho_0 c}$$

$$\xi = \frac{1}{j\omega} \frac{A}{\rho_0 c} e^{-jkx}$$

$$|\xi| = \frac{|A|}{\omega \rho_0 c} = \frac{2}{2\pi(100)415} = \underline{7.67 \times 10^{-6}} \text{ m}$$

(c) $u = \frac{A e^{-jkx}}{\rho_0 c} =$

$$|u| = \frac{|A|}{\rho_0 c} = \frac{2}{415} = \underline{0.0048} \text{ m/s}$$

(d) $P_e = \frac{|p|}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \underline{1.4142}$ Pa

(e) $L_p = 10 \log \frac{P_e^2}{P_{ref}^2} = 10 \log \frac{1.4142^2}{4 \times 10^{-10}}$ dB re $20 \mu Pa$

$= 96.9896$ dB re $20 \mu Pa$