Name: ______________________________

ME 513 --- Engineering Acoustics

Exam 1 – Fall 2015 --- 11/06/2015

Note: To help you complete this exam, you may refer to your class notes, your homework, solutions provided to you and other material distributed as part of the course either in-class or *via* the course website, but you may not refer to the text (Kinsler, Frey, Coppens and Sanders) or to any other acoustics text

- Problem 1: ___________/20
- Problem 2: ___________/20
- Problem 3: ___________/20

Problem 1.

(i) What is sound?

Small amplitude propagating pressure fluctuation in an elastic medium

(ii) For free oscillations to occur, a physical system must have both ___stiffness___ and ___mass___.

(iii) The motion of the mass in a SDOF system is governed by a second order, ordinary differential equation. As a result, the solution features ___two arbitrary constants___ that must be determined by application of initial conditions.

(iv) The time phasor $e^{-j\omega t}$ can be represented in the complex plane by a vector that rotates in the ___clockwise___ direction.

(v) What are the characteristics of a stiffness-like mechanical impedance?

- negative, imaginary and inversely proportional to frequency

(vi) What physical property of a SDOF system controls its forced response at frequencies well above the resonance frequency?

Its mass

(vii) The wave number can also be referred to as

The spatial frequency.

(viii) The restoring force acting on a segment of a tensioned string is not proportional to the slope of the segment: instead, it is proportional to

curvature.

(ix) In the calculation of sound pressure levels, why was the reference root mean square sound pressure chosen to be 2×10^{-5} Pa?

Lowest sound pressure audible by healthy young adult. As a result sound pressure levels for audible sounds are positive

(x) Why is a spherically-symmetric wave expanding into free space referred to as a one-dimensional wave?

The sound pressure depends only on radius

Problem 2.

A uniform, tensioned string (tension, T, and mass per unit length, ρ_L) is rigidly fixed at $x = 0$ and is terminated by a point mass (of mass, m) at $x = L$.

(i) Give an appropriate assumed solution for the transverse displacement of the string. Define quantities as necessary.

(ii) Draw a free body diagram of the forces acting at the string termination at $x = L$.

(iii) Give in equation form the boundary conditions that apply at $x = 0$ and $x = L$.

(iv) Use the boundary conditions in conjunction with the assumed solution to derive the transcendental characteristic equation that can be solved for the allowed wave numbers. Make sure that both terms in this equation are expressed in terms of (kL); also, choose to express the characteristic function in terms of cot rather than tan – finally, you should manipulate the results so that the frequency does not appear explicitly in the characteristic equation`.

(v) Sketch the solution of the characteristic equation. Clearly indicate the solution points on the sketch.

(vi) Show with reference to the sketch how this case can be made to approach the case of a rigid termination at $x = L$.

Problem 2

(i) $y = Ae^{-jkx}e^{j\omega t} + Be^{+jkx}e^{j\omega t}$

$$k = \frac{\omega}{c} \qquad c = \sqrt{T/\rho}$$

(ii)

(iii) at $x = 0$ $\quad y = 0$

at $x = L$ $\quad \sum f_y = ma$

$$T\sin\theta\Big|_{x=L} = m\frac{\partial^2 y}{\partial t^2}\Big|_{x=L}$$

$$-T\frac{\partial y}{\partial x}\Big|_{x=L} = m\frac{\partial^2 y}{\partial t^2}\Big|_{x=L}$$

(iv) Apply b.c. at $x = 0$

$$y(0) = 0 = Ae^{j\omega t} + Be^{j\omega t}$$

$$B = -A$$

$$\therefore y(x,t) = A(e^{-jkx} - e^{+jkx})e^{j\omega t}$$

$$y(x,t) = -2jA \sin kx \, e^{j\omega t}$$

$$\frac{\partial y}{\partial x} = -2jkA \cos kx \, e^{j\omega t}$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=L} = -2jkA \cos kL \, e^{j\omega t}$$

$$\frac{\partial^2 y}{\partial t^2} = +2j\omega^2 A \sin kx \, e^{j\omega t}$$

$$\left.\frac{\partial^2 y}{\partial t^2}\right|_{x=L} = 2j\omega^2 A \sin kL \, e^{j\omega t}$$

$$\therefore \; -\cancel{2}jkT\cancel{A} \cos kL \, \cancel{e^{j\omega t}} = \cancel{2}j\omega^2 m \cancel{A} \sin kL \, \cancel{e^{j\omega t}}$$

$$-kT \cos kL = \omega^2 m \sin kL$$

$$-\frac{kT}{\omega^2 m} = \tan kL$$

$$k = \frac{\omega}{c}$$

$$\tan kL = -\frac{k \rho_L \cancel{c^2}}{k^{\cancel{2}} \cancel{c^2} m}$$

$$\omega^2 = k^2 c^2$$

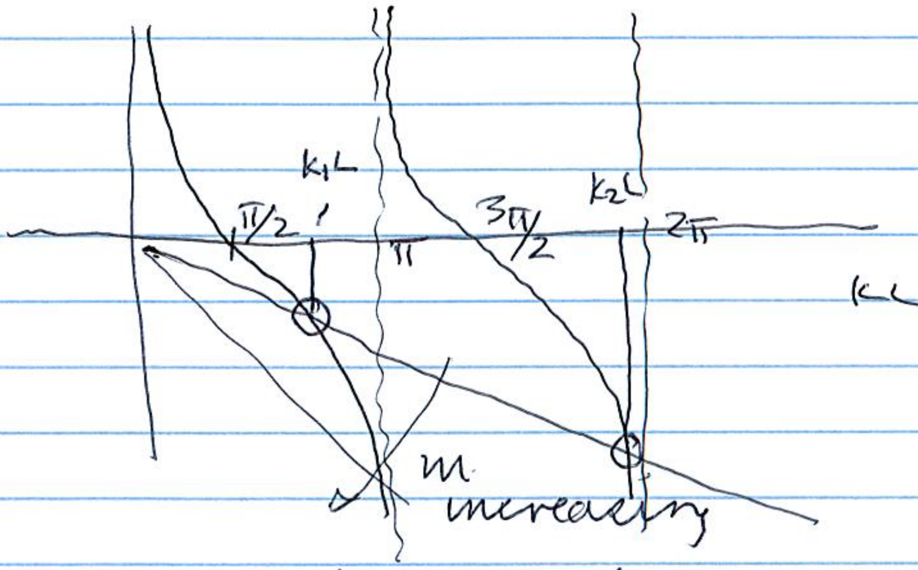
$$c = \sqrt{T/\rho_L}$$

$$\tan kL = -\frac{\rho_L L}{kL\, m} = -\left(\frac{\rho_L L}{m}\right)\frac{1}{kL}$$

$$c^2 = T/\rho_L \qquad T = c^2 \rho_L$$

$$-\left(\frac{m}{\rho_L L}\right) kL \approx \cot kL$$

(v)



(vi) As m is increased negative slope increases, so intersections points approach $k_nL = n\pi$, as for hard termination case

Problem 3.

A plane sound wave of complex amplitude A propagates towards an infinitely rigid surface at normal incidence. The surface is located at $z = 0$, and the plane wave approaches the surface from the region $z < 0$. The density of the acoustic medium is ρ_0, and the speed of sound in the acoustic medium is c.

When the incident wave hits the surface a reflected wave is created that has the same complex amplitude as the incident wave, but is, of course, propagating in the opposite direction.

(i) Give a complete expression for the sound pressure field in this case, defining quantities (such as the wave number, for example) as necessary. Please express the sound pressure field in the most compact form possible.

(ii) Derive by using the linearized momentum equation a corresponding expression for the particle velocity.

(iii) Calculate the time-averaged acoustic intensity flowing in the z-direction.

(iv) Find the location in front of the reflecting surface at which the magnitude of the particle velocity reaches its first maximum

(v) What is the value of the specific acoustic impedance at the location where the particle velocity reaches its first maximum

Problem 3

(i) $\tilde{p}(z,t) = A\left(e^{-jkz} + e^{+jkz}\right)e^{j\omega t}$

$= 2A \cos kz \, e^{j\omega t}$

(ii) $\tilde{u}_z = -\dfrac{1}{j\omega\rho_0}\dfrac{\partial\tilde{p}}{\partial z}$

$= -\dfrac{1}{j\omega\rho_0}\left(-2kA \sin kz\right)e^{j\omega t}$

$= \dfrac{2kA}{j\omega\rho_0} \sin kz \, e^{j\omega t}$

$= \dfrac{2A}{j\rho_0 c} \sin kz \, e^{j\omega t}$

(iii) $I_z = \frac{1}{2}\mathrm{Re}\{\tilde{p}\,\tilde{u}_z^*\}$

$= \frac{1}{2}\mathrm{Re}\left\{2A \cos kz \, e^{j\omega t}\left(\dfrac{2A^*}{-j\rho_0 c} \sin kz\right)e^{-j\omega t}\right\}$

$= \frac{1}{2}\mathrm{Re}\left\{\dfrac{4|A|^2 \cos kz \sin kz}{-j\rho_0 c}\right\}$

$= 0$

(iv) particle velocity is a maximum when $kz = -\frac{\pi}{2}$

$$z = -\frac{\pi}{2k} = -\frac{\pi\lambda}{2(2\pi)} = -\frac{\lambda}{4}$$

(v) What is the specific acoustic impedance at the ~~latter~~ location where the particle velocity is a maximum?

$$Z = \frac{\hat{p}}{u_z}\Bigg|_{z=-\frac{\lambda}{4}} = \frac{\cancel{p}\; 2A\cos(-\frac{\pi}{2})\,e^{j\omega t}}{\frac{2A}{j\rho_0 c}\sin(-\frac{\pi}{2})\,e^{j\omega t}}$$

$$= 0$$