LAB 6: SYSTEM IDENTIFICATION OF A SECOND ORDER SYSTEM

Overview Of ID Techniques

10/06/2008

Four techniques are used to identify the system parameters ω_n and ζ for the 2nd order system studied in this experiment. These techniques fall into two categories: (1) Time domain techniques and (2) Frequency domain techniques.

Time Domain Techniques

Both the time domain techniques require a step input to be given to the system. This is done in this lab by using a square wave from the function generator as input. The square wave can be thought of as a series of step inputs. The output is recorded using SYSTEMID.VI.

1. Percentage Overshoot Method

Input: Step Input
Output: As shown in Figure 1:

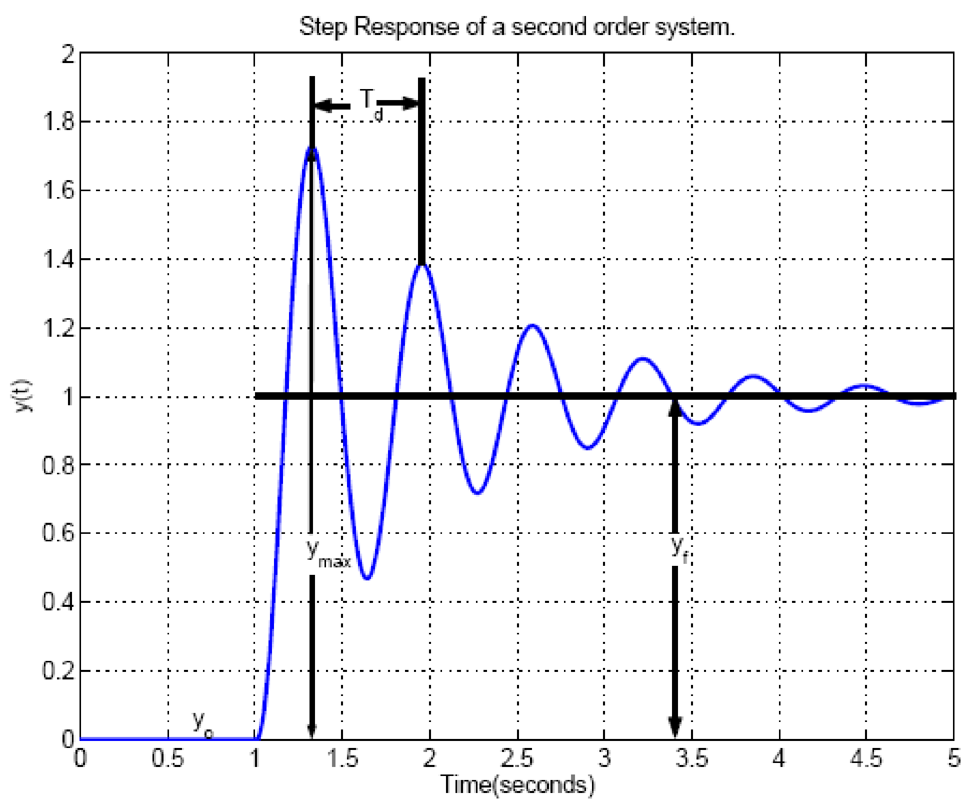

Figure 1: Step response of a second order system for percentage overshoot calculations
Numbers are for illustrative purposes only. y_0 is not equal to 0 in this experiment.

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Procedure:

- The analytical expression for the above step response (*y(t)*) is given by

$$y(t) = y_0 + (y_f - y_0)\left[1 - \frac{\exp(-\zeta\omega_n t)}{\sqrt{1-\zeta^2}}.\sin(\omega_d t + \phi)\right]$$

where $\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$. A version of this equation with y_0=0 is available in equation 16 in page 6-12 of the course packet.

- This can be algebraically manipulated to give:

$$\frac{y_{\max} - y_f}{y_f - y_0} = \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

where y_{max} is the amplitude of the first peak in the oscillations, y_f is the steady state value of the output and y_0 is the initial value of the output. All these are marked in Figure 1.

The above equation can be rearranged to solve for $\boxed{\zeta}$.

- If the time period of the oscillations is T_d, then the *damped natural frequency* is given by $\omega_d = \frac{2\pi}{T_d}$. From the damped natural frequency and damping ratio, the undamped natural frequency can be calculated using $\boxed{\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}}$.

Limitations: For this method to work, the output must show oscillations. This only happens when the system is underdamped i.e. only if $\boxed{\zeta < 1}$.

2. Log Decrement Method

Input: Step Input
Output: As shown in Figure 2

Procedure:

- Denote the first peak in the output as y_i (which is the same as y_{max} for the percentage overshoot method). The peaks after the first peak are labeled in order as y_{i+1}, y_{i+2}, …, y_{i+n}, …

- Define $\Delta = \frac{1}{n} ln\left[\frac{y_i - y_f}{y_{i+n} - y_f}\right]$ where n refers to the n^{th} peak after the first peak. Hence if $n = 2$, use $y_{i+n} = y_{i+2}$ in the denominator.

With this definition, it can be shown that:

$$\zeta = \frac{\Delta\sqrt{1-\zeta^2}}{2\pi}$$

The above equation can be arranged to solve for $\boxed{\zeta}$.

- Then, following the same procedure as in the percentage overshoot method, the undamped natural frequency $\boxed{\omega_n}$ can be evaluated.

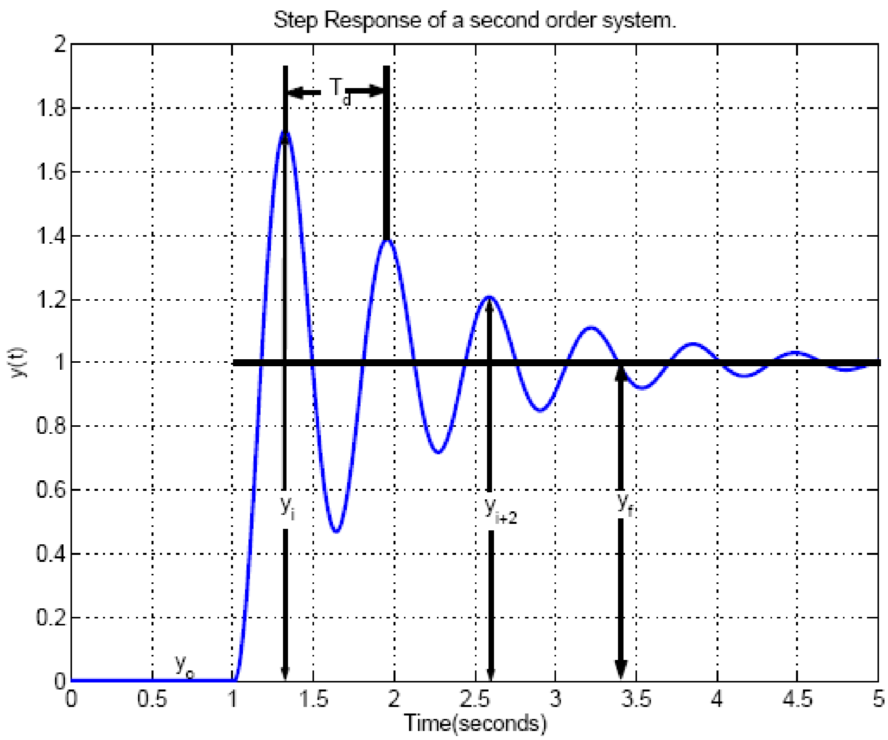

Figure 2: Step response of a second order system for log decrement calculations
Numbers are for illustrative purposes only. y_0 is not equal to 0 in this experiment.

Limitations: Same as for the percentage overshoot method.

Frequency Domain Techniques

Both the frequency domain techniques require the generation of the Bode plot for the system. In this lab, this is generated and recorded using BODE.VI.

1. Half-Power Method

Procedure:

The Bode plot must be converted from log – log scale to a linear scale for both the x and y axis. The magnitude plot in the Bode plot, plotted on a linear scale is shown in Figure 3.

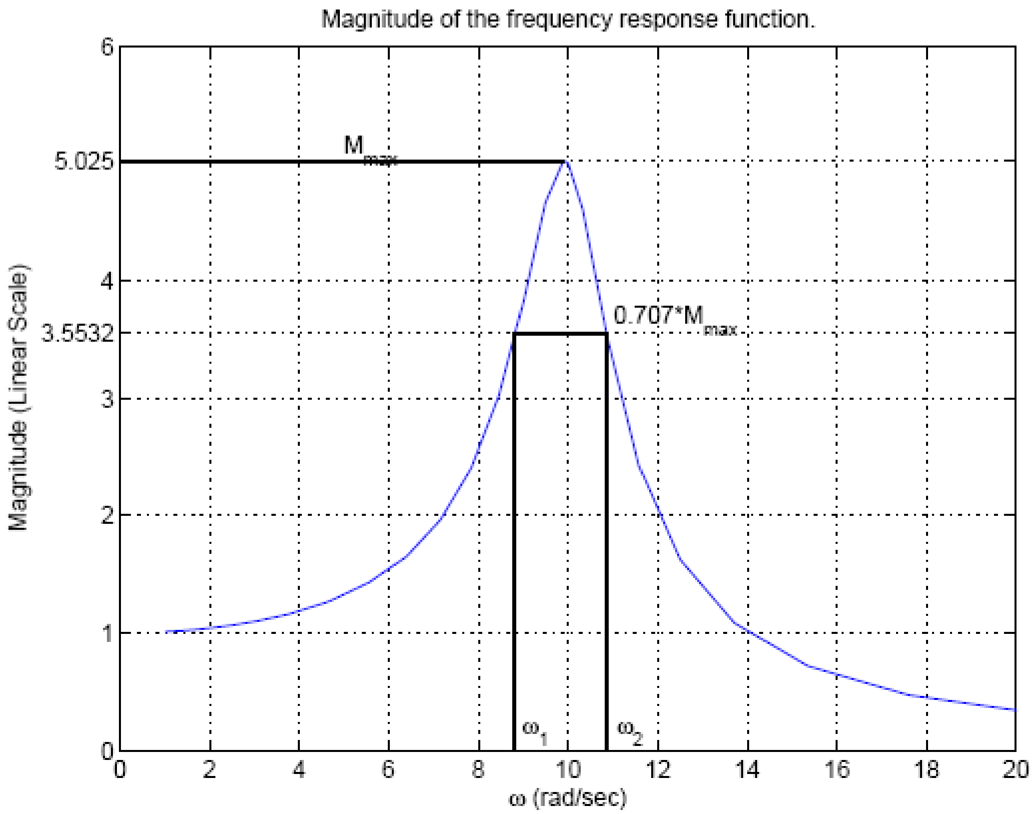

Figure 3: FRF magnitude plot in linear scale.
Numbers are for illustrative purposes only.

- In order to evaluate ω_n, refer to the phase plot of the Bode plot. The undamped natural frequency, $\boxed{\omega_n}$, is the frequency at which the phase ϕ = -90^o. Alternately, a Lissajous figure can be used in the lab to evaluate ω_n.

- Let the peak value of the frequency response function be denoted M_{max}. This peak occurs at a frequency called the *resonant natural frequency*, denoted by ω_r.

- Calculate $\frac{M_{max}}{\sqrt{2}} = 0.707 M_{max}$. Now find the two frequencies, ω_1 and ω_2, at which the FRF attains this magnitude. Then, the damping ratio can be calculated using $\boxed{\zeta = \frac{\omega_2 - \omega_1}{2\omega_n}}$

Limitations: For this method to work, the FRF magnitude plot must show a peak. This only happens when $\boxed{\zeta < \frac{1}{\sqrt{2}}}$..

2. Slope of Phase method

Procedure:

- Convert the phase plot of the Bode plot from log-log scale to linear scale in both phase and frequency. This is shown in Figure 4.
- Express phase in radians and frequency in radians/second.
- Evaluate the undamped natural frequency, $\boxed{\omega_n}$, in the same manner as for the half-power method.
- The damping ratio is evaluated using $\boxed{\zeta = \frac{-1}{\omega_n}\left\{\left[\frac{d\phi}{d\omega}\right]_{\omega=\omega_n}\right\}^{-1}}$ where $\left[\frac{d\phi}{d\omega}\right]_{\omega=\omega_n}$ is the slope of the phase plot at $\omega = \omega_n$ or at $\phi = -\pi/2$ *radians*.

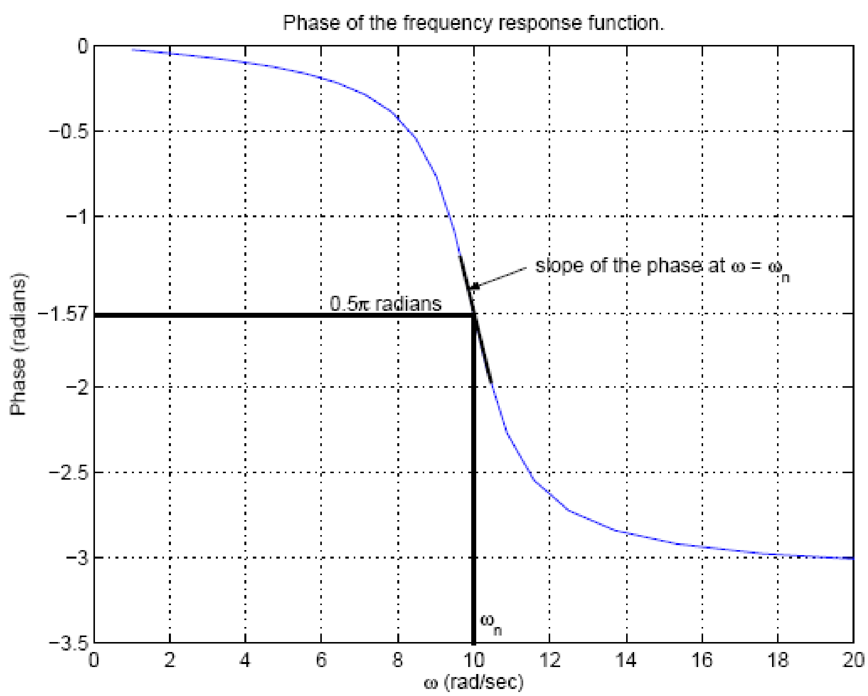

Figure 4: FRF phase vs. frequency in linear scale
Numbers are for illustrative purposes only.

Limitations:None