ME 363 Homework #3

1. (a).

$$h_0 - h_f = \mu^2 R$$

First roll: $\mu = \sqrt{\frac{2\text{-}1.5}{15}} = 0.183$

Second roll: $\mu = \sqrt{\frac{1.5\text{-}1.5\times 0.75}{15}} = 0.158$

Third roll: $\mu = \sqrt{\frac{1.125\text{-}1.125\times 0.75}{15}} = 0.137$

$\therefore$ minimum $\mu = 0.183$

(b).

$$V_0 \cdot w_0 \cdot t_0 = V_1 \cdot w_1 \cdot t_1 = V_2 \cdot w_2 \cdot t_2 = V_3 \cdot w_3 \cdot t_3$$

$$L_0 \cdot w_0 \cdot t_0 = L_3 \cdot w_3 \cdot t_3$$

$$t_3 = 0.75 \times 0.75 \times 0.75 t_0$$

$$w_3 = (1.03)^3 w_0$$

$$L_3 = \frac{w_0 t_0}{w_3 t_3} L_0 = \frac{12}{(0.75\times 1.03)^3} = 26.1\ ft$$

$$V_3 = \frac{V_0}{(0.75\times 1.03)^3} = \frac{40}{0.46} = 86.8\ ft/\min$$

(c).

$$L = \sqrt{R(h_0 - h_1)} = \sqrt{15\times 0.5} = 2.74\ in$$

$$V_r = \frac{V_0 + V_1}{2} = \frac{V_0 + \frac{V_0}{0.75\times 1.03}}{2} = 1.147 V_0 = 45.9\ ft/\min$$

$$\dot{\varepsilon} = \frac{V_r}{L}\ln\frac{1}{0.75} = \frac{45.9\times 12/60}{2.74}\ln\frac{1}{0.75} = 0.96$$

$$\sigma = C\dot{\varepsilon}^m = 24{,}000(0.96)^{0.08} = 23{,}929\ psi$$

$$Y'_{avg} = \frac{2}{\sqrt{3}}\times 23{,}929 = 27{,}631\ psi$$

$$F = 2.74\times 10\times 27{,}631\left(1 + \frac{0.183\times 2.74}{2\times 1.75}\right) = 863{,}000\ lbs$$

2.

(a)

$$K = 180\ MPa \quad n = 0.2$$

$$\frac{\pi}{4} d_0^2 h_0 = \frac{\pi}{4} d_f^2 h_f$$

$$h_f = \frac{d_0^2}{d_f^2} h_0 = \frac{1}{2^2}(2) = 0.5\ in$$

$$\varepsilon = \ln \frac{h_1}{h_0} = \ln \frac{0.5}{2} = -1.386$$

$$\sigma_f = K\varepsilon^n = 180(1.386)^{0.2} = 192.1\ MPa$$

$$P_a = \sigma_f \left(1 + \frac{\mu d_1}{3h_1}\right) = 192.1\left(1 + \frac{0.3 \times 2.0}{3 \times 0.5}\right) = 268.9\ MPa$$

$$F = P_a \cdot \frac{\pi}{4} d_1^2 = 268.9 \cdot \frac{\pi}{4}\left(2 \times 25.4\right)^2 = 545\ kN$$

(b)

$$\dot{\varepsilon} = \frac{v}{h} = \frac{0.5 \times 10^3}{0.5 \times 25.4} = 39.4$$

$$\sigma_f = C \cdot \dot{\varepsilon}^m = 80 \cdot 39.4^{0.07} = 103.5\ MPa$$

$$P_a = \sigma_f \left(1 + \frac{0.3 \times 2.0}{3 \times 0.5}\right) = 103.5 \times 1.4 = 144.9\ MPa$$

$$F = 144.9 \times \frac{\pi}{4} \times \left(2 \times 25.4\right)^2 = 293.7\ kN$$

3.

$$R = \frac{A_0}{A_f} = \frac{d_0^2}{d_1^2} = \frac{100^2}{40^2} = 6.25$$

$$\varepsilon = \ln \frac{A_0}{A_f} = \ln 6.25 = 1.83$$

$$\dot{\varepsilon} = \frac{6 \times 0.2 \times (0.1)^2 \times \tan 30°}{(0.1)^3 - (0.04)^3} 1.83 = 13.55$$

$$\sigma_{fm} = C\dot{\varepsilon}^m = 120 \times (13.55)^{0.1} = 155.7 \ MPa$$

$$\sigma_e = \sigma_{fm} \left(1.7 \ln R + \frac{2L}{D_0} \right)$$

$$= 155.7 \left(1.7 \times 1.83 + \frac{2 \times 100}{100} \right) = 796 \ MPa$$

$$F = \sigma_e A_0 = 796 \times \frac{\pi}{4} \times (100)^2 = 6.25 \ MN$$

$\therefore$ 10 MN one

4.

(*a*)

$$\varepsilon = \ln\frac{A_0}{A_f} = 2\ln\frac{3.0}{2.5} = 0.365$$

$$\sigma_{fm} = \frac{K}{\varepsilon}\left(\frac{\varepsilon^{n+1}}{n+1}\right) = \frac{500}{0.365}\left(\frac{0.365^{0.3+1}}{0.3+1}\right) = 284\ MPa$$

$$\sigma_d = \sigma_{fm}\left[\left(1+\frac{\mu}{\alpha}\right)\ln\frac{A_0}{A_f} + \frac{2}{3}\alpha\right]$$

$$= 284\left[\left(1+\frac{0.07}{0.262}\right)0.365 + \frac{2}{3}0.262\right] = 181\ MPa$$

(*b*)

$$F_d = 181\times10^6 \times \frac{\pi}{4} \times (2.5)^2 \times 10^{-6} = 888\ N$$

(*c*)

$$\frac{1.5}{3.0} = (0.61)^n$$

$$n \cong 1.4$$

$\therefore$ 2 steps