Name: solution

ME 513 --- Engineering Acoustics

Exam 1 – Fall 2013 --- 10/23/2013

Note: To help you complete this exam, you may refer to your class notes, your homework, solutions provided to you and other material distributed as part of the course either in-class or *via* the course website, but you may not refer to the text (Kinsler, Frey, Coppens and Sanders) or to any other acoustics text

- Problem 1: ____________/20
- Problem 2: ____________/20
- Problem 3: ____________/20

Problem 1.

(i) What is sound?

A small amplitude pressure (or particle velocity or density) fluctuation propagating in an elastic medium.

(ii) What is the restoring force equation for a linear, SDOF system?

$$f = -sx - R_m \frac{dx}{dt}$$

(iii) An "allowed" solution of a SDOF is one that satisfies the governing equations, but which includes two constants that must be determined by application of initial conditions.

(iv) The real part of a complex solution corresponds to The physical solution.

(v) According to the principle of superposition, the response of a SDOF system when driven at two frequencies simultaneously is

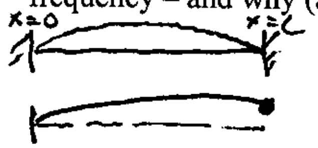
The sum of the responses to the two frequencies applied one at a time.

(vi) When driven at its natural frequency, the input mechanical impedance of a SDOF system is equal to:

$$Z_M = R_m$$

(vii) The wave number plays the same role with respect to space that the radian frequency does with respect to time.

(viii) Imagine a tensioned string of length L stretched between two rigid supports. Then imagine the same tensioned string held rigidly at one end by terminated by a point mass at the other end. Which of these strings would have the lower first natural frequency – and why (a sketch may help)?



when constrained by a point mass, wavelength of 1st mode is longer => lower natural frequency

(ix) Why is the "convective acceleration" normally neglected when deriving the wave equation in air?

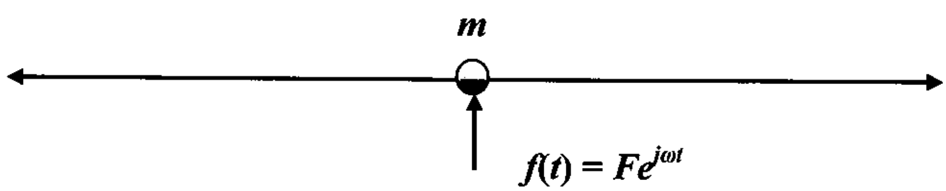
It is a term non-linear in small quantities and can be neglected compared to first order terms

The particle velocity of the sound field generated by a spherically symmetric source is purely radial – why?

Since the sound field is spherically symmetric there are no pressure gradients in the θ or ϕ directions so there can be no particle velocities in those directions

Problem 2.

A point mass, m, is attached to an infinite, uniform, tensioned string (tension, T, and mass per unit length, ρ_L) as shown in the figure below. The mass is attached at $x = 0$; positive x is to the right, and negative x is to the left. At the point at which the mass is attached, the string is driven by an applied transverse force, $f(t) = Fe^{j\omega t}$.



(i) Give appropriate assumed solutions for the transverse displacement of the string on both sides of the mass. Define quantities as necessary.

(ii) Draw a free body diagram of the forces acting on the mass.

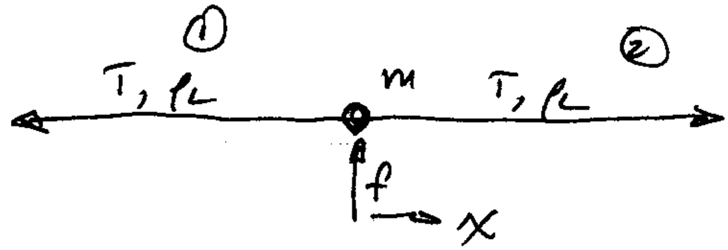
(iii) Give in equation form the boundary conditions that apply at the location of the mass.

(iv) Use the boundary conditions in conjunction with the assumed solutions to derive the solutions for the transverse displacement of the string on the two sides of the mass.

(v) Calculate the input mechanical impedance experienced by the applied transverse force.

(vi) What is the natural frequency for this system?

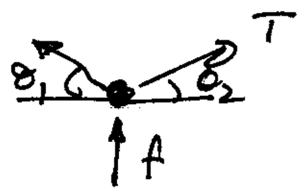
Problem 2



(i) $y_1(x) = A e^{+jkx} e^{j\omega t}$ $\qquad$ $y_2(x) = B e^{-jkx} e^{j\omega t}$

$k = \omega / c \qquad c = \sqrt{T/\rho_L}$

(ii)



(iii) b.c. (i) $\quad y_1 = y_2 \quad$ at $x = 0$

bc (ii) $T \sin\theta_1 + T \sin\theta_2 + f = m a_1$

$$-T \left.\frac{\partial y_1}{\partial x}\right|_{x=0} + T \left.\frac{\partial y_2}{\partial x}\right|_{x=0} + f = m \frac{\partial^2 y_1}{\partial t^2}$$

(iv) $\dfrac{\partial y_1}{\partial x} = +jkAe^{+jkx} e^{j\omega t}$ $\qquad$ $\dfrac{\partial y_1}{\partial t} = j\omega A e^{+jkx} e^{j\omega t}$

$\dfrac{\partial y_2}{\partial x} = -jkBe^{-jkx} e^{j\omega t}$ $\qquad$ $\dfrac{\partial^2 y_1}{\partial t^2} = -\omega^2 A e^{+jkx} e^{j\omega t}$

Apply b.c. (i) $y_1 = y_2$ at $x = 0$

$$A = B$$

Apply b.c. (ii)

$$-jkTA - jkTB + F = -\omega^2 m A$$

since $B = A$

$$-\omega^2 m A + 2jkTA = F$$

$$A = \frac{F}{2jkT - \omega^2 m}$$

$$y_1 = \frac{F}{2jkT - \omega^2 m} e^{+jkx} e^{j\omega t}$$

$$y_2 = \frac{F}{2jkT - \omega^2 m} e^{-jkx} e^{j\omega t}$$

(v) $Z_{mech} = \dfrac{F_e^{j\omega t}}{\left.\dfrac{\partial y_1}{\partial t}\right|_{x=0}}$

$$\frac{dy_1}{dt} = \frac{j\omega F}{2jkT - \omega^2 m} e^{jkx} e^{j\omega t}$$

$$= \frac{j\omega F}{j\omega(2\frac{T}{c} + j\omega m)} e^{jkx} e^{j\omega t}$$

$$c^2 = \frac{T}{\rho_L}$$

$$= \frac{j\omega F}{j\omega(2\rho_L \frac{c^2}{c} + j\omega m)} e^{jkx} e^{j\omega t}$$

$$= \frac{F}{2\rho_L c + j\omega m} e^{jkx} e^{j\omega t}$$

$\therefore$ impedance

$$Z_{mech} = \frac{F e^{j\omega t}}{\frac{F e^{j\omega t}}{2\rho_L c + j\omega m}}$$

$$= 2\rho_L c + j\omega m$$

(vi) Natural frequency - condition required to make

$$Im\{Z_{mech} = 0\}$$

$Im\{Z_{mech}\} = 0$ when $\underline{\omega = 0}$ natural freq

Problem 3.

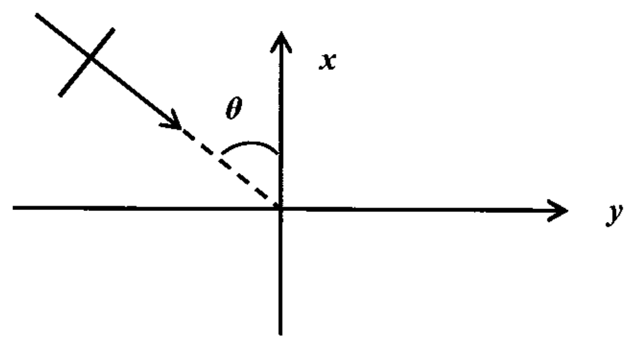
A plane sound wave is propagating in *free-space* in the direction shown in the sketch below.

(i) Give a complete expression for the sound pressure field, defining quantities (such as the wave numbers, for example) as necessary.

(ii) Derive by using the linearized momentum equation an expression for the vector particle velocity.

(iii) Give an expression for the specific acoustic impedance normal to the surface $x = 0$.

(iv) Derive an expression for the time-averaged acoustic intensity field in the x-direction, and show that the energy flow crossing the surface $x = 0$ goes to zero when θ goes to $\pi/2$.



Problem 3

(i) $p = Ae^{j(\omega t + k_x x - k_y y)}$

$$k_x = k\cos\theta \qquad k_y = k\sin\theta \qquad k = \frac{\omega}{c}$$

(ii) $$\bar{u} = -\frac{1}{j\omega\rho_0}\nabla\tilde{p} = -\frac{1}{j\omega\rho_0}\left(\frac{\partial\tilde{p}}{\partial x}\bar{i} + \frac{\partial\tilde{p}}{\partial y}\bar{j}\right)$$

$$= -\frac{1}{j\omega\rho_0}\left(jk_x\tilde{p}\,\bar{i} - jk_y\tilde{p}\,\bar{j}\right)$$

$$= \frac{k}{\omega\rho_0}\tilde{p}\left(-\cos\theta\,\bar{i} + \sin\theta\,\bar{j}\right)$$

$$= \frac{\tilde{p}}{\rho_0 c}\left(-\cos\theta\,\bar{i} + \sin\theta\,\bar{j}\right)$$

(iii) The surface $x = 0$ is the y-z plane.
The impedance normal to that surface is

$$z = \frac{\tilde{p}}{\tilde{u}_x}\bigg|_{x=0} = \frac{\tilde{p}}{-\frac{\tilde{p}}{\rho_0 c}\cos\theta} = -\frac{\rho_0 c}{\cos\theta}$$

(iv) $I_x = \frac{1}{2}\mathrm{Re}\{\tilde{p}\,\tilde{u}_x^*\}$ at $x=0$

$$= \frac{1}{2}\mathrm{Re}\left\{A\,\cancel{e^{-jk_y y}}\,\cancel{e^{j\omega t}}\,\frac{A^*}{\rho_0 c}\,\cancel{e^{+jk_y y}}\,\cancel{e^{-j\omega t}}(-\cos\theta)\right\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{\frac{|A|^2}{\rho_0 c}(-\cos\theta)\right\}$$

$$= -\cos\theta\,\frac{|A|^2}{2\rho_0 c}$$

or $I_x = \frac{1}{2}\mathrm{Re}\{\tilde{p}\,\tilde{u}_x^*\}$

$$= \frac{1}{2}\mathrm{Re}\left\{\tilde{p}\left(-\frac{\cos\theta}{\rho_0 c}\tilde{p}\right)^*\right\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{-\cos\theta\,\frac{|\tilde{p}|^2}{\rho_0 c}\right\}$$

$$= -\cos\theta\,\frac{|\tilde{p}|^2}{2\rho_0 c}$$

Since $\cos\theta \to 0$ when $\theta \to \pi/2$

$I_x \to 0$ when $\theta = \frac{\pi}{2}$