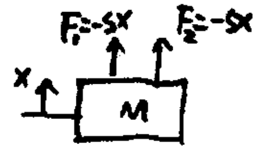
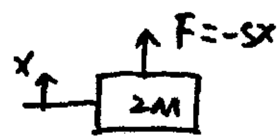
1.2.1

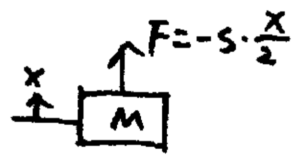
(a)
$$M\frac{d^2X}{dt^2} = -SX - SX$$
$$M\frac{d^2X}{dt^2} + 2SX = 0$$
$$\Rightarrow \omega_0 = \sqrt{\frac{2S}{M}} \Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{2S}{M}}$$



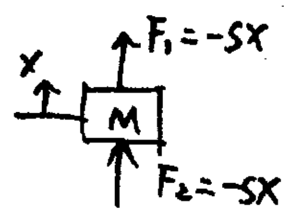
(b)
$$(2M)\frac{d^2X}{dt^2} = -SX$$
$$2M\frac{d^2X}{dt^2} + SX = 0$$
$$\Rightarrow \omega_0 = \sqrt{\frac{S}{2M}} \Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{S}{2M}}$$



(c)
$$M\frac{d^2X}{dt^2} = -S\cdot\left(\frac{X}{2}\right)$$
$$M\frac{d^2X}{dt^2} + \frac{S}{2}X = 0$$
$$\Rightarrow \omega_0 = \sqrt{\frac{S}{2M}} \Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{S}{2M}}$$



(d)
$$M\frac{d^2X}{dt^2} = -SX - SX$$
$$M\frac{d^2X}{dt^2} + 2SX = 0$$
$$\Rightarrow \omega_0 = \sqrt{\frac{2S}{M}} \Rightarrow f_0 = \frac{1}{2\pi}\sqrt{\frac{2S}{M}}$$



1.3.2 $\omega_0 = 5$ rad/s $\quad x_0 = 0.03$ m

$u_0 = 0$

simple oscillator

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0$$

General Solution

$$x(t) = A_1 \cos\omega_0 t + A_2 \sin\omega_0 t$$

$$x(0) = A_1 = 0.03$$

$$\dot{x}(t) = -\omega_0 A_1 \sin\omega_0 t + \omega_0 A_2 \cos\omega_0 t$$

$$\ddot{x}(t) = -\omega_0^2 A_1 \cos\omega_0 t - \omega_0^2 A_2 \sin\omega_0 t$$

(a) $\rightarrow \ddot{x}(0) = -\omega_0^2 A_1 = -25(0.03) = -0.75\ m/s^2$

$$\dot{x}(0) = +\omega_0 A_2 = 0 \rightarrow A_2 = 0$$

so $x(t) = A_1 \cos\omega_0 t = 0.03 \cos\omega_0 t$

(b) Amplitude $= 0.03$ m

velocity $\dot{x}(t) = -\omega_0 A_1 \sin\omega_0 t$

(c) so max speed $= \omega_0 A_1 = 0.15$ m/s

1.5.1 $\tilde{x} = \tilde{A} e^{j\omega t}$ $\qquad \tilde{A} = a + ib$

$= |\tilde{A}| e^{j\phi} e^{j\omega t}$ $\qquad \phi = \tan^{-1}\left(\frac{b}{a}\right)$

$= |\hat{A}| e^{j(\omega t + \phi)}$

$= |\hat{A}| \left(\cos(\omega t + \phi) + j \sin(\omega t + \phi)\right)$

let $|\hat{A}| = A$

$$Re\{\tilde{x}\} = A\cos(\omega t + \phi) = x$$

$$\text{so} \quad x^2 = A^2 \cos^2(\omega t + \phi)$$

$$\tilde{x} = A e^{j(\omega t + \phi)}$$

$$\tilde{x}^2 = A^2 e^{2j(\omega t + \phi)}$$

$$= A^2\left(\cos 2(\omega t + \phi) + j \sin 2(\omega t + \phi)\right)$$

$$Re\{\tilde{x}^2\} = A^2 \cos 2(\omega t + \phi) \neq A^2 \cos^2(\omega t + \phi)$$

$$\therefore x^2 \neq Re\{\tilde{x}^2\}$$

1.5.3.

(a). $\mathrm{Re}\{\hat{A}\hat{B}\} = \mathrm{Re}\{AB\exp[j(2\omega t+\theta+\phi)]\}$
$= AB\cos(2\omega t+\theta+\phi)$

(b). $\mathrm{Re}\{\tilde{A}/\hat{B}\} = \mathrm{Re}\{\frac{A}{B}\exp[j(\theta-\phi)]\}$
$= \frac{A}{B}\cos(\theta-\phi).$

(c). $\mathrm{Re}\{\hat{A}\}\cdot\mathrm{Re}\{\hat{B}\} = A\cos(\omega t+\theta)\cdot B\cos(\omega t+\phi)$
$= AB\cos(\omega t+\theta)\cos(\omega t+\phi).$

(d) $\mathrm{Phase}\{\hat{A}\tilde{B}\} = \mathrm{Phase}\{AB\exp[j(2\omega t+\theta+\phi)]\}$
$= 2\omega t+\theta+\phi$

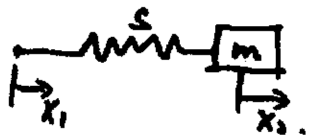
(e). $\mathrm{Phase}\{\tilde{A}/\hat{B}\} = \mathrm{Phase}\{\frac{A}{B}\exp[j(\theta-\phi)]\}$
$= \theta-\phi.$

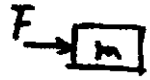
1.7.5.

(a).

x_1: Displacement of the table.

x_2: Displacement of the mass.





restoring force eqn:

$$f = \cancel{-}s(x_2 - x_1)$$

Eqn of motion:

$$f = m\ddot{x}_2$$

So the governing Eqn:

$$m\ddot{x}_2 + s(x_2 - x_1) = 0. \qquad \text{— (1)}$$

Given: $\ddot{x}_1 = A\exp(j\omega t)$ (Consider steady state only).

$$\Rightarrow \ddot{x}_1 = +\frac{A}{(j\omega)^2}\exp(j\omega t)$$

$$= -\frac{A}{\omega^2}\exp(j\omega t).$$

substitute into governing Eqn:

$$m\ddot{x}_2 + s x_2 = -\frac{As}{\omega^2}\exp(j\omega t). \qquad \text{— (2)}$$

Assume the solution is in the form:

$$x_2 = A_2\exp(j\omega t)$$

substitute into (2).

$$mA_2(-w^2)\cancel{\exp(jwt)} + sA_2\cancel{\exp(jwt)} = \frac{-As}{w^2}\cancel{\exp(jwt)}.$$

$$A_2 = \frac{-\frac{As}{w^2}}{s-mw^2} = \frac{\frac{As}{w^2}}{mw^2-s}$$

Thus: $\cancel{\ddot{x}_2 = \frac{A_2}{-w^2}\exp(jwt)}$

$$\ddot{x}_2 = (-w^2)A_2\exp(jwt)$$

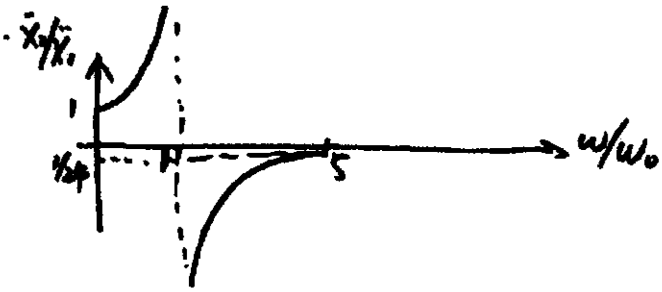
$$= \frac{As}{s-mw^2}\exp(jwt).$$

And: $$\frac{\ddot{x}_2}{\ddot{x}_1} = \frac{\frac{As}{s-mw^2}\exp(jwt)}{A\exp(jwt)} = \frac{s}{s-mw^2}$$

$$= \frac{1}{(1-w^2\frac{m}{s})} = \frac{1}{1-(w/w_0)^2}$$

with: $w_0^2 = s/m$.

(b).



(c). The system acts as a good vibration isolator at high frequency when $w/w_0 \gg 1$.