SHAPE SYNTHESIS

OF

HIGH-PERFORMANCE

MACHINE PARTS

AND

JOINTS

By

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Based on notes from
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Much of the activity that takes place during the design process focuses on analysis of existing parts and existing machinery. There is very little attention focused on the synthesis of parts and joints and certainly there is very little information available in the literature addressing the shape synthesis of parts and joints.

The purpose of this document is to provide guidelines for the shape synthesis of high-performance machine parts and of joints. Although these rules represent good design practice for all machinery, they especially apply to high performance machines, which require high strength-to-weight ratios, and machines for which manufacturing cost is not an overriding consideration. Examples will be given throughout this document to illustrate this.

Two terms which will be used are part and joint. **Part** refers to individual components manufactured from a single block of raw material or a single molding. The main body of the part transfers loads between two or more joint areas on the part. A **joint** is a location on a machine at which two or more parts are fastened together.

1.0 General Synthesis Goals

Two primary principles which govern the shape synthesis of a part assert that (1) the size and shape should be chosen to induce a uniform stress or load distribution pattern over as much of the body as possible, and (2) the weight or volume of material used should be a minimum, consistent with cost, manufacturing processes, and other constraints. If the stresses (force per unit area) are indeed uniform throughout the part, and the material used is a minimum, then the stresses in the part will be at their maximum safe level which represents efficient design.

In order to achieve these goals of uniform stress and minimum weight, it is necessary to consider the stress patterns that are present throughout the part as a result of applied forces and the geometry of the part. As external loads are applied molecular bonds within the material develop tensile and compressive and shear forces to transmit the load throughout the part. These internal load distributions are stress. For simple shapes and loading patterns, known stress patterns are introduced throughout the part. Because of this, it is useful to study stress patterns in simple-shaped parts because we can infer which of these stress patterns are inefficient, and, therefore, are more likely to cause failure in more complex parts.

Strong stress patterns are those in which the large percentage of the material is stressed uniformly. Simple tension and compression are good examples of strong stress patterns, because the normal stresses are uniform across the cross section of a part loaded in tension and compression. Other examples of strong stress patterns include transverse shear stresses, torsional shear stresses in hollow tubes, and the normal stresses associated with bending of an I-beam cross-section. (See Figure 1.)

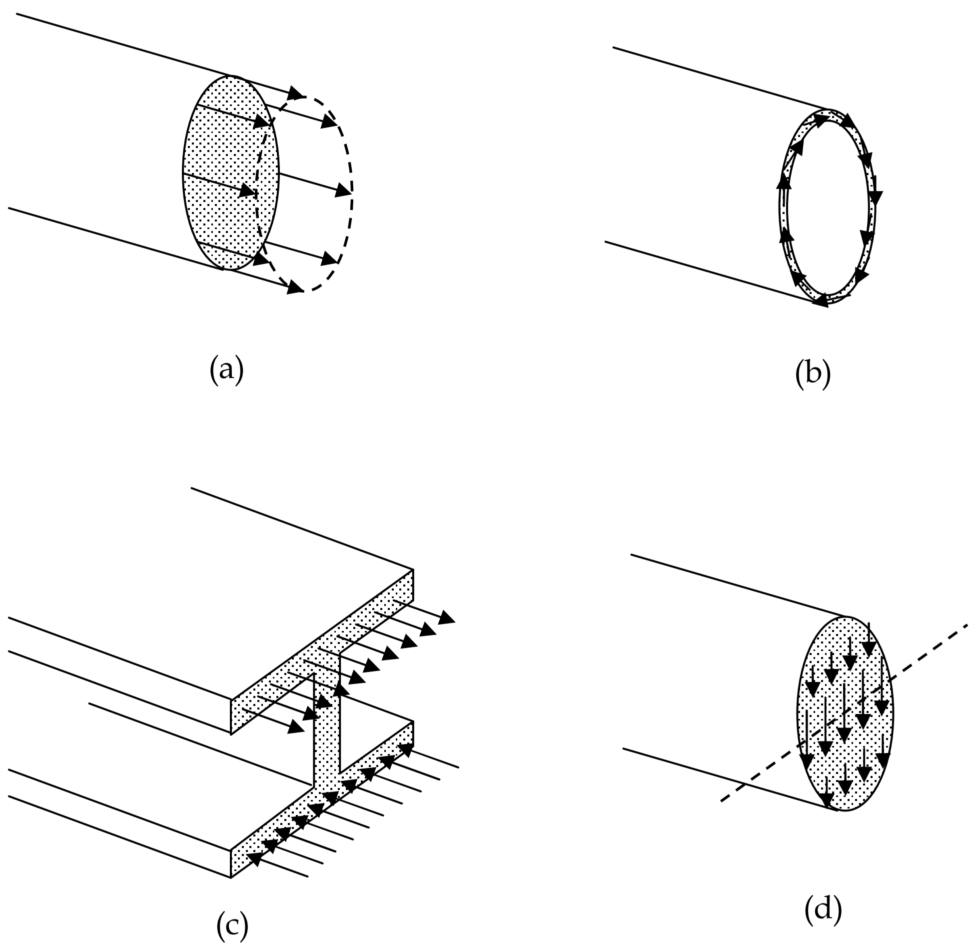


Figure 1. Strong stress patterns: a) tension/compression, b) torsion of a hollow tube, c) I-beam bending, and d) transverse shear

Weak stress patterns, on the other hand, include those for which stresses are not uniformly distributed throughout a section. Spot contact or Hertz contact stresses are one of the most serious of the weak stress patterns, because forces are highly localized over a small contact area. Other weak stress patterns include the normal stresses resulting from bending of non-I-beam sections, and torsion of solid prismatic cross sections. Notice that the weak stress pattern do not have uniform stress levels throughout the part. (See Figure 2.)

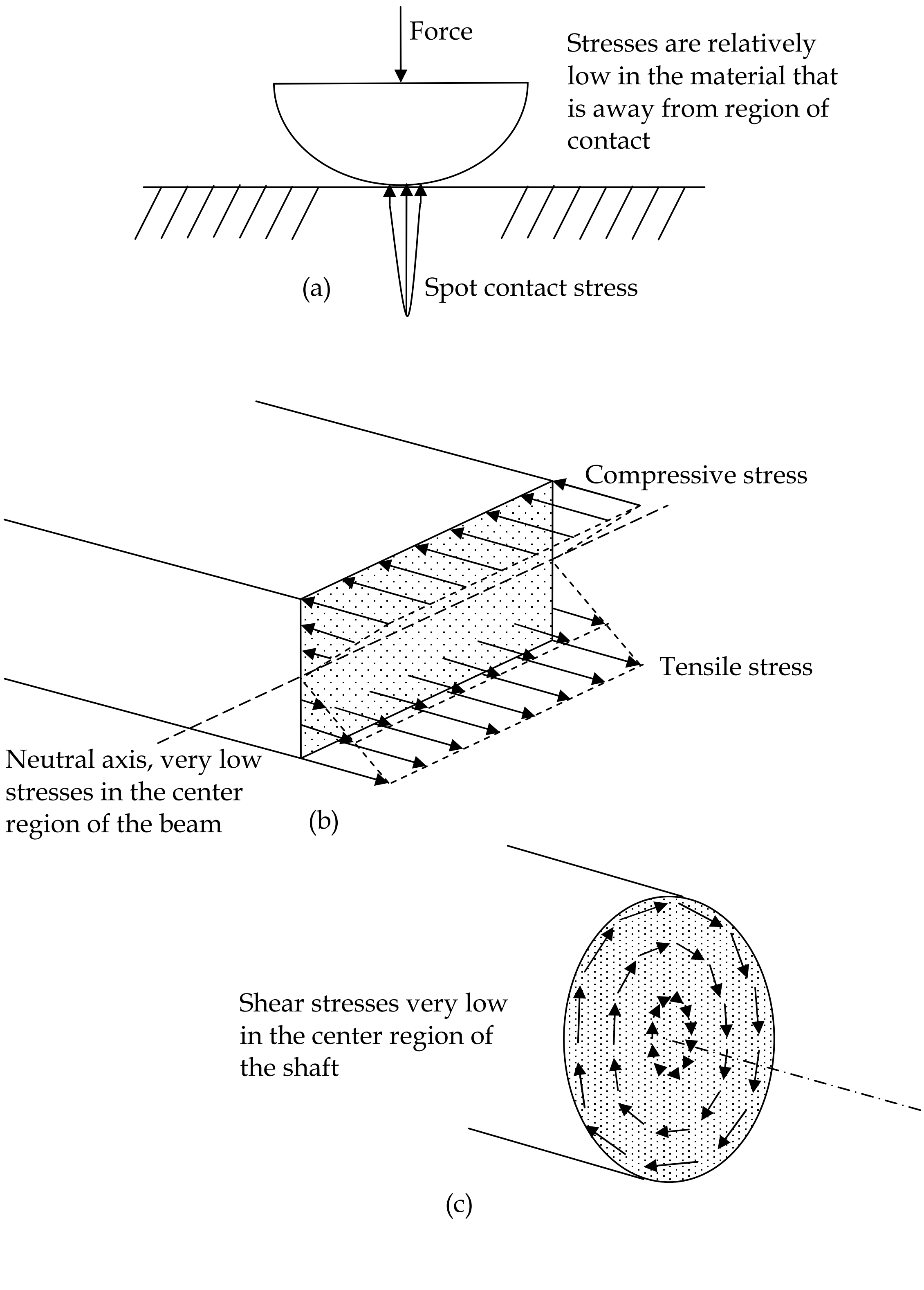
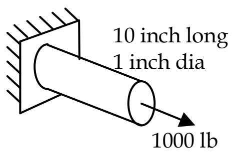
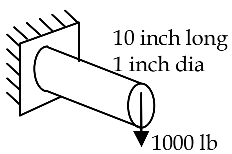
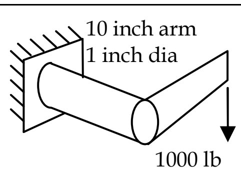
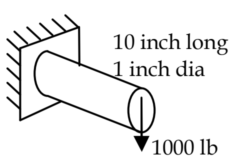
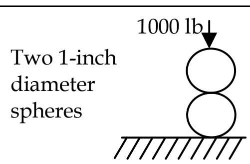


Figure 2. Weak stress patterns: (a) spot contact, (b) bending of solid sections, (c) torsion of solid sections. Weak stress patterns have large amounts of material on the section with relatively low stresses, therefore they are inefficient.

To illustrate the great differences in these strong and weak stress patterns, consider the following examples, in which 1000 pounds of load is carried through a 10 inch long beam of 1 inch diameter by means of the different stress patterns. Table 1 below summarizes the different load patterns and the stresses that result. Note that compared to tension, the strongest stress pattern, transverse shear has 1.33 times the stress, torsion has 40 times the stress, bending has 80 times more stress, and spot contact has 742 times higher stresses. This illustrates the great design efficiency that can be expected if tension, compression, and shear stress patterns can dominate designs.

Table 1. Sample stress calculations showing the dramatic differences between strong and weak stress patterns.

Stress Pattern	Figure	Equation	Stress, psi	Stress ratio
Tension		$\sigma = \frac{F}{A}$	1,270	1.0
Transverse Shear		$\tau = \frac{4}{3}\frac{V}{A}$	1,700	1.3
Torsion		$\tau = \frac{Tr}{J}$	50,930	40.0
Bending		$\sigma = \frac{Mc}{I}$	101,900	80.0
Spot Contact		$\sigma = \frac{3F}{2\pi a^2}$ $a = \sqrt[3]{\frac{3Fd}{8}\frac{\left(1-\nu^2\right)}{E}}$	944,000	742.0

In order to achieve uniform stress throughout a part it is desirable to seek loading patterns and part geometry that will tend to cause tension and compression, transverse shear, hollow-tube torsion, or I-beam bending, since these are strong stress patterns. Part geometries which tend to load material in bending or torsion or tend to localize forces in small areas should be avoided since these induce weak stress patterns and do not uniformly stress the part.

2.0 Part Shape Synthesis Rules

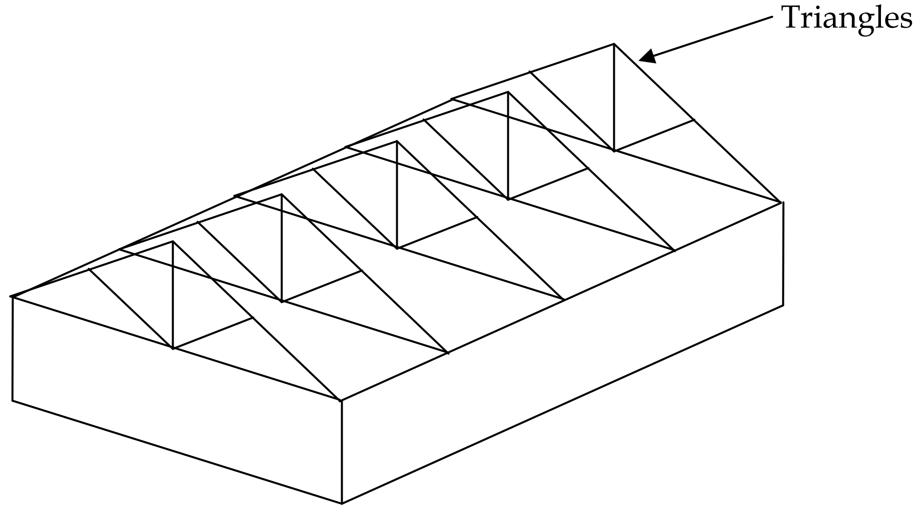
To achieve strong stress patterns then, there are several rules which will help the designer produce stronger parts. Following is a list of these rules with a brief description of them. Also included are examples of each of the principles.

2.1 *Triangle Principle*.

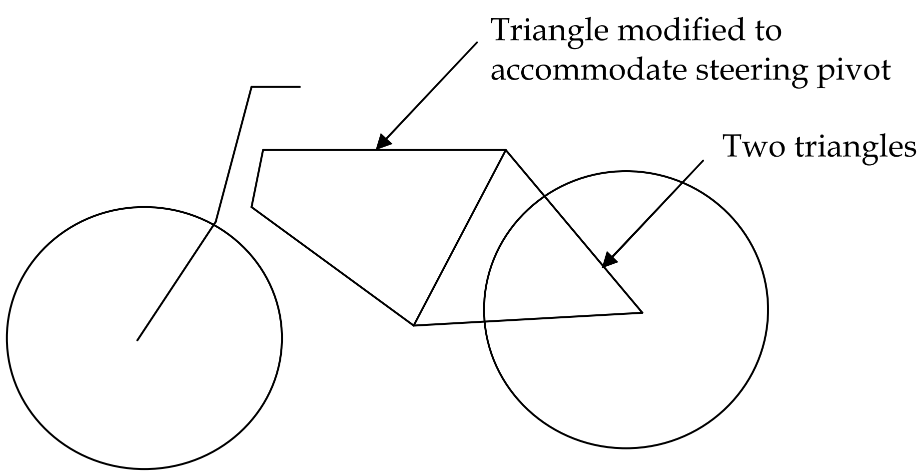
The Triangle Principle states that a triangle loaded at its corners is the most efficient two-dimensional structure. This result is achieved because in a triangle members are loaded in pure tension and pure compression only which is the most efficient stress pattern. Examples of the Triangle Principle include a bicycle frame, the A arms in an automobile front-suspension system, and the trusses used in residential roof structures. (See Figure 3.)

2.2 *Tetrahedron Principle*.

The Tetrahedron Principle states that a Tetrahedron loaded at its corners is the most efficient structure for transferring an arbitrary force in three-dimensional space to a plane. Again, as for the triangle, this is achieved because the members of the Tetrahedron are loaded only in pure tension and pure compression and bending moments are eliminated. Examples of the Tetrahedron Principle include antenna towers which are usually riveted steel beam structures shaped into Tetrahedrons, bicycle wheels, and jack stands which are used to support automobiles during repairs. It is interesting to note that a cardboard box which is folded shut and glued securely forms many tetrahedrons with the material along the diagonals of each of its walls. Also, the corrugated walls of the box illustrate the I-Beam Principle. (See Section 2.5.) Because of these proper applications of the Form Synthesis Rules, the cardboard box tends to be a very strong shape even though the material itself is not strong.



(a)



(b)

Figure 3. Triangle Principle, (a) roof trusses illustrates strong 2-dimensional shape, (b) bicycle frame with modified triangle in front to accommodate steering

2.3 *Metal Removal Principle*.

The Metal Removal Principle states that any material with stresses significantly below the maximum safe stress for the material being used can be removed to make the design more efficient. It is useful in applying the Metal Removal Principle to visualize the transmission of forces throughout the part as a flowing fluid. Consequently by knowing where forces are applied to a part and in what direction they are applied the most useful material will be that which directly connects the applied forces in a smooth and continuous way as would flowing fluid. Thus the Metal Removal Principle and the Force Flow Principle are closely related. Examples of the Metal Removal Principle include the automobile piston shown in Figure 4 and the universal joint shown in Figure 5. Note that a significant amount of material has been removed from both of these objects and that the material which remains has been removed so that the parts will be lighter in weight yet still be strong to carry the loads in as a direct a path as possible.

2.4 *Force Flow Principle*.

As mentioned above, the Force Flow Principle states that internal forces can be visualized as a flowing fluid, which will smoothly flow around sharp corners and other shape changes in a part. Consequently the internal sharp corners shown below in Figure 6 represents stress concentrations which cause a disruption in the force flow and thereby cause increased stresses. An automobile connecting rod is a good example of the Force Flow Principle because the shape of the rod is very smoothly contoured to transmit forces from the piston to the crankshaft of the engine. An automobile connecting rod is shown in Figure 7.

2.5 *I-Beam Principle*.

We have already seen that bending tends to be a weak stress pattern. However, it is also true that functional requirements sometimes dictate that bending is necessary. When bending is required, it is important that the shape be an I-beam like shape. Thus, the I-Beam Principle states that for structures loaded in bending it is more efficient to have the majority of the material near the outer fibers of the structure, that is, far from the neutral axis of bending. As will become more clear in the Anti-buckling Principle, the I-Beam Principle taken to an extreme leads to excessively thin and excessively large structures, which are subject to buckling and which may also be too large to adequately serve the function of the part. When the direction of the bending moment is not known, a hollow circular cross section is an ideal shape for the I-Beam Principle, because it has an equally large area moment of inertia about any cross-sectional axis. However, when the direction of the largest bending moments are known, an I-Beam C-channel or Z-shaped channel as shown in Figure 8 is a useful shape.

A standard house wall is a good example of the I-Beam Principle because it includes two-by-four or possibly two-by-six studs which serve as the center web of the

I-beam with sheathing both on the outside and the inside of the wall. The sheathing loads are pure tension or pure compression as lateral forces are applied to the wall. A second example of the I-Beam Principle is the beams used to support bridge structures. These are, in fact, I-Beams and are I-Beam shapes because the primary bending moment load directions are known.

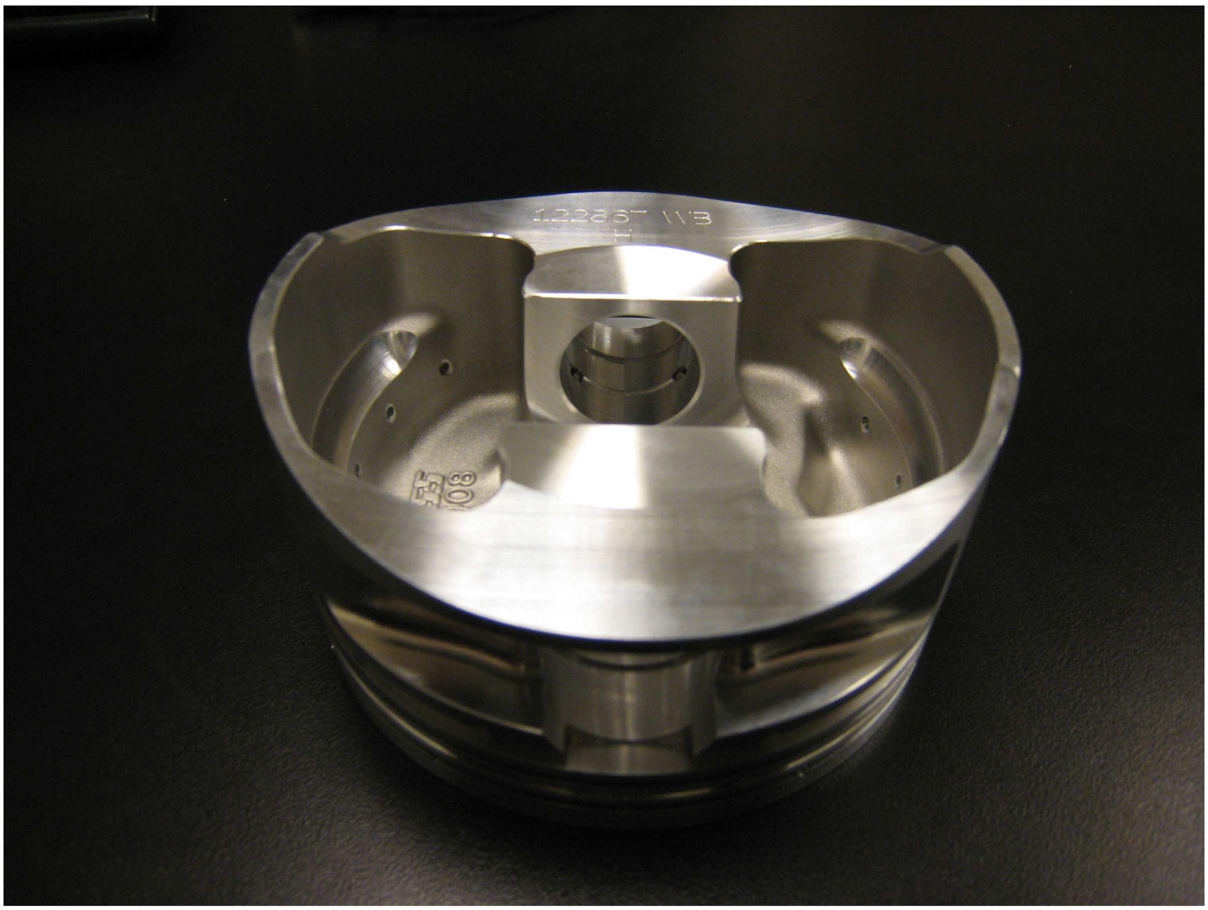
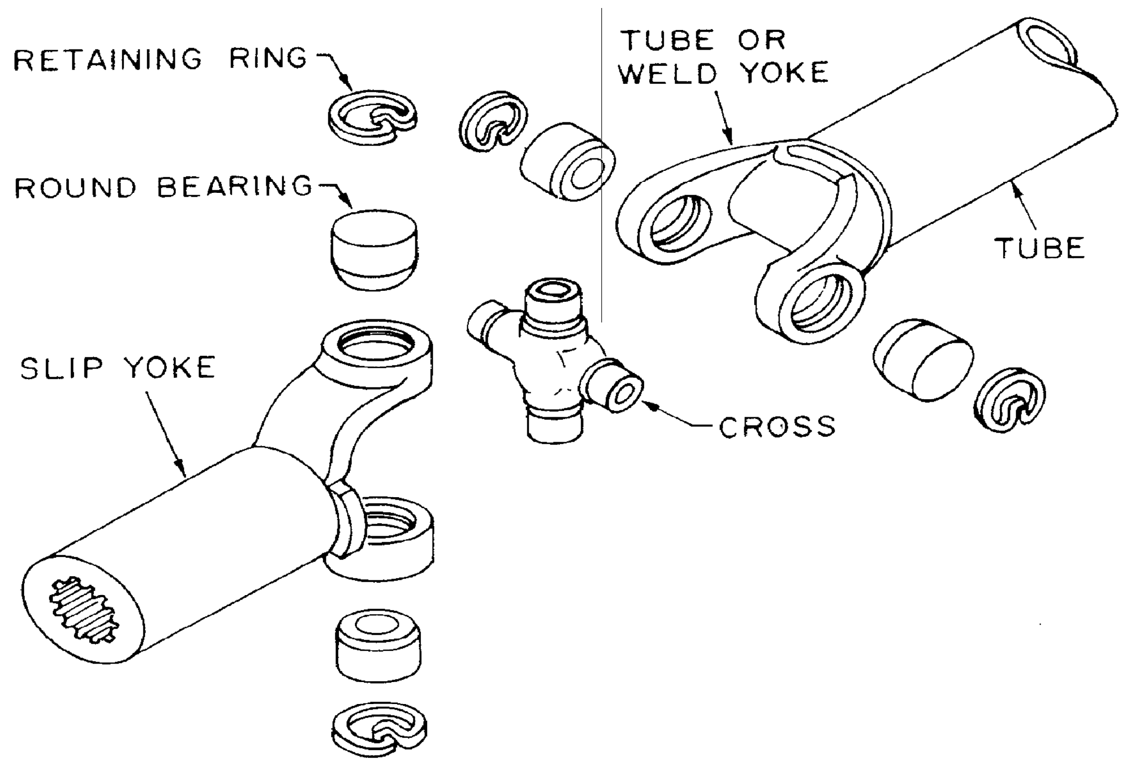
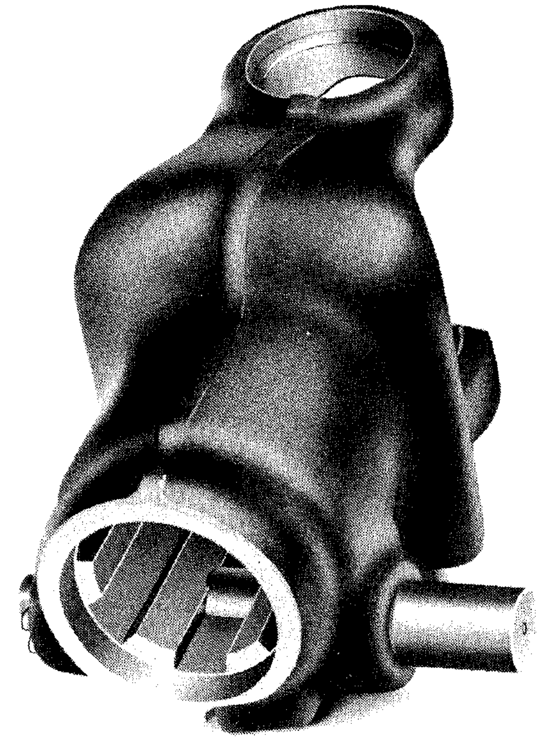


Figure 4. View of the underneath side of a piston showing metal removal. Material is left around the wrist pin holes for strength. Material has been removed both inside and outside of the piston.



(a)



(b)

Figure 5. Universal joint showing metal removal, (a) shape of yokes blended with tubes, and (b) in smooth contours and rounded corners in yoke.

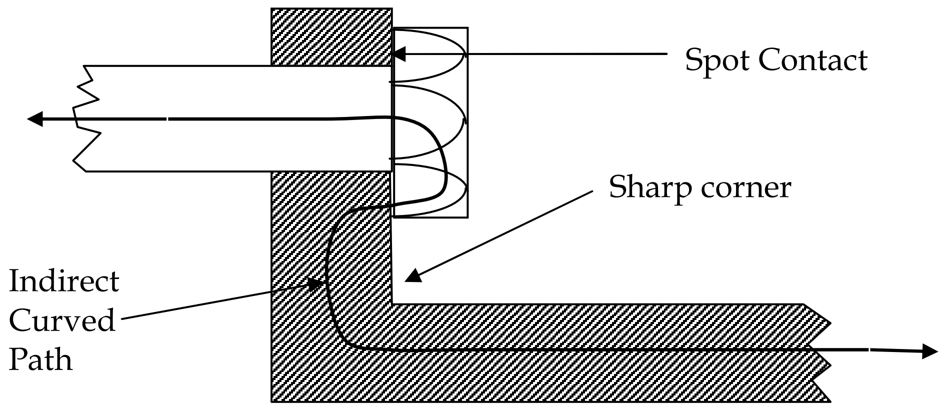


Figure 6. Force flow interrupted by sharp corners, curved paths, and reduced surface areas.

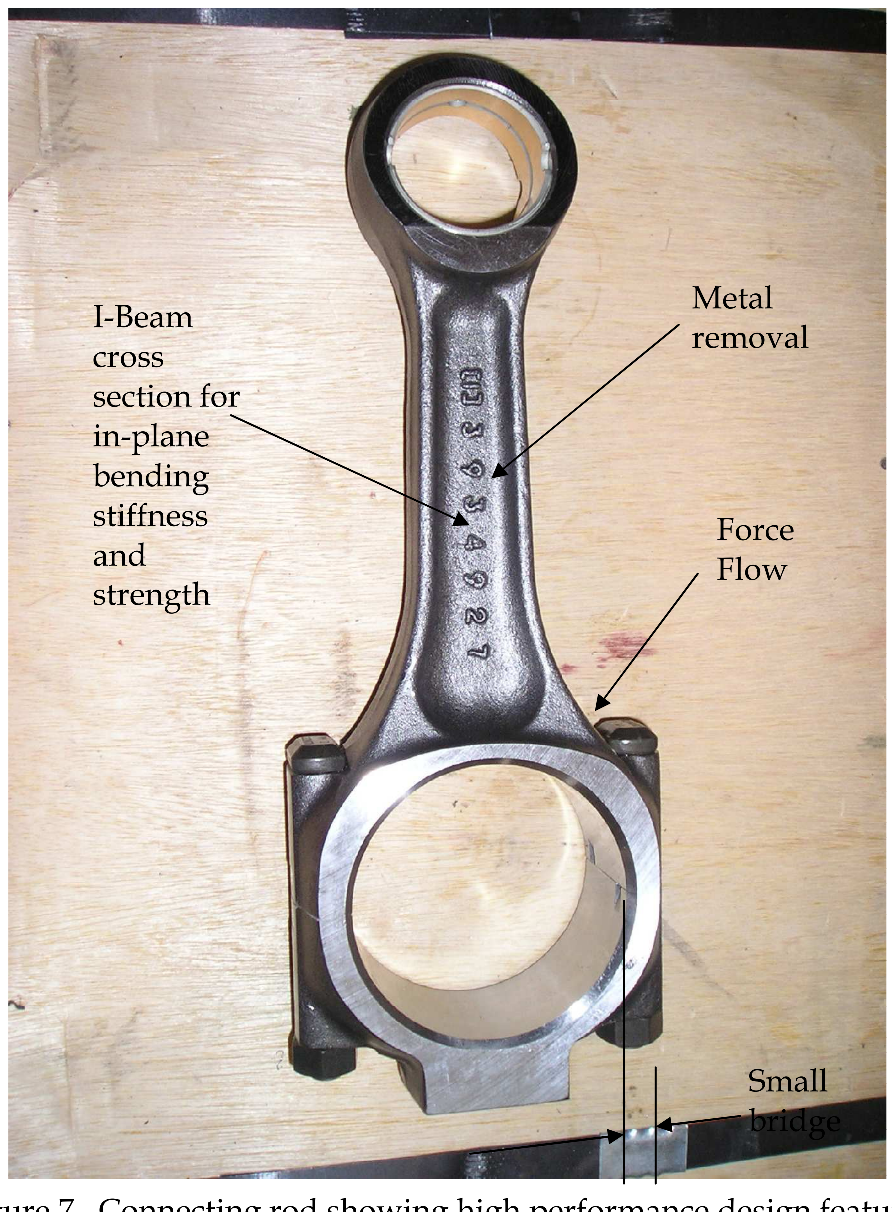


Figure 7. Connecting rod showing high performance design features

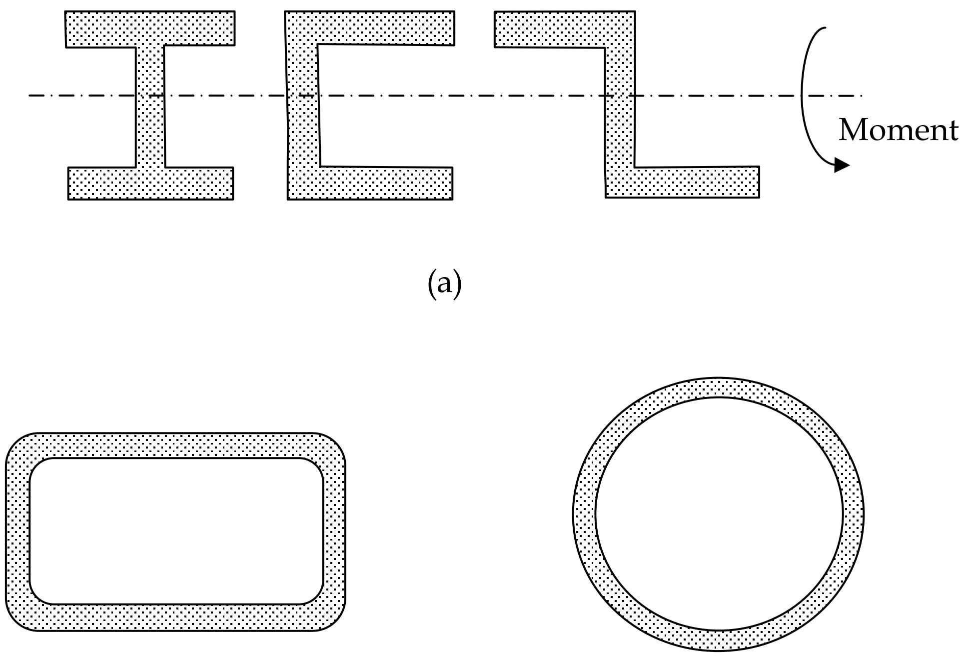


Figure 8. Good shapes for bending moments: (a) when axis of moment is known, and (b) when axis of moment is not known

2.6 *Torsion-of-a-Hollow-Tube Principle*

The corollary to the I-Beam Principle is the Torsion Principle. For moments about the axis of a long, slender beam, it is important to increase the polar moment of inertia of the material. Consequently, hollow tubes are the most efficient shapes for carrying torsional loads because nearly uniform shear stress exists on the material cross section of a large-radius thin-walled, hollow tube. However, as was true with the I-Beam Principle, taking the Hollow-Tube Principle to an extreme also leads to buckling tendencies. A classic example of the Hollow-Tube Principle is the drive shaft of a rear-wheel drive automobile which is a hollow tube. Bending moments are eliminated from the shaft by the universal joints at the ends of the shaft, and axial loads are eliminated by a spline slip joint.

The Hollow-Tube Principle is simple to apply when the cross-sectional shapes are circular. However, for non-circular cross sections it is more difficult to determine ideal cross-sectional shapes for transmitting torsion. For these cases, the membrane analogy is useful. The membrane analogy makes use of the fact that the differential equations and boundary conditions which describe stress and stiffness in a prismatic member loaded in torsion are the same as those which describe the slope and displacement of a thin pressurized membrane fitted over a hole of the same shape as that of the cross section of the prismatic member. Therefore, by visualizing the deflections of the membrane to different boundary conditions and pressures we can gain insight to the response of the prismatic section loaded in torsion. From the membrane analogy it can be shown then that the following are true.

1. The direction of the maximum stress at any point in the cross section of a prismatic member is the same as the tangent to the contour line of the same point on the corresponding pressurized membrane.
2. The magnitude of the shear stress at any point on the cross-section is proportional to the slope of the membrane at 90 degrees from the contour line.
3. The torsional stiffness is proportional to the volume enclosed by the membrane between its pressurized and unpressurized locations.

Thus, by visualizing a pressurized membrane stretched over a proposed cross-section, the shear stresses can be inferred. Because of these three analogues, several important general principles can be drawn from the membrane analogy. These include:

1. Of two cross sections having the same area the one that is more nearly circular will be stiffer in torsion. This is evident since whichever of the two sections has a longer perimeter for the same area will have a larger tendency to tie the membrane down at its edges, and reduce the volume enclosed, which represents the stiffness.
2. Since stiffness is analogous to the volume under the membrane, it is clear that small inward or outward protrusions in the perimeter of the section will have little effect on the stiffness of the section since it only slight affects the volume under the membrane.
3. The torsional strength and stiffness of I-beams, Z-beams, C-channels, and similar sections is relatively insensitive to the shape of the section, provided that the wall thicknesses are equal.
4. Although small inward or outward protrusions have little effect on the stiffness of the section, they can have a great effect on the local strength of a member in torsion. In particular small inward notches cause large stress concentrations. Therefore manufacturers of channels and angle irons typically put a radius on the interior corners of these sections.
5. For arbitrary cross sections, the largest possible inscribed circle in the section will intersect the surface of the cross section at points of highest shear stress. Of these points, the one which is most sharply inward protruding, that is, has the most sharply concave shape when viewed from outside, will be the location of the highest shear stress.

2.7 *Anti-buckling Principle.*

Buckling is a phenomena which is a consequence of the great strength difference between stress patterns. Parts which buckle are typically loaded in a configuration which causes a strong stress pattern such as compression or torsion. However, slight displacements of such members can cause the stress pattern to change from these strong stress patterns into bending. Consider, for example, the column shown in Figure 9. Initially the column is loaded in compression and a uniform stress pattern results. However, a slight offset at the center of the column can cause weak bending stresses in the column. Further loading causes a catastrophic failure because the bending stresses increase as the lateral deflection of the center of the column increases.

The Anti-buckling Principle states that supplementary shapes should be added to parts which are sensitive to buckling to prevent it. Figure 10 shows examples of buckling which can occur in hollow tubes in torsion, and I-beams in bending. Note that in each case strong stress patterns are converted into weak stress patterns as the parts buckle. In the case of the tube, compressive loads are converted into bending loads as small local buckles take place. In the case of the beam, compressive loads are also converted into bending loads as local buckling takes place on the flange. Figure 11 show supplementary shapes which have been added to prevent local buckling.

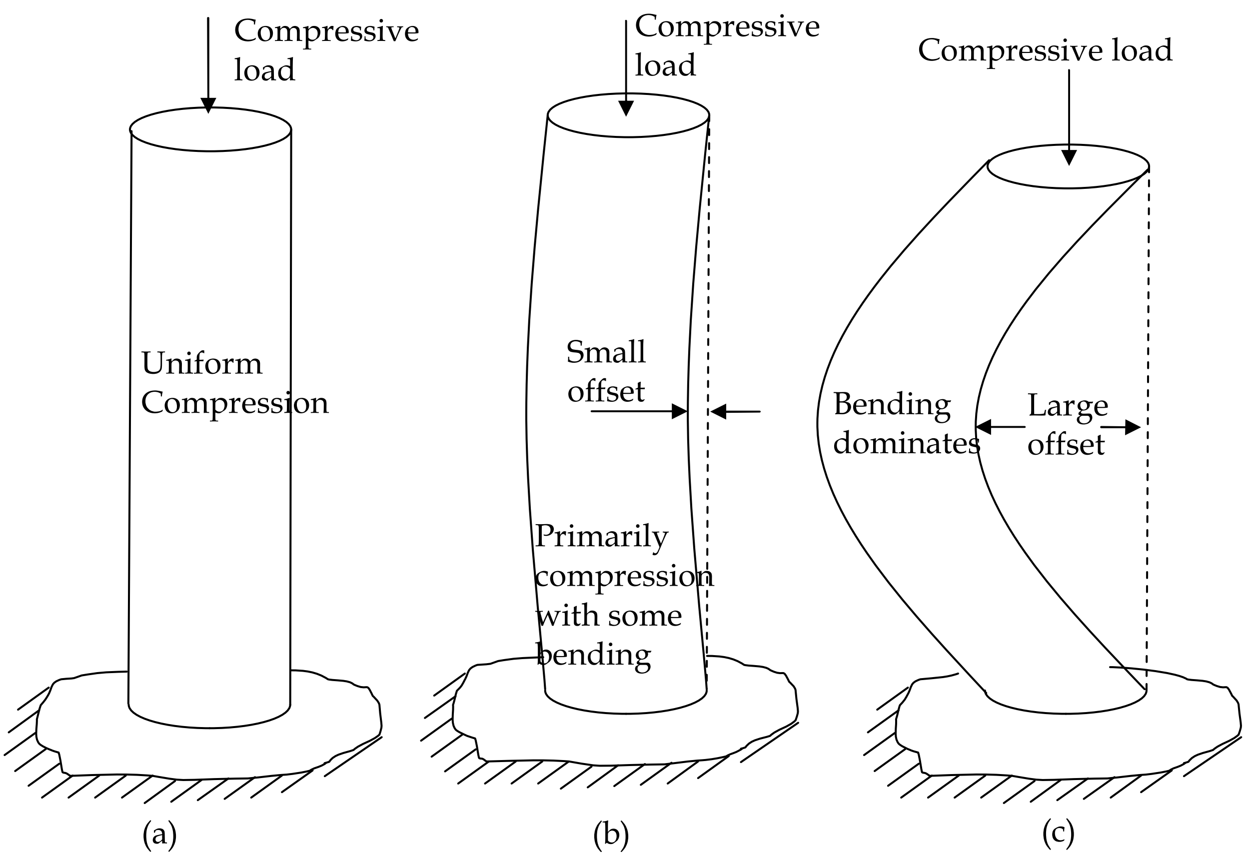


Figure 9. Column buckling showing stress pattern shift from strong stress pattern dominated by compressive stresses, (a) to a weaker stress pattern with mostly compressive stresses but some bending stress, (b) to a very weak stress pattern dominated by bending stress, (c)

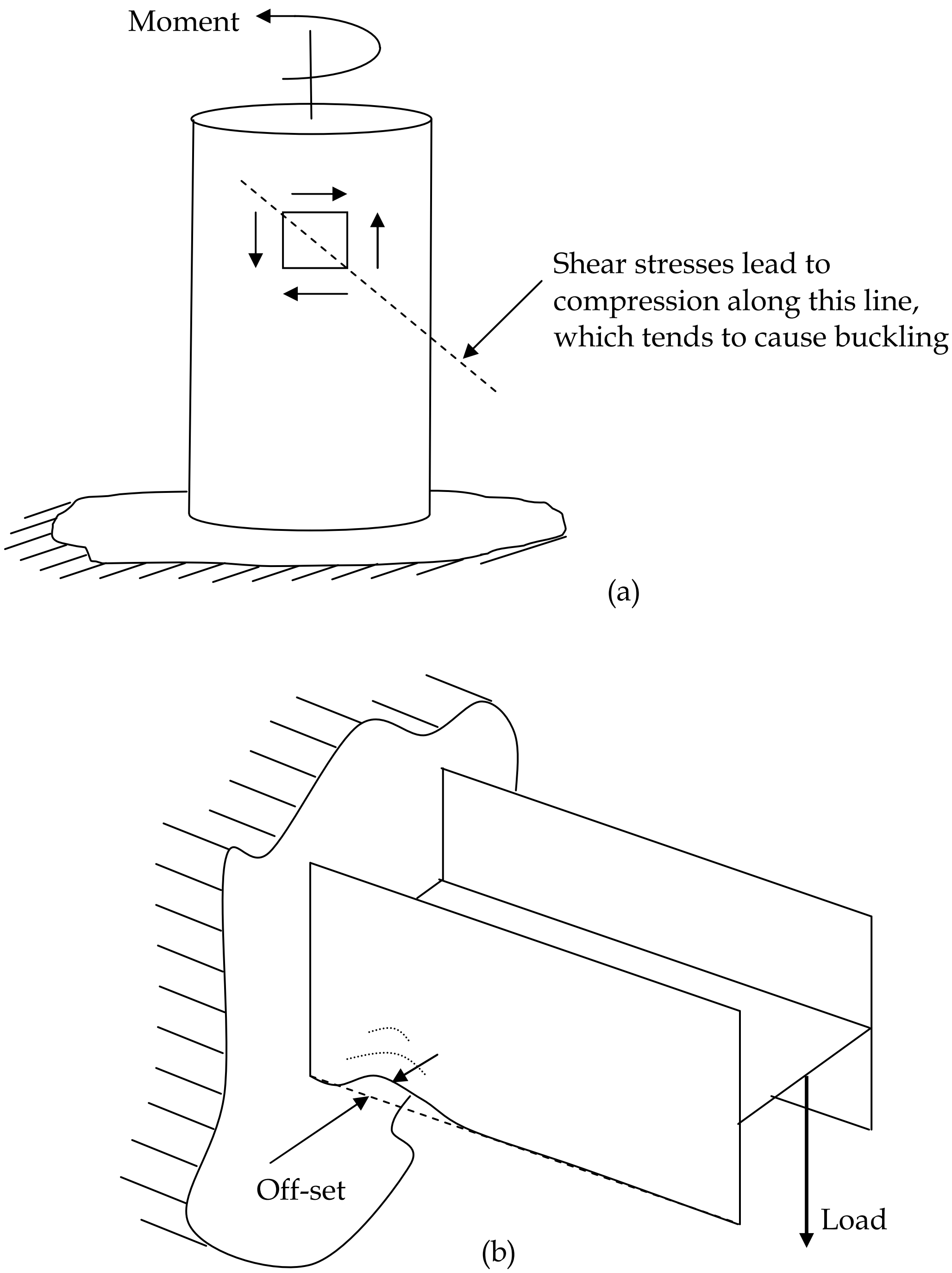


Figure 10. Local buckling due to: (a) torsion, and (b) compression in an I-beam.

2.8 *Direct Path Principle*.

The Direct Path Principle states that material should be placed in a direct line between the points of force application. This principle is based on the fact that material so placed is loaded in pure tension or compression with minimal bending. A good example of the Direct Path Principle is the tie-rod used in an automobile steering system. Another example are the bicycle spokes which hold the rim to the hub in a bicycle wheel. These spokes align directly between the rim and axle bearings. The axle bearings are also spaced apart to gain leverage on the loads (see the Leverage Principle) but not too far for functional reasons (space and weight). Pre-load is applied so that the spokes are never loaded in compression which would cause buckling.

An interesting violation of the Direct Path Principle which was mentioned earlier is the C-clamp. It has severe bending loads which are necessary because of the functional requirements of the clamp. Consequently the I-Beam Principle is applied where the bending exists. Joints also are inherently a violation of the Direct Path Principle as well as several other principles which have been discussed.

2.9 *The Leverage Principle*.

The Leverage Principle states that material should be placed in a part such that it will exert leverage on the loads. Figure 12 shows different designs for a cantilever beam which illustrate the principle of leverage. In Figure 12a no leverage is generated by the material in the beam and bending stresses are extremely high. In Figure 12b a small amount of leverage is obtained by the angle between the upper and lower members of the beam. Figure 12c shows an extreme application of the Leverage Principle. When the Leverage Principle is applied in extreme, not only is an excessive amount of material required, but also the parts become thinner because of the increased leverage and buckling becomes a problem.

Figure 13 illustrates this in the torsion of tubes. In Figure 13a the Leverage Principle is not applied well because large percentage of the material in torsion is placed near the center of the shaft where torsional stresses are low. In Figure 13b the Leverage Principle is applied in excess in an extremely thin tube of very large diameter results. In this case, torsional buckling could occur as we saw in the Anti-buckling Principle.

Near-optimal applications of the Leverage Principle are shown in Figures 12(d) and 13(c). Optimum is determined by the specific strength needs and loads for each design.

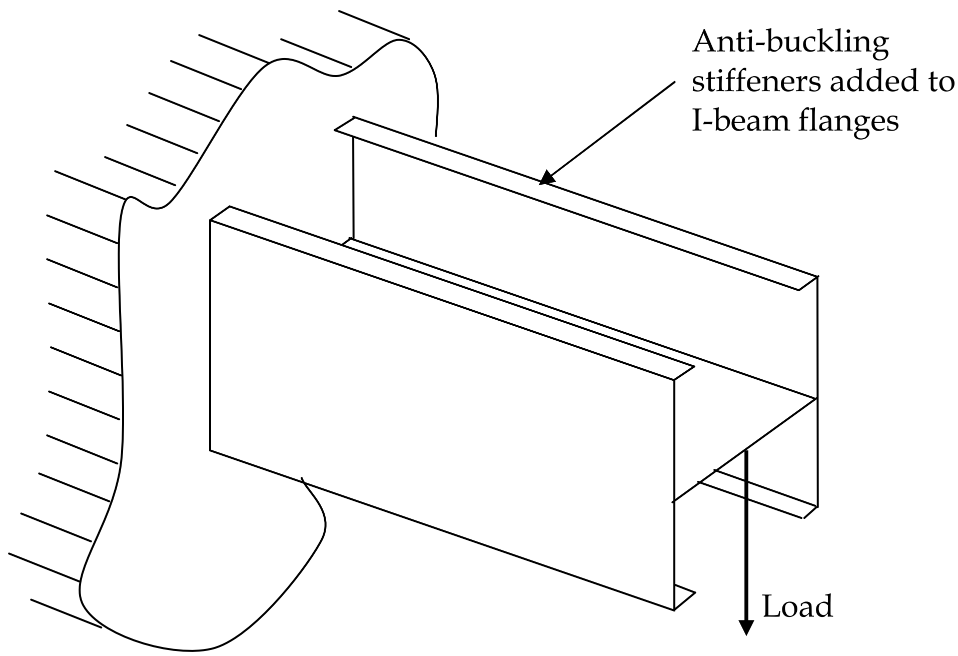


Figure 11. Anti-buckling stiffeners added to I-beam flanges.

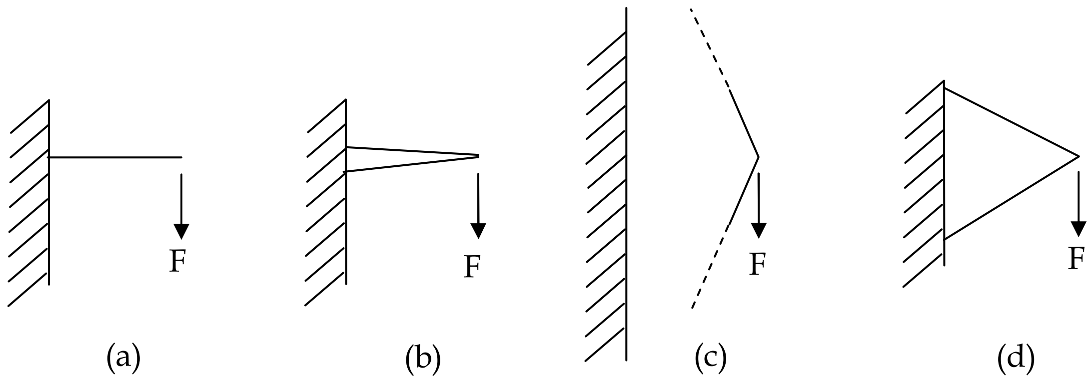


Figure 12. Leverage Principle: a) No Leverage, b) Too Little, c) Too Much, and d) about right, compromise between lower loads and excessive material.

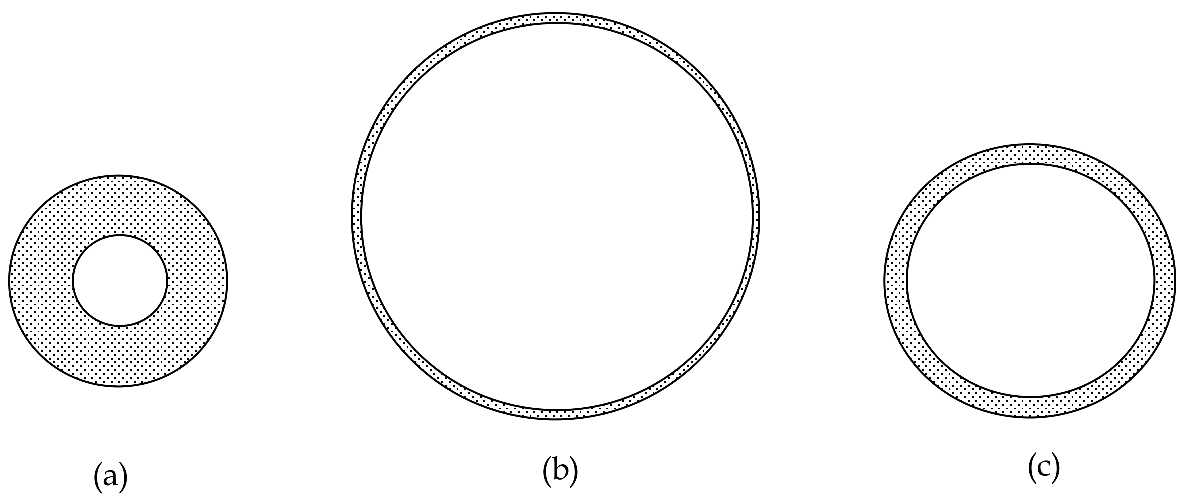


Figure 13. Leverage principle for torsion, three tubes with equal polar area moment of inertia, J: a) Not enough leverage, tube will be to heavy (b) too much leverage, tube will be fragile to dents and subject to buckling, and c) looks about right.

2.10 *Uniform Size Principle*.

The goal of an efficient design is to obtain a uniform, high-stress level in all areas of the machine part. It is logical to assume, then, that any two parts, or any two portions of the same part which are similarly loaded must be roughly uniform in size or else there is little chance that the parts will be uniformly stressed. This is the Uniform Size Principle. Observation of suspension components on automobiles indicate that many of the parts particularly in the steering components of the front wheels are approximately the same size.

2.11 *Overlap and Redundancy*

Many of these principles overlap and are interrelated. For example, metal removal from the center of a plate may make it a triangle. Also, the I-beam Principle is valid because material in the outer fibers of the beam follow the Leverage Principle. Nevertheless, because it is often hard to visualize the application of these principles, it is useful to consider each of them separately.

2.12 *Reasons for Violating the Form Synthesis Rules*.

It is often desirable and necessary to violate the principles of form synthesis and incorporate the weaker stress patterns of bending torsion and spot contact. Following are several circumstances under which it is necessary to use these weaker stress patterns.

<u>When space is limited</u>. Contact stresses could be lower in transmissions and axles if gear teeth and roller bearings were made larger. However, to conserve space they are made small and designed to live with high contact stresses. Similarly, wear problems could be reduced and thinner oil could be used if journal bearings in automobile engines were much larger. However, to save space their size is limited.

<u>Cost Limitations</u>. A trailer tongue is often not reinforced with triangles, because this would be excessively expensive, for the product.

<u>Manufacturing Process Limitations.</u> Automobile rear axles should theoretically be made out of hollow tubes, but because it is desirable to manufacture the wheel attachment flange as an integral part of the axle this is not done. This requires a forging process and hence the hollow tube is impractical.

<u>Functional Requirements</u>. Probably one of the most common reason for violating the rules is the need to meet the functional requirements of the design. Consider, for example, the C-clamp shown in Figure 14, which is used to temporarily squeeze two pieces of material together. A C-clamp violates the Direct Path Principle because it places material on the backside of the clamp in bending rather than using tension and

compression. However, this is necessary because the function of the clamp requires space for the pieces which it will clamp together.

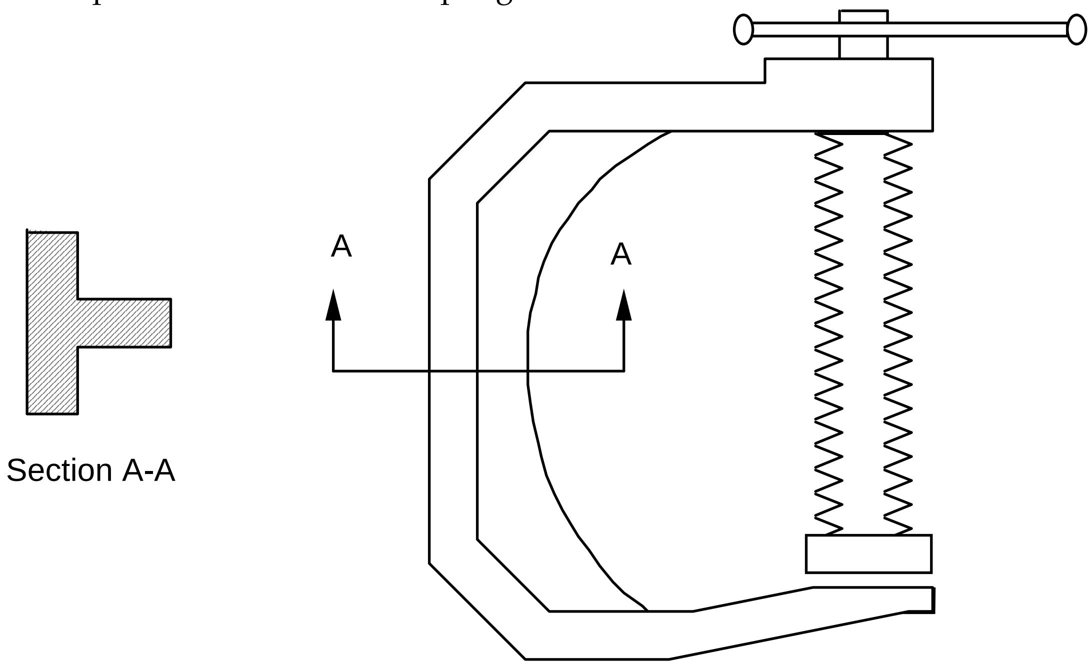


Figure 14. C-Clamp.

Another example in which functional requirements dictate a violation of the principles of form synthesis are roller and ball bearings. In these devices extremely high contact stresses are present. However, this is necessary in order to allow reduced friction and smooth power transmission in the presence of limited wear.

<u>Aesthetics</u>. Often a design is made weaker by using less efficient stress patterns in order to produce a more aesthetically pleasing product. Body parts on automobiles are good examples of this.

<u>Joints and Fasteners</u>. Probably the most common reason for violating the part synthesis rules arises from the need to fasten parts together. Joints are needed to allow machine assembly, to allow for future maintenance, to allow motion between parts, and to allow dissimilar materials. Though joints are necessary, they are flagrant violators of many of the principles stated above. Consider, for example, the joint shown in Figure 15 in which a bolt fastens two plates together and are loaded as shown. The Force Flow Principle is violated because the forces between the two plates must follow an indirect path going through the bolt to connect the two pieces together. The Direct Path Principle is also violated as is the Leverage Principle. The bolted joint on the connecting rod shown in Figure 7 illustrated an effort to minimize these violations.

When two or more parts are joined together, weak stress patterns are common in the joint area. The design of efficient joints is such an important aspect of machine design that rules and guidelines just for joint design have been developed.

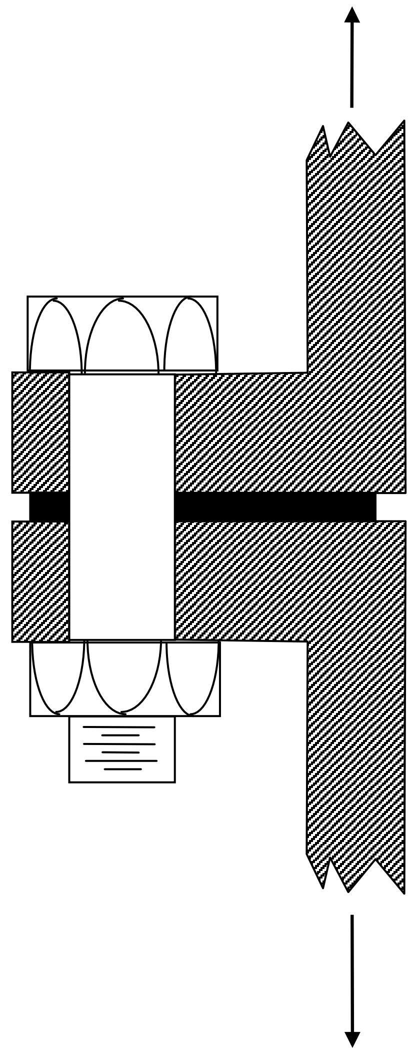


Figure 15. Bolted joint illustrating violation of the direct path and short bridge principles.

3.0 Joint Synthesis.

We have asserted that a part usually has distinct portions which can be called 1) the body and 2) the joint elements. Sections 1 and 2 dealt with stress patterns in the body portions only. Stress patterns in the area of the joint will be considered here.

It is necessary to construct a complex machine using many parts to facilitate the manufacture of the parts. The joints provide a means of assembling the machine and transmitting forces from one part to another. A joint is a break in the continuity of the machine structure. This break in continuity decreases the strength of the structure.

There are three major types of joints in machinery: 1) welded joints in which two or more parts are melted at the joint and allowed to re-solidify as one part, 2) adhesive joints in which parts are fastened together by a surface bonding agent which usually does not alter either part at the joint interface, and 3) mechanical interlock joints. Welded joints can be as strong as the original parts if sufficient filler material is added, and adhesive joints can be strong if they are loaded in shear and if the surface area of the bond is large. The remainder of this document will address in more detail mechanical interlock joints.

Most joints in machinery are of the mechanical interlock type. They force the stresses to detour around the break, often through indirect and restricted alternate paths. Consequently, the joints are inherently weak, and they must be reinforced by adding otherwise unneeded material and special shapes. Usually, the presence of the joint 1) forces the stress flow to divide and pass over discrete, narrow concentrated bridges before rejoining to form a uniform stress field on the other side of the joint, and 2) a joint induces the weak stress patterns of bending, torsion, and spot contact in this detour bridge path.

Consider, for example, the joint which connects the two parts of the connecting rod that was shown in Figure 7. When the connecting rod is loaded in tension, the stress pattern in the body of the connecting rod is pure tension. In order for the loads to be transferred through the bottom of the connecting rod and into the crankshaft journal, the loads must leave the center line of the connecting rod and travel out to the bolted flange. This causes bending in the bottom portion of the connecting rod. Further, in order for the forces to be transferred through the bolt they must reverse direction since the contact between the head of the bolt and the flange of the connecting rod can only sustain compressive loads. Consequently, the forces reverse direction going through the compressive region between the head of the bolt and the flange and then reverse direction again to precede through the shank of the bolt. This causes bending stresses in the flange of the rod and the head of the bolt. The forces again reverse direction at the bottom end of the bolt to pass through the compression zone between the nut and the bottom flange, and then reverse again to reform as tensile loads in the bottom portion of the connecting rod journal. Thus the need for a joint in the connecting rod and the functional requirement that the bottom end of the rod contain a journal bearing causes a serious disruption in the uniform stress field of pure tension in the connecting rod.

Because of the serious disruption of uniform stress fields caused by joint elements, it is important to focus attention on the design of joints to try to eliminate failure in joint regions. Section 4.0 will describe general principles of joint design.

4.0 General Principles of Joint Design.

The most efficient joint is one which disrupts the uniform stress field within the part a minimal amount. Although all joints disrupt the force flow to some extent, high joint efficiency can be achieved by application of some general principles. The following general principles will help improve the efficiency of joint design when properly applied.

4.1 *Many-Small-Fasteners Principle.*

Whenever thin sheets of uniformly stressed material meet at long interfaces, and wherever there is not a special reason to use a few large fasteners, many small closely-spaced fasteners should be used. Spot welds in automobile bodies and rivet joints which hold airplane wing skins onto the wing structure are two examples of this principle. Also, sewn garments illustrate the use of many small fasteners, because a very weak piece of thread is used to make many small mechanical interlock connections between two pieces of cloth to be sewn together. The arguments which support the Many-Small-Fasteners Principle are:

1. There is a minimal change in the design of the original parts when many small fasteners are used.

2. The many parallel force-flow paths which result from many small fasteners allows a minimum of alteration of the uniform stress field within the part. Consequently, there is less under-stressed material within the part.

3. The use of many small fasteners is desirable for gasketed joints where a uniform squeeze force is required to seal two parts. When many small fasteners are used a very large total force can be transferred between two parts. Also, since joint elements are small less power is required to fasten each individual element.

4. Using many small fasteners incorporates redundancy in the design. If one fastener fails, the total reduction in strength of the joint is small.

4.2 *Few-Large-Fastener Principle.*

Functional requirements, manufacturing and assembly requirements, and other considerations sometimes make it desirable or necessary to use a few large fasteners in place of many small ones. For example, the engine of an automobile is typically fastened to the frame by three or four engine mounts. There are several reasons for this including 1) the need to isolate engine vibrations from the rest of the vehicle, and 2) the need to keep the elastic warping of the frame of the automobile from imposing excessive loads on the engine and thereby damaging it. Other cases where a few large fasteners should be used in place of many small fasteners include joints where mobility is required such as between the drive shaft and the output shaft of the transmission in an automobile. In this case a universal joint is used and spot contact or very localized contact is used to transmit the torque. Also, sometimes parts need to be disassembled for maintenance. A few large fasteners makes this task easy. An example of this is the lug bolts which hold a wheel to an axle.

When a few large fasteners are used to join two bodies together, close attention must be paid to the design of the joint. When there are only a few joint elements, the design requires extra material in the region of the joint to strengthen it. Also, there are usually shape merging regions near the joint to transfer the loads from the thinner body of the part to the region of the joint. The joint design principles which follow address the case when a few large fasteners are needed, and the shape of the joint elements must be designed with care in order to be efficient.

Some joints are more efficient because both the Many-Small-Fastener and the Few-Large-Fastener Principles are applied simultaneously. Consider the threads between a bolt and a nut as an example which illustrates this. Between the bolt and the nut is a joint which consists of many interconnected threads which if stretched out straight would amount to a long joint. However, the bolt itself is a localized joint on the scale of the overall part. Consequently each bolt joint illustrates both of these joint principles. A button sewn on a garment also illustrates this. Many small threads are concentrated at the button, where a single discrete fastener, the button, is used to temporarily fasten the cloth together. These few large fasteners are needed because the joint must be frequently disassembled, but many small fasteners are used to hold the few large ones together.

4.3 *Shape Merging Principle*.

When a few large fasteners are used to transmit loads to specific points on a structure it is necessary to increase the strength of the structure in that area. Notice in the connecting rod in Figure 7 that the material has side ribs which not only prevent buckling but also help distribute the loads from the body of the rod to the joint elements. In general, shape merging involves the transition of material between a region of concentrated load to a region of smaller distributed load. The region of concentrated load is called the compact region. The region of smaller distributed load is called the thin-walled region. The transition material, which includes ribs, fans, and tapers, is known as the merge region. Figure 16 illustrates the use of fans, tapers, and ribs to help transfer load in the shape-merging region.

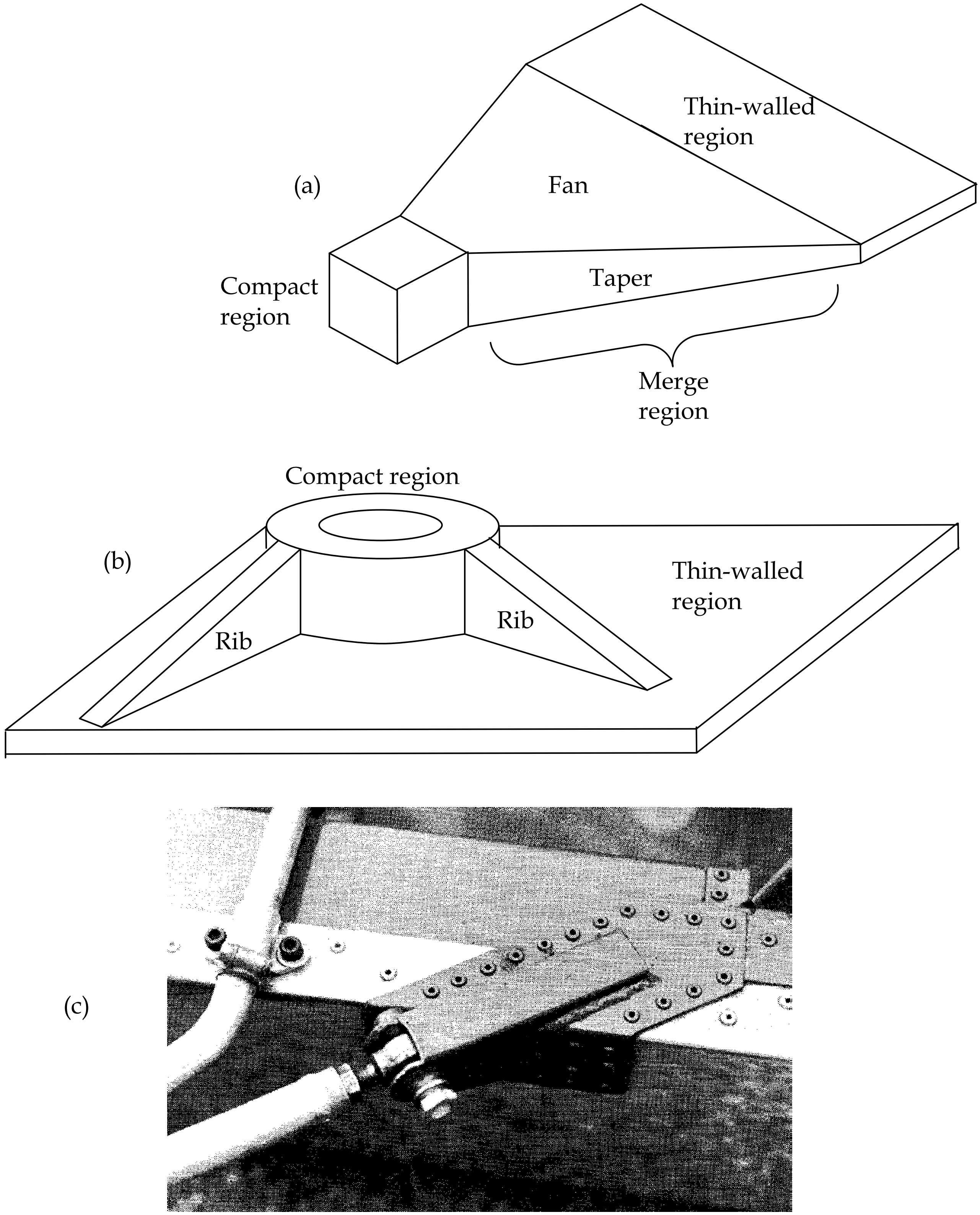


Figure 16. Shape merging principle showing: a) fans and tapers, b) ribs, and (c) example connection between suspension link and race car chassis.

4.4 *Mating Surface Principle.*

The Mating Surface Principle states that when two surfaces are in contact a large surface area is preferable to a small surface area. The larger surface area will distribute the loads at the joint to a large amount of body material in the next part and thereby induce a more uniform stress pattern. As a result of the mating surface principle, the region under the bolt heads for high-performance parts should be slightly thicker and slightly wider than the head of the bolt to increase the strength of the joint. Figure 17 also illustrates two examples of the Mating Surface Principle where degrees of freedom are required in the joint. In Figure 17(a) a cam joint has surface contact between the cam and the follower, reducing the contact stresses. In Figure 17(b) the spherical joint between the shoe and the connecting rod of a compressor reduces the contact stresses also. Table 2 summarizes four types of surface contact conditions and indicates the relative damage potential as well as examples of each type of surface contact.

4.5 *The Small Bridge Principle.*

As was shown in the connecting rod example in Figure 7, joints often convert uniform tensile and compressive stress fields into bending stress fields which are weak stress patterns. To help reduce the bending stresses as much as possible, it is desirable to have the axis of the bolt as close to the line of action of the tensile forces as possible. The principle that results from this design goal is called the Small Bridge Principle. The Small Bridge Principle states that when a cantilever is required to carry loads, it should be as short as possible. Notice that the bolt in Figure 7 passes as closely as possible to the surface of the journal bearing on the end of the connecting rod. This helps reduce the bending stresses in the connecting rod flange under the bolt.

4.6 *Shape Specialization Principle.*

It is often desirable in a joint to have the geometry of the mating joint elements specialized such that the loads in different directions are transmitted through the joint by different parts of the joint. Consider, for example, the bolted joint shown in Figure 18. The two base parts are shaped such that any transverse loads are taken by the mechanical interlock shown. The function of the bolts is merely to hold the two pieces together and to take loads perpendicular to the plane of the base part. Another example of Shape Specialization is shown in Figure 19. In this part a small piece of material has been added to the clevis to prevent large bending loads in the vertical portions of the clevis as shown.

A drive-shaft from a rear-wheel-drive automobile also illustrates shape specialization. In this case, the spline joint in the drive shaft permits free changes in length in the drive shaft to accommodate suspension travel without putting large axial loads on the drive-shaft and its bearings. This spline joint could be eliminated if the rest of the drive shaft were much stronger. But it is well worth the added cost and complexity to include the joint so that axial loads are reduced in the bearings.

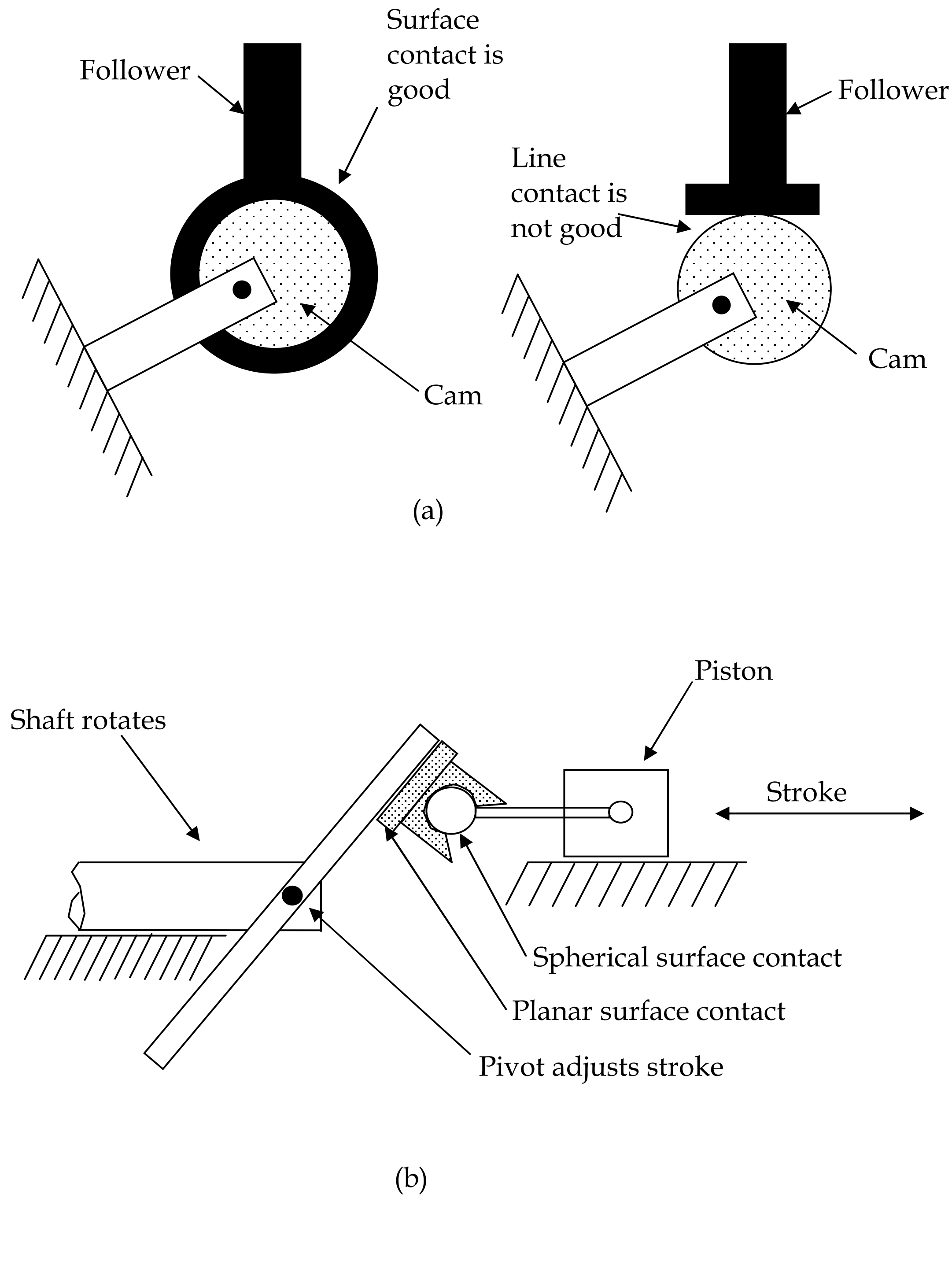
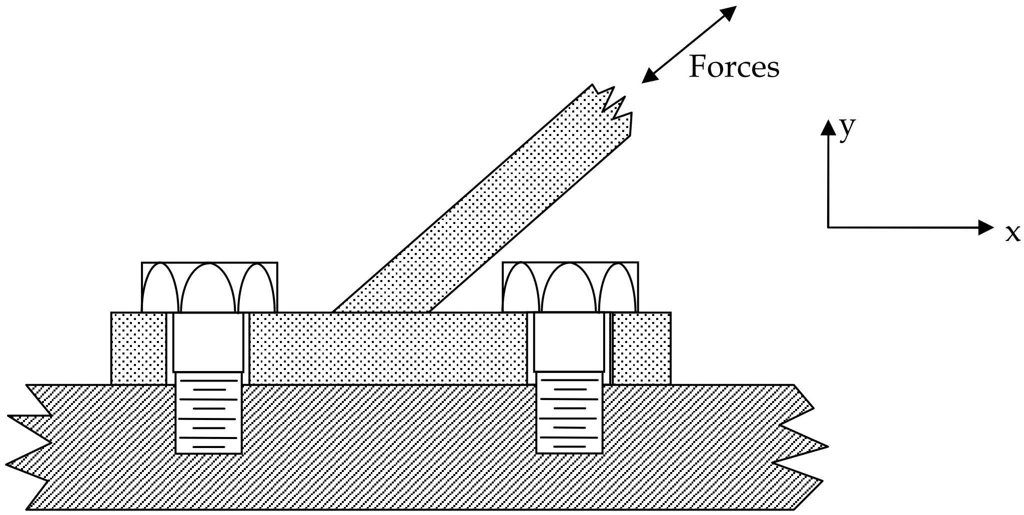


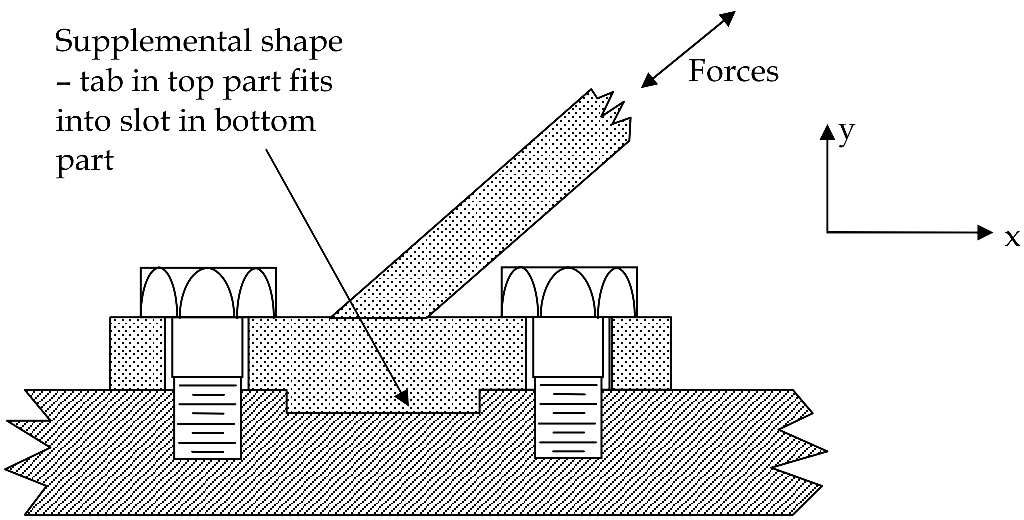
Figure 17. Mating surface principle showing: a) a circular cam, and b) a variable-stroke compressor.

Table 2. Four surface contact types

Contact Type	Damage Potential	Examples
Point or line contact with sliding	Very high	Cam with flat-faced follower Gear tooth
Point or line contact with no sliding	High	Ball Bearing Cam and roller follower
Surface contact with sliding	Moderate	Journal bearing Piston/cylinder
Surface contact with no sliding	Low	Bolted joint Press-fit Riveted joint



(a)



(b)

Figure 18. Joint design illustrating: (a) no shape specialization, bolts carry x-direction (shear) and y-direction (tension) forces, and (b) shape specialization, bottom tab carries x-direction forces, and bolts carry y-direction forces.

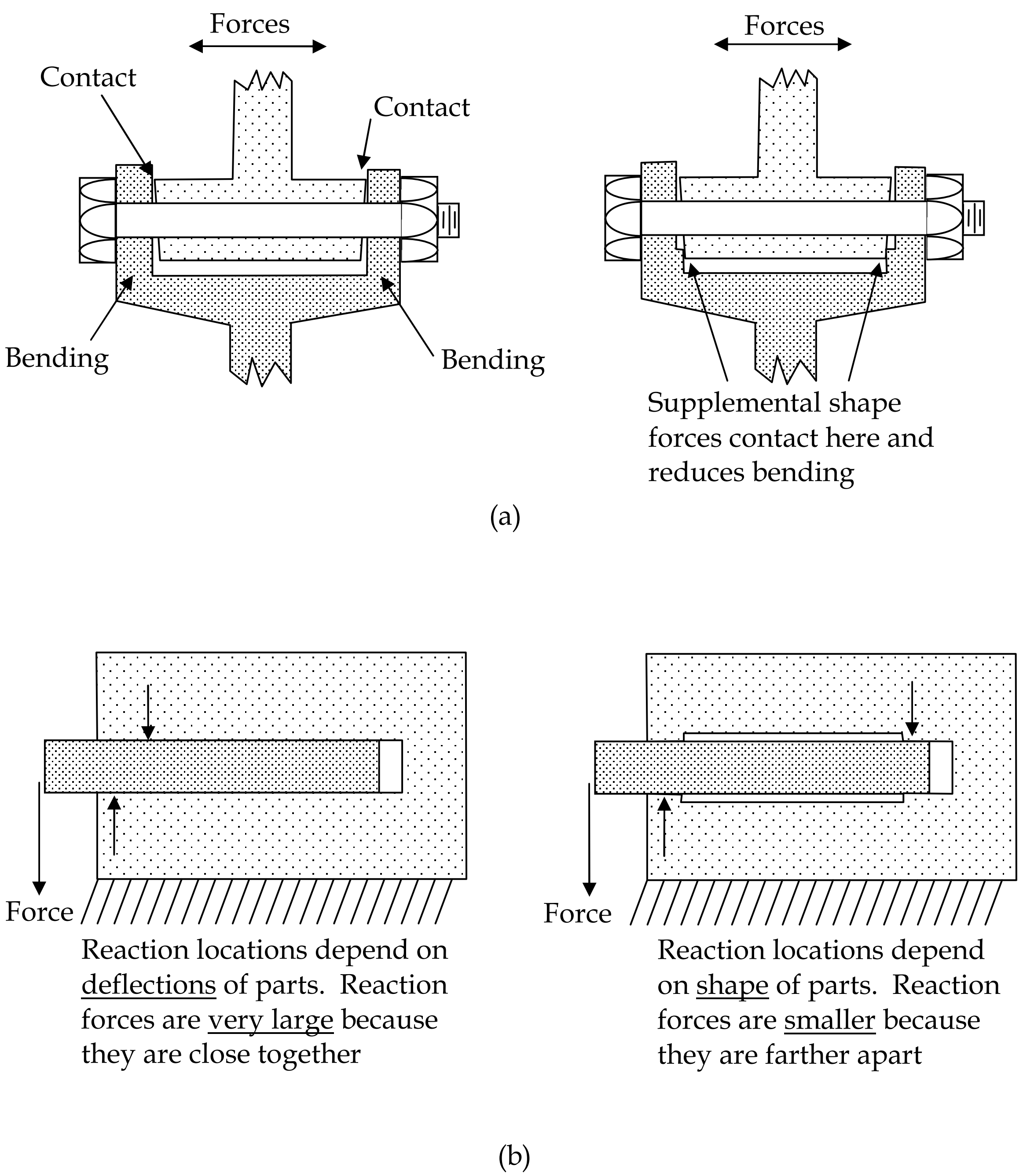


Figure 19. Supplemental shapes used to: (a) reduce bending in a clevis joint, and (b) reduce bending stresses in a pin by shifting the load bearing points.

4.7 *Elastic Matching Principle*.

The Elastic Matching Principle states that the joint elements should be shaped such that the elastic properties of the elements allow a maximum of the material on both sides of the joint to be equally stressed. Figure 20 shows two joints in which the Elastic Matching Principle has been applied. In Figure 20(a) a bolted joint has carefully shaped nut and thread portions which allow the deflections at the ends of the threads in the bolt and nut to be such that the threads in the middle of the joint section will also carry significant load. In Figure 20(b) the connection between a flange and a hollow tube are made such that equal loads are carried in each of the 21 rivets that connect them. Not only is the size, spacing, and number of rivets on each plane important here, but also the thickness of the flange material between each row of rivets determines the elastic deflection between each row of rivets and consequently the load in each of the rivets.

4.8 *Preload*

Joints must be designed to transmit loads between parts. However, the loads on the joint elements are usually made up of two components: the external, applied, load, and the internal, preload. Good joint design incorporates preload, a load built into the joint, present even when there are no external loads being carried from one part to the other.

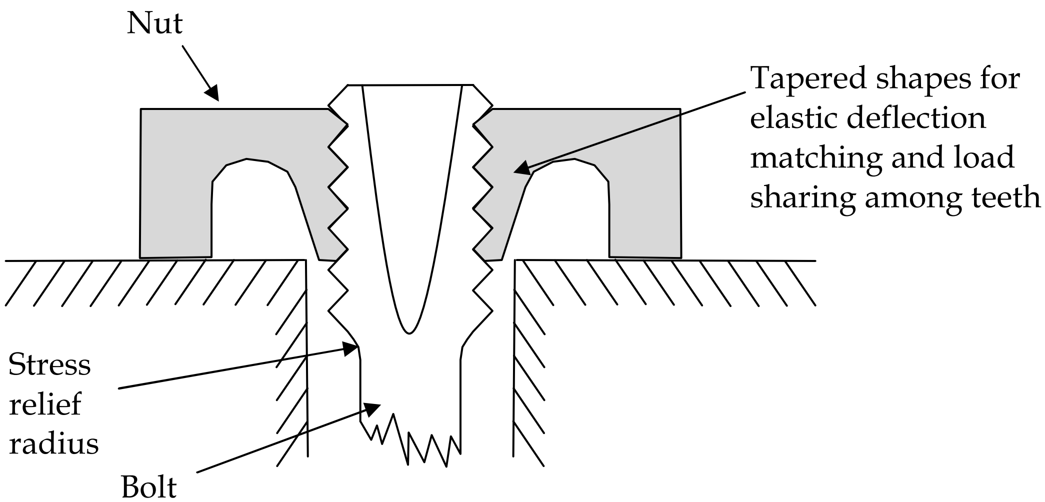
Consider a bolted joint for example. When two parts are fastened with a bolt, the bolt is usually tightened snug, and then further tightened with wrenches. In fact, great care is taken to torque the bolts to the proper tightness during assembly. This pre-tension, or preload, stresses the bolt and the joint areas of the parts which it squeezes together, even though the main body of the parts have no loads applied to them yet.

Preload is good because it helps eliminate variability in the loads that parts carry. Bicycle spokes are preloaded in tension so that even when no one is sitting on it, the spokes are stressed. As rider loads are applied, the spokes on the top of the rim receive more tension, and the spokes on the bottom of the rim receive less. If the preload is sufficient, no spoke will ever be loaded in compression. If it did, it surely would buckle, because it is so slender. Because of the preload, supplemental shapes that prevent buckling, and add weight, are not needed.

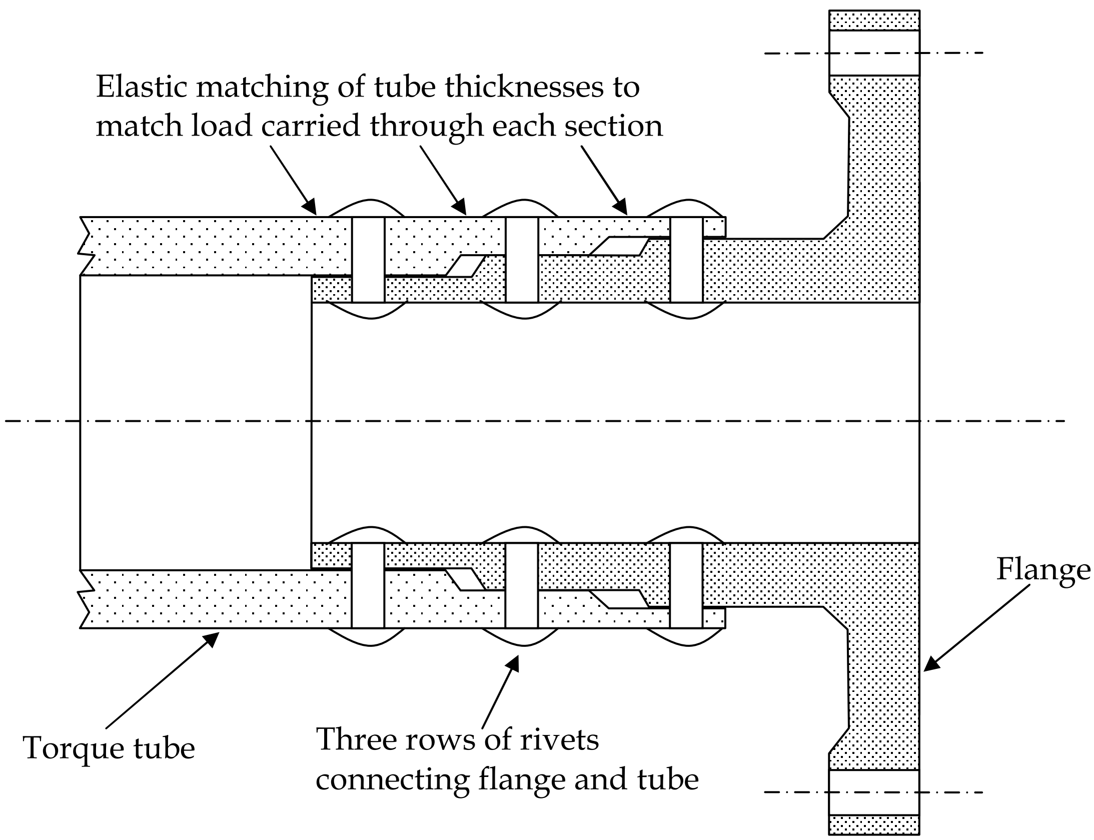
Preload also reduces load variations in joints which must seal. When a lid is screwed tightly onto the top of a jar, a compressive preload is introduced between the lid and the jar. If pressure builds up inside of the jar, the force required to seal the jar is still maintained, provided the preload is greater than the applied separation pressure.

Preload also prevents rattles and vibration in joints. It reduces wear in joints by eliminating small relative motions. It also effects the fatigue life of joints by shifting the damage-causing alternating components of the applied loads onto the fasteners, which can easily be designed to survive these loads.

Joints should be preloaded.



(a)



(b)

Figure 20. Elastic matching in: (a) bolted and nut, and (b) riveted joint.

4.9 *Joint Locking Devices*.

Mechanical interlock joints should be designed so that the normally applied loads tend to make the joint tighter (or at least do not tend to loosen it). However, it is clear that if the joint has been assembled, it could conceivably loosen itself and come apart if the joint is not in some way locked. There are several joint-locking principles which can be applied to prevent this.

There are three components of a locked mechanical interlock joint. The first of these is the major joint element itself. The major joint elements are those which carry the primary load of the joint which is almost always compressive. Most joints have in addition to the major joint elements an intermediate locking element which is capable of applying forces orthogonal to the direction of the major forces in the joint. Consequently, forces perpendicular to the direction of the applied loads in the joint are necessary to fasten the joint. Consider, for example, a simple bolt and nut which fastens two plates together. The primary loads in the bolt are along its axis, but the forces which are necessary to tighten the bolt are perpendicular the bolt axis. Thus, the loads in the bolt do not tend to loosen it. (Because of this, fine-thread bolts will have less tendency to self-loosen because of the smaller helix angle of the threads)

High-performance joints often have a third element in the joint, a final locking device. The loads required to insert or remove the final locking device again are orthogonal to the loads which are required to insert or remove the intermediate locking elements. An example of a final locking device would be a cotter pin used in a castellated nut shown in Figure 21. The final locking device for a high-performance bolted joint could be held by either friction, a spring action, or plastic deformation.

To illustrate the joint-locking device principles of joint design consider the joint shown in Figure 21 in which a wheel is fastened to an axle. The primary forces transmitted between the wheel and the axle consist of forces in the plane of the wheel as well as moments about the axle. The forces in the plane of the wheel are taken by direct contact between the wheel and the axle. The moments about the axle are taken by the mechanical interlock in the key way between the wheel and the axle. These elements constitute the major joint elements. The entire joint is held in place by a nut threaded on to the axle shaft as shown in the figure. The mechanical interlock between the nut and the axle shaft constitutes the intermediate locking device. It is held in place by friction which is augmented by the angle of the threads. The final joint-locking device is the cotter pin which passes through the nut and the axle shaft. This cotter pin is held in place by plastic deformation as the ends of the pin are bent over.

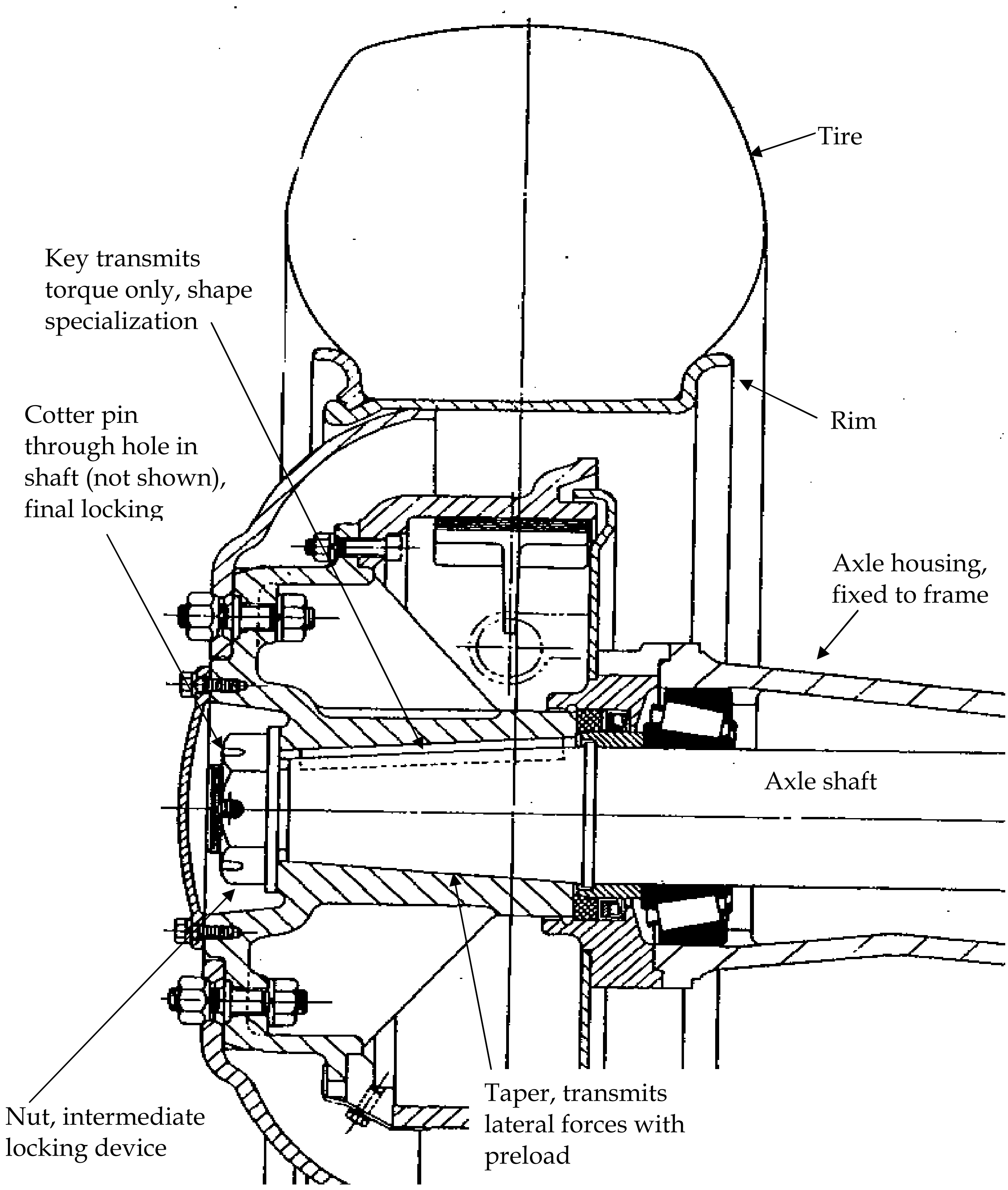


Figure 21. Wheel mounted on a tapered axle shaft with a key, held on by a castellated nut, and a Cotter Pin (not shown).

5.0 Overall Procedure for Form Synthesis of Parts and Joints.

Having presented many design rules and principles for the design of parts and joint elements, it is now possible to list a procedure for the design of high-strength, high-performance parts. Since design is an iterative process, the order of this procedure is not sacred. However, it should serve as a good starting point, and if nothing else, a checklist.

DESIGN THE BODY OF THE PART FIRST

1. Note where the given forces are. Note their positions and directions.

2. Conceive where metal must be to transmit these forces using uniform stress as much as possible.

3. Consider the strong stress-patterns: tension, compression, transverse-shear, torsion in a hollow tube, and bending of I-beam-like cross sections; and the weak stress-patterns: bending, torsion and spot contact. Use the strong patterns; avoid the weak.

4. Consider the corollary principles: triangle, tetrahedron, mating-surface.

5. Use the leverage principle, if possible.

6. Use anti-buckling stiffeners, if compressive forces are present.

7. Use supplementary shapes.

8. Use shape-merging sections where needed around joints.

9. Avoid stress concentration (internal sharp corners).

10. Remove metal at under-stressed regions.

11. Check the proposed design for "roughly-uniform-size".

12. Position tetrahedrons to have the external forces at their corners.

13. Consider manufacturing processes and cost-of-production, and modify the ideal shapes accordingly.

14. Iterate and refine. Visualize the flow of stresses through the body of the part and improve the shapes again, according to form-synthesis principles, but remember the constraints of manufacturing processes and cost.

DESIGN THE JOINT ELEMENTS SECOND.

1. Select overall joint type, i.e.: many small fasteners, few large fasteners, or a combination.

2. Conceive overall joint configuration using uniform stress as much as possible and applying the joint-element synthesis principles.

3. Consider the principle of elastic matching.

4. Use preloading wherever feasible.

5. The actual mating surfaces must be larger than the cross-section of the equivalent part, especially if there is to be relative sliding motion.

6. Design the bridges, as much as possible, to use strong-type stress patterns.

7. Use short bridges.

8. Design intermediate stress-gathering ribs, etc., to blend gradually into the thinner, uniformly-stressed regions of the part removed from the discrete bridges.

9. Modify shapes to minimize stress-concentration effects.

10. Remove metal where not needed.

11. Consider manufacturing processes and cost-of-production, and modify ideal shapes accordingly.

12. Iterate and refine. Visualize the flow of stresses through the joint elements and improve the shapes again according to form-synthesis principles, but remember the constraints of manufacturing processes and cost.