Solution to Problem 1.
(i) The sum of the moments about the right-hand bearing C can be written as

$$\Sigma M_C = 0 \tag{1a}$$

that is

$$14R_o - 7F_A - 2F_B = 0 \tag{1b}$$

Substituting the maximum forces at A and B, that is, $F_A = 1300$ lb and $F_B = 500$ lb into Eq. (1b), the maximum reaction force at bearing O is

$$R_o = 721.43 \text{ lb} \tag{2}$$

The sum of the forces in the Y-direction can be written as

$$\Sigma F_y = 0 \tag{3a}$$

that is

$$R_o + R_C - F_A + F_B = 0 \tag{3b}$$

Substituting the maximum force at A and B and Eq. (2) into Eq. (3b), the maximum reaction force at bearing C is

$$R_C = 1078.57 \text{ lb} \tag{4}$$

The shear force diagram showing the maximum forces on the bar is shown in Figure 1a.

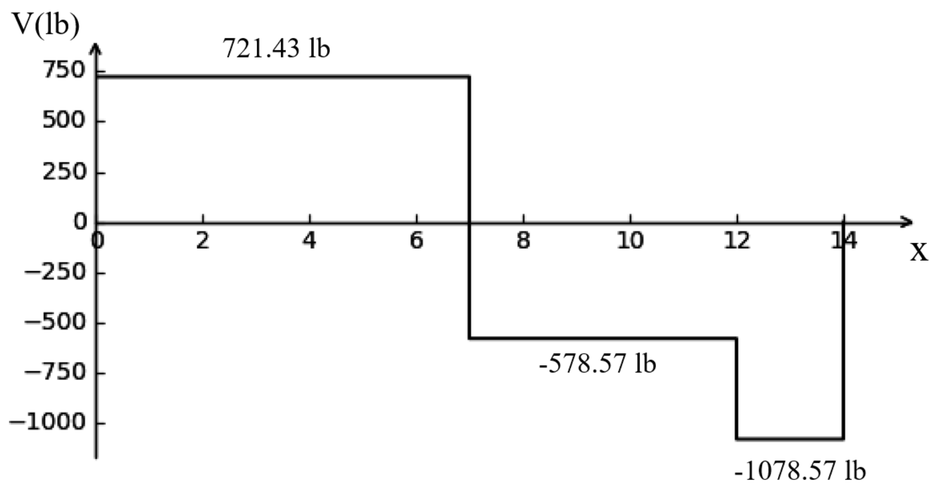

Figure 1a. The shear force diagram of the bar.

The maximum and minimum bending moments at section A are

$$M_{A_{max}} = 7\ R_O = 7 \times 721.43 = 5050.01 \text{ lb} \cdot \text{in} \quad \text{and} \quad M_{A_{min}} = 0 \tag{5a}$$

and the maximum and minimum bending moments at section B are

$$M_{B_{max}} = 12\ R_O - 5\ F_A = 2157.16 \text{ lb} \cdot \text{in} \quad \text{and} \quad M_{B_{min}} = 0 \tag{5b}$$

The bending moment diagram of the bar is shown in Figure 1b.

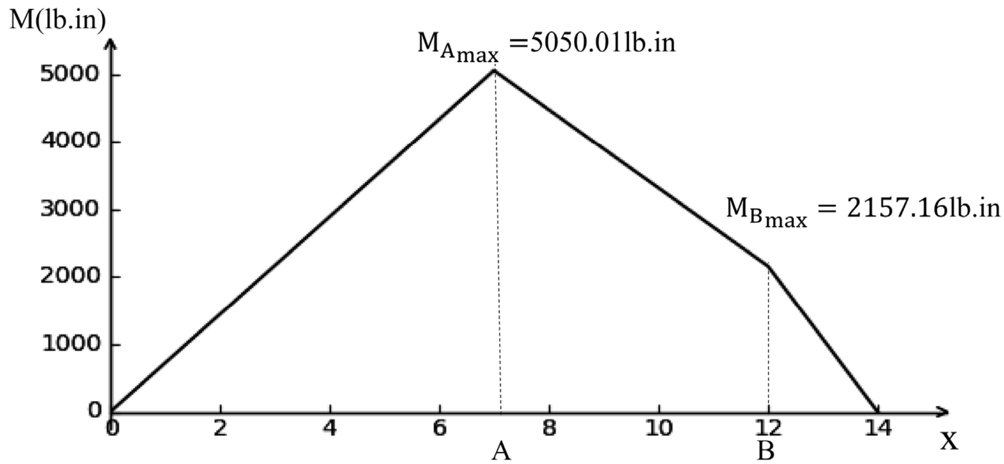

Figure 1b. The bending moment diagram of the bar.

(ii) Note that the bending moment at section A is the largest and the diameter at section A is the smallest and the stress concentration effects are only at section A. Therefore, the critical section of the bar is section A. The critical element is at the outer edge of the critical section.

The factor of safety guarding against yielding using the Langer line can be written from Eq. (6-43), see page 330, as

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} \tag{6}$$

The yield strength of AISI 1030 cold drawn steel, see Table A-20, page 1056, is

$$S_y = 64 \text{ kpsi} \tag{7}$$

From Eq. (5a), the alternating component of the bending moment at section A is

$$M_{A_a} = \frac{|M_{A_{max}} - M_{A_{min}}|}{2} = 2525 \text{lb.in} \tag{8a}$$

and the mean component of the bending moment at section A is

$$M_{A_m} = \frac{(M_{A_{max}} + M_{A_{min}})}{2} = 2525 \text{lb.in} \tag{8b}$$

The alternating and mean components of the normal stress at section A are

$$\sigma_a = \frac{M_{A_a} c}{I} = \frac{32 M_{A_a}}{\pi d^3} \quad \text{and} \quad \sigma_m = \frac{M_{A_m} c}{I} = \frac{32 M_{A_m}}{\pi d^3} \tag{9}$$

Substituting Eqs. (8a) and (8b) into Eq. (9), the alternating and mean components of the normal stress are

$$\sigma_a = \frac{2525 \times 32}{\pi 1.6^3} = 6.28 \text{ kpsi} \quad \text{and} \quad \sigma_m = \frac{2525 \times 32}{\pi 1.6^3} = 6.28 \text{ kpsi} \tag{10}$$

Note that the effects of stress concentration are neglected here. Substituting Eqs. (7) and (10) into Eq. (6), the factor of safety guarding against yielding is

$$n_y = \frac{64}{6.28+6.28} = 5.1 \tag{11}$$

(iii) The Gerber parabola criterion of fatigue failure for infinite life from Eq. (6-48), page 334, can be written as

$$n_f = \frac{1}{2}(\frac{\sigma_a}{S_e})(\frac{S_{ut}}{\sigma_m})^2 \left\{-1+\left[1+(\frac{2\,\sigma_m\,S_e}{\sigma_a\,S_{ut}})^2\right]^{1/2}\right\} \tag{12}$$

The fully corrected endurance limit of the bar can be written from Eq. (6-17), see page 309, can be written as

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{13}$$

The uncorrected endurance limit of the bar from Eq. (6-10), see page 305, is

$$S'_e = 0.5\ S_{ut} \tag{14}$$

The ultimate tensile strength of AISI 1030 cold drawn steel, from Table A-20, see page1056, is

$$S_{ut} = 76\ \text{kpsi} \tag{15}$$

Substituting Eq. (15) into Eq. (14), the uncorrected endurance limit of the bar is

$$S'_e = 0.5 \times 76 = 38\ \text{kpsi} \tag{16}$$

The surface modification factor from Eq. (6-18), see page 311, is

$$k_a = aS_{ut}^b = 2.70 \times 76^{-0.265} = 0.857 \tag{17}$$

The size modification factor. Since the round bar is not rotating then the equivalent diameter can be written from Eq. (6-23), see page 313, as

$$d_e = 0.370\ d = 0.370 \times 1.6 = 0.592\ \text{in} \tag{18}$$

Then the size modification factor can be written from Eq. (6-19), see page 312, as

$$k_b = 0.879 d^{-0.107} = 0.879 \times 0.592^{-0.107} = 0.930 \tag{19}$$

Substituting Eqs. (16), (17), and (19) into Eq. (13), the fully corrected endurance strength of the bar is

$$S_e = 0.857 \times 0.930 \times 38 = 30.29\ \text{kpsi} \tag{20}$$

The fatigue stress concentration factor for the critical element from Eq. (6-32), see page 321, is

$$K_f = 1 + q(K_t - 1) \tag{21a}$$

Given D = 2.4 in, d = 1.6 in, and r = 0.4 in, the theoretical stress concentration factor for the critical element from Figure A-15-14, see page 1046, is

$$K_t = 1.5 \tag{21b}$$

Substituting the notch sensitivity $q = 0.88$ and Eq. (21b) into Eq. (21a), the fatigue stress concentration factor is

$$K_f = 1 + 0.88 \times (1.5 - 1) = 1.44 \tag{22}$$

The alternating and mean components of the normal stress on the critical element, including the fatigue stress concentration factor at the groove, can be written as

$$\sigma_a = K_f \sigma_{a,nom} \qquad \text{and} \qquad \sigma_m = K_f \sigma_{m,nom} \tag{23}$$

Substituting Eqs. (10) and (22) into Eq. (23), the alternating and mean components of the normal stress on the critical element are

$$\sigma_a = 1.44 \times 6.28 = 9.04 \text{ kpsi} \qquad \text{and} \qquad \sigma_m = 1.44 \times 6.28 = 9.04 \text{ kpsi} \tag{24}$$

Substituting Eqs. (15), (20), and (24) into Eq. (12), the factor of safety as predicted by the Gerber parabola failure criterion for infinite life can be written as

$$N_f = \frac{1}{2}\left(\frac{9.04}{30.29}\right)\left(\frac{76}{9.04}\right)^2 \left\{-1 + \left[1 + \left(\frac{2\times 9.04 \times 30.29}{9.04 \times 76}\right)^2\right]^{1/2}\right\} \tag{25a}$$

Therefore, the fatigue factor of safety as predicted by the Gerber parabola criterion of failure for infinite life is

$$N_f = 2.9 \tag{25b}$$

Solution to Problem 2. The free body diagram of the shaft is shown in Figure 1.

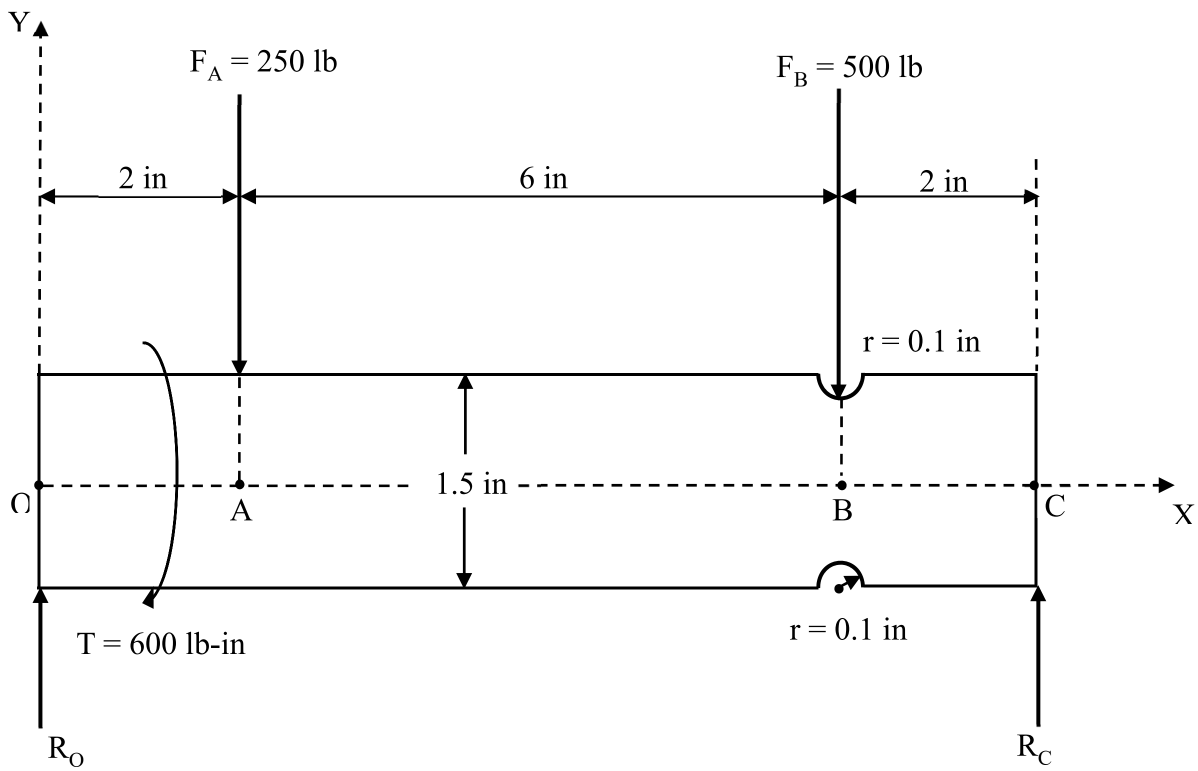

Figure 1. The free body diagram of the shaft. (Not drawn to scale).

The sum of the moments about the Z-axis through section O is

$$\Sigma M_O = 0 \qquad (1a)$$

that is

$$-2 \text{ in} \times F_A - 8 \text{ in} \times F_B + 10 \text{ in} \times R_C = 0 \qquad (1b)$$

Substituting $F_A = 250$ lb and $F_B = 500$ lb into Eq. (1b), the reaction force at section C is

$$R_C = 450 \text{ lb} \qquad (1c)$$

The sum of the forces in the Y-direction can be written as

$$\Sigma F_y = 0 \qquad (2a)$$

that is

$$R_O - F_A - F_B + R_C = 0 \qquad (2b)$$

Substituting $F_A = 250$ lb and $F_B = 500$ lb and Eq. (1c) into Eq. (2b), the reaction force at O is

$$R_O = 300 \text{ lb} \qquad (2c)$$

(a) Section 1 (0 in < x < 2 in). A cut through this section gives the free body diagram shown in Figure 2.

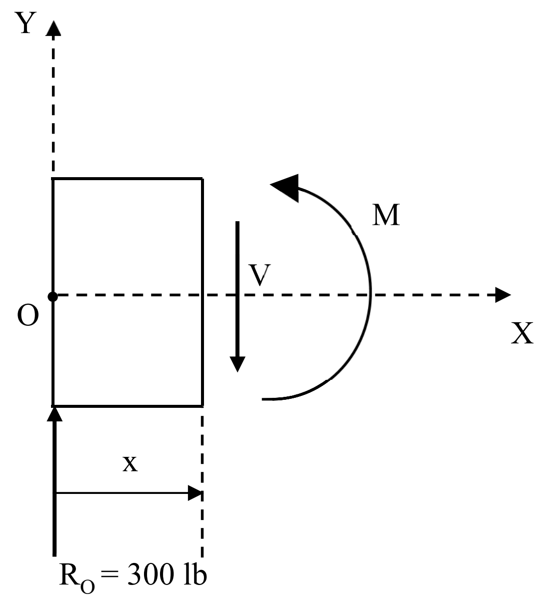

Figure 2. The free body diagram for the section of the shaft between O and A.

The sum of the forces in the Y-direction can be written as

$$\Sigma F_y = 0 \tag{3a}$$

that is

$$R_O - V = 0 \tag{3b}$$

Substituting Eq. $(2c)$ into $(3b)$, the shear force is

$$V = 300 \text{ lb} \tag{3c}$$

The sum of the moments about the Z-axis at section O can be written as

$$\Sigma M_O = 0 \tag{4a}$$

that is

$$+M - \mathrm{x} \cdot V = 0 \tag{4b}$$

Substituting Eq. $(3c)$ into $(4b)$, the bending moment is

$$M = (300 \text{ x}) \text{ lb-in} \tag{4c}$$

Therefore, the bending moment at section A is

$$M_A = 300 \times 2 \text{ lb-in} = 600 \text{ lb-in} \tag{5}$$

(b) Section 2 (2 in < x < 8 in). A cut through this section gives the free body diagram shown in Figure 3.

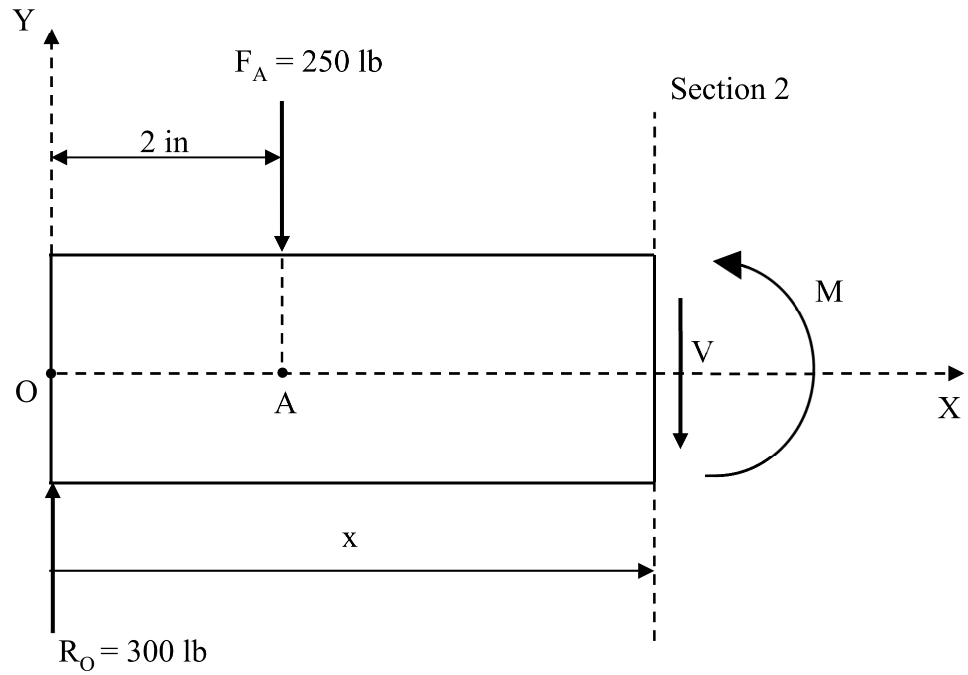

Figure 3. The free body diagram for the section of the shaft between A and B.

The sum of the forces in the Y-direction can be written as

$$\Sigma F_y = 0 \tag{6a}$$

that is

$$R_O - F_A - V = 0 \tag{6b}$$

Substituting Eq. (2c) and $F_A = 250$ lb into $(6b)$, the shear force is

$$V = 50\text{ lb} \tag{6c}$$

The sum of the moments about the Z-axis at section O can be written as

$$\Sigma M_O = 0 \tag{7a}$$

that is

$$+M - \text{x} \cdot V - F_A \times 2\text{ in} = 0 \tag{7b}$$

Substituting Eq. $(6c)$ and $F_A = 250$ lb into $(7b)$, the bending moment is

$$M = (50\text{ x} + 500)\text{ lb-in} \tag{7c}$$

Therefore, the bending moment at section B is

$$M_B = (50 \times 8 + 500)\text{ lb-in} = 900\text{ lb-in} \tag{8}$$

(c) Section 3 (8 in < x < 10 in). A cut through this section gives the free body diagram shown in Figure 4.

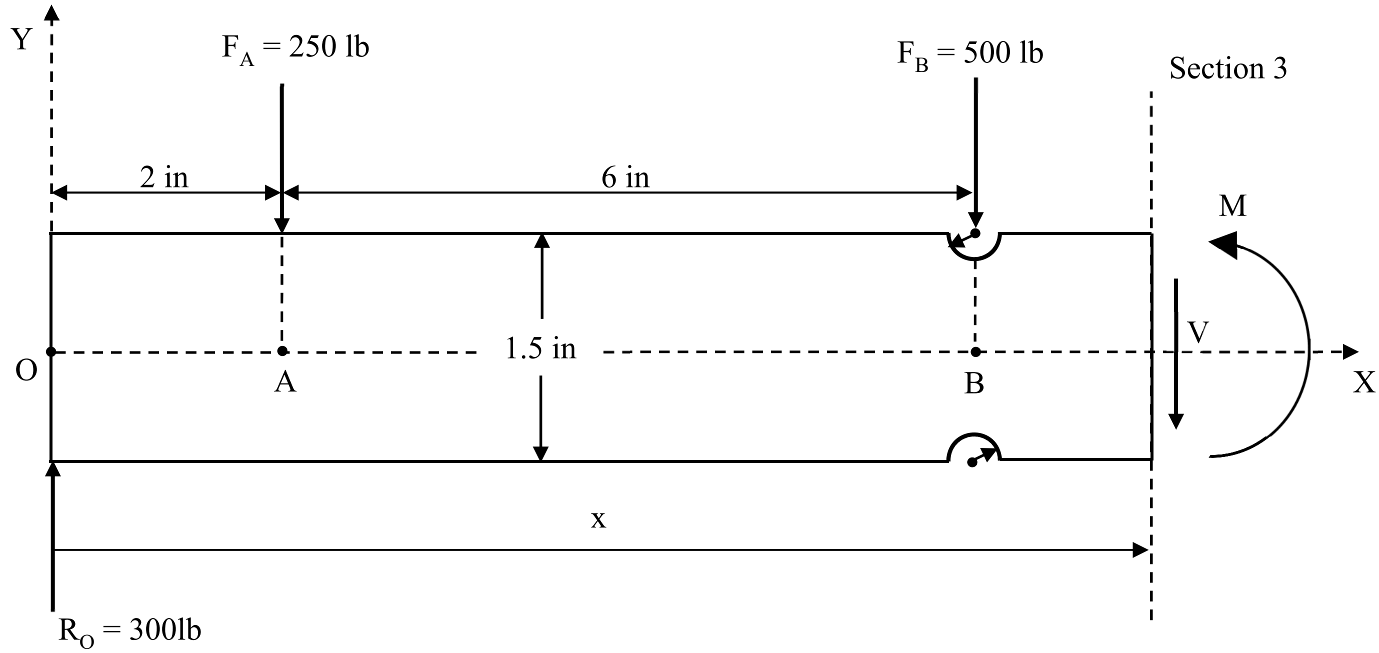

Figure 4. The free body diagram for the section of the shaft between B and C

The sum of the forces in the Y-direction can be written as

$$\Sigma F_y = 0 \tag{9a}$$

that is

$$R_O - F_A - F_B - V = 0 \tag{9b}$$

Substituting Eq. (2c), $F_A = 250$ lb and $F_B = 500$ lb into Eq. $(9b)$, the shear force is

$$V = -450 \text{ lb} \tag{9c}$$

The sum of the moments about the Z-axis at section O can be written as

$$\Sigma M_O = 0 \tag{10a}$$

that is

$$+M - \text{x} \cdot V - F_A \times 2 \text{ in} - F_B \times 8 \text{ in} = 0 \tag{10b}$$

Substituting Eq. $(9c)$, $F_A = 250$ lb and $F_B = 500$ lb into Eq. $(10b)$, the bending moment is

$$M = (-450 \text{ x} + 4500) \text{ lb-in} \tag{10c}$$

The shear force diagram of the shaft in the X-Y plane is shown as Figure 5.

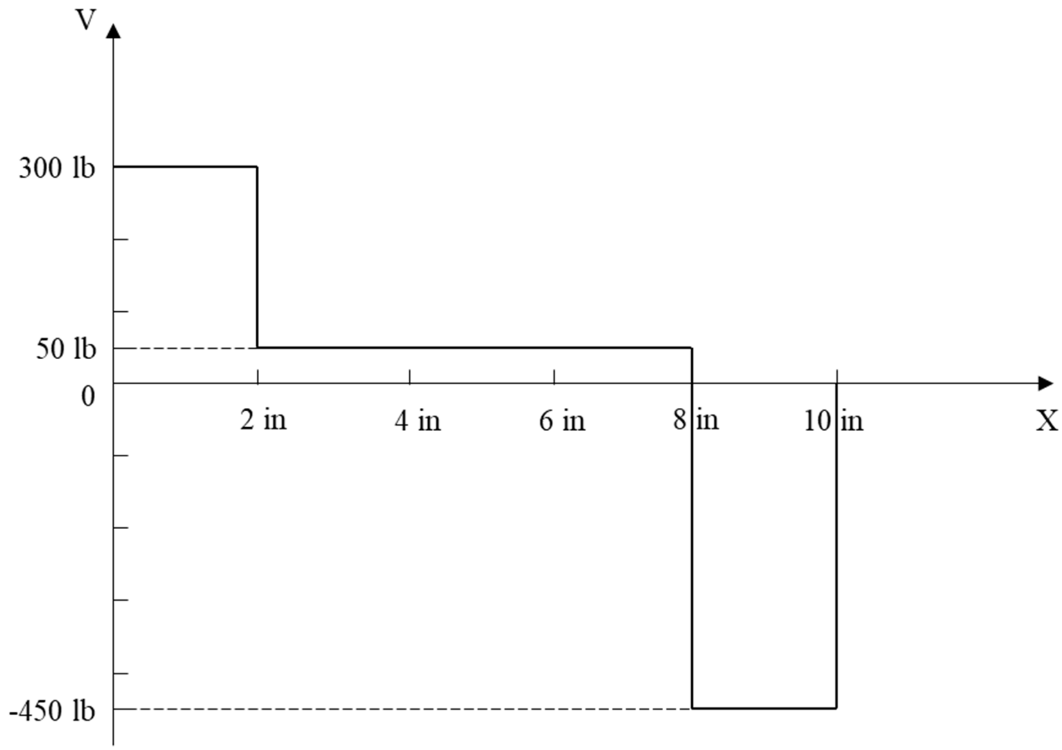

Figure 5. The shear force diagram for the shaft.

The bending moment diagram for the shaft is shown in Figure 6.

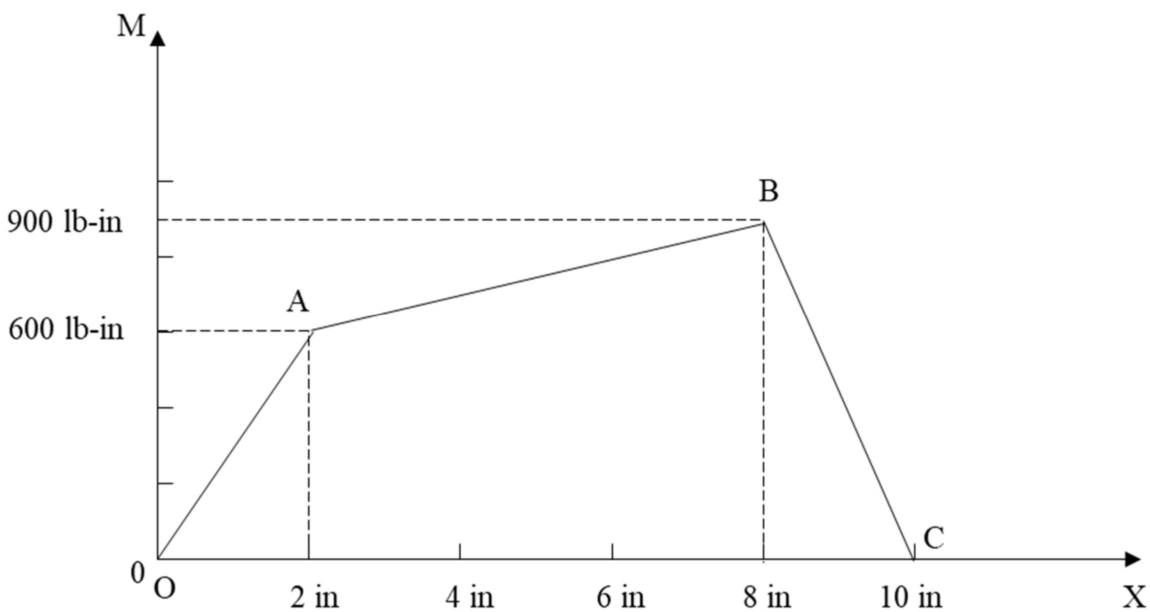

Figure 6. The bending moment diagram for the shaft.

(i) The reaction forces at bearings O and C are given by Eqs. (2c) and (1c), that is

$$R_O = 300 \text{ lb} \qquad \text{and} \qquad R_C = 450 \text{ lb} \tag{11a}$$

The bending moments at sections A and B are given by Eqs. (6) and (8), that is

$$M_A = 600 \text{ lb-in} \qquad \text{and} \qquad M_B = 950 \text{ lb-in} \tag{11b}$$

(ii) The ultimate tensile strength of AISI 1020 CD steel shaft, from Table A-20, see page 1056, is

$$S_{ut} = 68 \text{ kpsi} \tag{12}$$

Therefore, the uncorrected endurance limit can be written from Eq. (6-10), see page 305, as

$$S'_e = 0.5S_{ut} = 34 \text{ kpsi} \tag{13}$$

The fully corrected endurance limit can be written from Eq. (6-17), see page 309, is

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{14}$$

The surface modification factor, see Eq. (6-18), page 311, can be written as

$$k_a = aS_{ut}^b \tag{15a}$$

For machined finish, the coefficient and the exponent from Table 6-2, see page 311, are

$$a = 2.70 \text{ kpsi} \qquad \text{and} \qquad b = -0.265 \tag{15b}$$

Substituting Eqs. (12) and (15b) into Eq. (15a), the surface modification factor is

$$k_a = 2.70 \times 68^{-0.265} = 0.883 \tag{15c}$$

The size factor, see Eq. (6-19), page 312, is

$$k_b = 0.879d^{-0.107} = 0.879 \times 1.3^{-0.107} = 0.855 \tag{16a}$$

The loading factor for combined loading, see page 314, is

$$k_c = 1 \tag{16b}$$

The remaining modification factors are specified as

$$k_d = k_e = k_f = 1 \tag{16c}$$

Substituting Eqs. (13), (15c), and (16c) into Eq. (14), the fully corrected endurance limit is

$$S_e = 25.669 \text{ kpsi} \tag{17}$$

(iii) The largest bending moment is at section B, see Figure 6, and the stress raiser is at section B. Therefore, the critical element is on the circumference of the shaft at section B.

The Goodman failure criterion can be written from Eq. (6-40), see page 329, as

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} \tag{18}$$

For the critical element E (which is at the circumference of the shaft at the critical section B), the normal stress due to bending can be written as

$$(\sigma_x)_{bending} = \frac{M_B y}{I} = \frac{M_B \frac{d}{2}}{I} \tag{19a}$$

where the diameter of the shaft at section B is

$$d = D - 2r = 1.5 \text{ in} - 2 \times 0.1 \text{ in} = 1.3 \text{ in} \tag{19b}$$

The second moment of area of the shaft at section B is

$$I = \frac{\pi d^4}{64} \tag{19c}$$

Substituting Eqs. (19b) and (19c) into Eq. (19a), the normal stress due to bending is

$$(\sigma_x)_{bending} = \frac{32 M_B}{\pi d^3} = \frac{32 \times 900 \text{ lb-in}}{\pi \times (1.3 \text{ in})^3} = 4.173 \text{ kpsi} \tag{20}$$

Since the shaft is rotating then the normal stress due to bending is fully reversed, that is, the mean stress is zero. The alternating and midrange components of the normal stress due to bending are

$$\left|(\sigma_x)_{bending}\right| = 4.173 \text{ kpsi} \qquad \text{and} \qquad (\sigma_m)_{bending} = 0 \tag{21}$$

The shear stress due to the torque acting on the critical element E can be written as

$$(\tau)_{torsion} = \frac{Tc}{J} \tag{22a}$$

Substituting $J = \frac{1}{32}\pi d^4$ and $c = \frac{d}{2}$ into Eq. (22a), the torsional shear stress is

$$(\tau)_{torsion} = \frac{16T}{\pi d^3} = \frac{16 \times 600 \text{ lb-in}}{\pi \times (1.3 \text{ in})^3} = 1.391 \text{ kpsi} \tag{22b}$$

Since the torque is constant then the alternating and midrange components of the shear stress are

$$(\tau_a)_{torsion} = 0 \qquad \text{and} \qquad (\tau_m)_{torsion} = 1.391 \text{ kpsi} \tag{23}$$

The von Mises alternating component of stress and the von Mises mean component of stress for the critical stress element E can be written from Eqs. (6-66) and (6-67) see page 348, as

$$\sigma'_a = \left\{\left[\left(K_f\right)_{bending}(\sigma_a)_{bending}\right]^2 + 3\left[\left(K_{fs}\right)_{torsion}(\tau_a)_{torsion}\right]^2\right\}^{\frac{1}{2}} \tag{24a}$$

and

$$\sigma_m' = \left\{\left[\left(K_f\right)_{bending}\left(\sigma_m\right)_{bending}\right]^2 + 3\left[\left(K_{fs}\right)_{torsion}\left(\tau_m\right)_{torsion}\right]^2\right\}^{\frac{1}{2}} \tag{24b}$$

Substituting Eqs. (21) and (23) and $K_f = K_{fm} = 1.57$ and $K_{fs} = K_{fs_m} = 1.33$ into Eqs. (24a) and (24b), the von Mises alternating component of stress is

$$\sigma_a' = 6.552 \text{ kpsi} \tag{25a}$$

and the von Mises mean component of stress is

$$\sigma_m' = 3.204 \text{ kpsi} \tag{25b}$$

Substituting Eqs. (12), (17), (25a), and (25b) into Eq. (18), the fatigue factor of safety can be written as

$$n_f = \frac{1}{\frac{6.552}{25.669} + \frac{3.204}{68}} \tag{26a}$$

Therefore, the fatigue factor of safety is

$$n_f = 3.3 \tag{26b}$$

Alternative approach. The fatigue factor of safety from the Goodman criterion can be written from Eq. (7-7), see page 382, as

$$\frac{1}{n_f} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{\frac{1}{2}} + \frac{1}{S_{ut}}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{\frac{1}{2}}\right\} \tag{27}$$

Since the shaft is rotating and the torque is constant, at the critical element, then the midrange and alternating components of the bending moment are

$$M_m = 0 \qquad \text{and} \qquad M_a = |M_B| = 900 \text{ lb-in} \tag{28a}$$

The midrange and alternating components of the torque are

$$T_m = T = 600 \text{ lb-in} \qquad \text{and} \qquad T_a = 0 \tag{28b}$$

Substituting Eqs. (28b) and $K_f = K_{fm} = 1.57$ and $K_{fs} = K_{fs_m} = 1.33$ into Eq. (27), the fatigue factor of safety from the Goodman criterion can be written as

$$n_f = \frac{1}{\frac{16}{\pi d^3}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)} \tag{29}$$

where the coefficients are

$$A = \sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} = 2826 \text{ lb-in} \tag{30a}$$

and

$$B = \sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2} = 1382.177 \text{ lb-in} \tag{30b}$$

Substituting Eqs. (30b) into Eq. (29), the fatigue factor of safety from the Goodman criterion of failure is

$$n_f = 3.3 \tag{31}$$

Note that this answer is in complete agreement with Eq. (26b).

Solution to Problem 3. The free body diagram of the bar is shown in Figure 1.

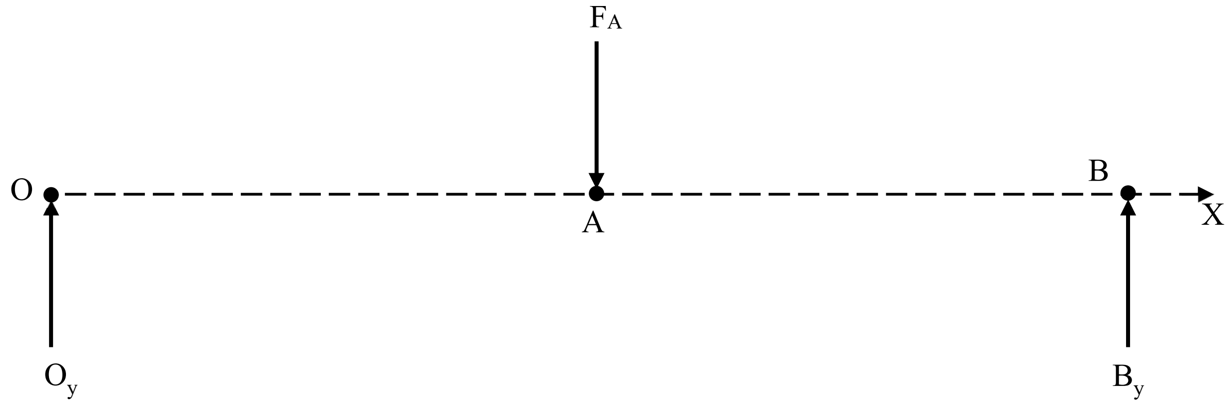

Figure 1. The free body diagram of the bar.

The sum of the moments about bearing O can be written as

$$-8\,F + 16\,B_y = 0 \tag{1a}$$

Rearranging this equation, the vertical reaction force at bearing C can be written as

$$B_y = 0.5\,F \tag{1b}$$

The sum of the forces in the Y-direction can be written as

$$O_y + B_y - F = 0 \tag{1c}$$

Substituting Eq. (1b) into Eq. (1c), and rearranging, the vertical reaction force at bearing O can be written as

$$O_y = 0.5\,F \tag{1d}$$

The bending moment diagram for the bar is shown in Figure 2.

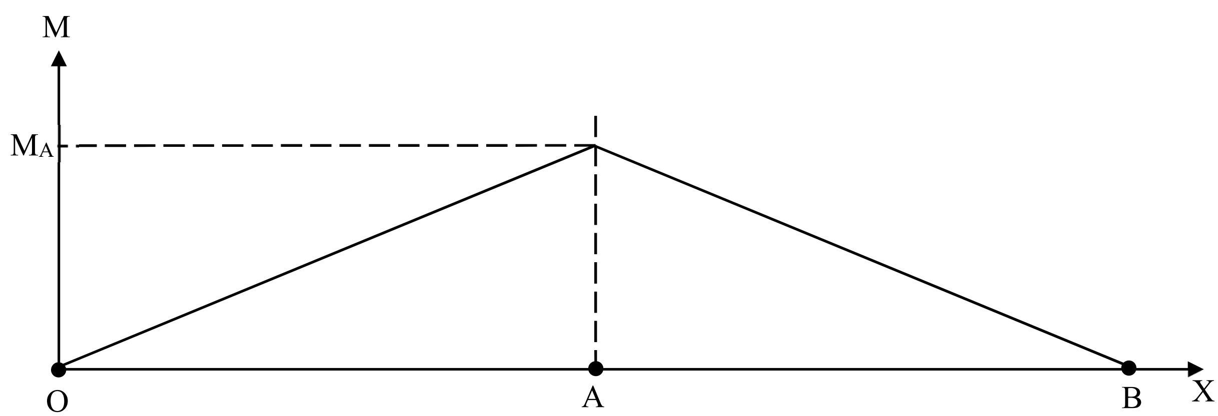

Figure 2. The bending moment diagram for the bar.

The critical section is section A because (i) the bending moment is the largest; and (ii) the effects of stress concentration. The sum of the moments about the critical section A can be written as

$$M_A - 8\,O_y = 0 \tag{2a}$$

Substituting Eq. (1d) into Eq. (2a), and rearranging, the bending moment at the critical section A can be written as

$$M_A = 4\,F \tag{2b}$$

Therefore, the maximum and the minimum bending moments at the critical section A are

$$M_{A,max} = 8.0 \text{ kip-in} \qquad \text{and} \qquad M_{A,min} = -\,4.0 \text{ kip-in} \tag{2c}$$

Part I. (a). For a circular cross-section, the maximum normal stress due to the bending moment M can be written as

$$\sigma = \frac{32\,M}{\pi\,d^3} \tag{3a}$$

The critical element is at the circumference of the bar at the critical section A. The nominal values of the maximum and minimum bending stresses acting on the critical element are

$$\sigma_{\max} = \frac{32\times 8.0}{\pi\times 2.0^3} = 10.186 \text{ kpsi} \qquad \text{and} \qquad \sigma_{\min} = \frac{32\times(-4.0)}{\pi\times 2.0^3} = -5.093 \text{ kpsi} \tag{3b}$$

(b) The theoretical stress concentration for a round bar with a shoulder fillet is given by Figure A-15-9, see page 1044. The geometry is

$$\frac{D}{d} = \frac{3.0}{2.0} = 1.5 \qquad \text{and} \qquad \frac{r}{d} = \frac{0.05}{2.0} = 0.025 \tag{4a}$$

Therefore, the theoretical stress concentration factor is

$$K_t = 2.6 \tag{4b}$$

For the fillet radius r = 0.05 inches and the ultimate tensile strength S_{ut} = 150 kpsi, the notch sensitivity from Figure 6-26, see page 321, is

$$q = 0.87 \tag{5a}$$

Alternative Procedure: The notch sensitivity can be written from Eq. (6-33), see page 322, as

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} \tag{5b}$$

The Neuber constant for bending can be written from Eq. (6-35), see page 322, as

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3\,\sqrt{\text{in}} \tag{5c}$$

Substituting the ultimate tensile strength S_{ut} = 150 kpsi into Eq. (5c), the Neuber constant is

$$\sqrt{a} = 0.034\,\sqrt{\text{in}} \tag{5d}$$

Substituting the specified notch radius and Eq. (5d) into Eq. (5b), the notch sensitivity is

$$q = 0.869 \tag{5e}$$

Note that the answer given by Eq. (5e) is in good agreement with Eq. (5a).

The fatigue stress concentration factor can be written from Eq. (6-32), see page 321, as

$$K_f = 1 + q(K_t - 1) \tag{6a}$$

Substituting Eqs. (4b) and (5a) into Eq. (6a), the fatigue stress concentration factor is

$$K_f = 1 + 0.87(2.6 - 1) = 2.392 \tag{6b}$$

Check: Substituting Eqs. (4b) and (5e) into Eq. (6a), the fatigue stress concentration factor is

$$K_f = 1 + 0.869(2.6 - 1) = 2.391 \tag{6c}$$

The ASME elliptic failure criterion can be written from Eq. (6-52), see page 335, as

$$\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2 = \frac{1}{n_f^2} \tag{7a}$$

The mean and alternating components of the normal stress can be written from Eqs. (6-38) and (6-39), see page 327, as

$$\sigma_m = K_f \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \text{and} \qquad \sigma_a = K_f \frac{|\sigma_{max} - \sigma_{min}|}{2} \tag{7b}$$

Substituting Eqs. (3b) and (6b) into Eq. (7b), the mean and alternating components of the normal stress are

$$\sigma_m = 6.09 \text{ kpsi} \qquad \text{and} \qquad \sigma_a = 18.26 \text{ kpsi} \tag{7c}$$

The ultimate tensile strength and the yield strength of the bar are specified, respectively, as

$$S_{ut} = 150 \text{ kpsi} \qquad \text{and} \qquad S_y = 90 \text{ kpsi} \tag{8a}$$

The uncorrected endurance limit can be written from Eq. (6-10), see page 305, as

$$S'_e = 0.5 S_{ut} = 75 \text{ kpsi} \tag{8b}$$

The fully-corrected endurance limit can be written from Eq. (6-17), see page 309, as

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{8c}$$

Substituting Eq. (8b) and the Marin factors into Eq. (8c), the fully-corrected endurance limit is

$$S_e = 0.716 \times 75 = 53.7 \text{ kpsi} \tag{8d}$$

Substituting Eqs. (7c), (8a), and (8d) into Eq. (7a), the fatigue factor of safety can be written as

$$n_f = \left(\left(\frac{18.26}{53.7}\right)^2 + \left(\frac{6.09}{90}\right)^2\right)^{-1/2} = 2.9 \tag{9}$$

Part II. (a). Increasing the fillet radius to r = 0.15 inches affects the fatigue stress concentration factor. The geometry is

$$\frac{D}{d} = \frac{3.0}{2.0} = 1.5 \qquad \text{and} \qquad \frac{r}{d} = \frac{0.15}{2.0} = 0.075 \tag{10a}$$

The theoretical stress concentration factor from Figure A-15-9, see page 1044, is

$$K_t = 1.81 \tag{10b}$$

For the fillet radius r = 0.15 inches and the ultimate tensile strength S_{ut} = 150 kpsi, the notch sensitivity from Figure 6-26, see page 321, is

$$q = 0.91 \tag{11a}$$

Alternative Procedure. Since the ultimate strength is unchanged then the Neuber constant is as given by Eq. (5d). Therefore, the notch sensitivity obtained from Eqs. (5b) and (5d), with r = 0.15 inches, is

$$q = 0.92 \tag{11b}$$

Substituting Eq. (11a) into Eq. (6a), the new fatigue stress concentration factor is

$$K_f = 1 + 0.91(1.81 - 1) = 1.737 \tag{11c}$$

The nominal maximum and minimum bending stresses are given by Eq. (3b). Therefore, substituting Eqs. (3b) and (11c) into Eq. (7b), the mean and alternating components of the normal stress are

$$\sigma_m = 4.42 \text{ kpsi} \qquad \text{and} \qquad \sigma_a = 13.27 \text{ kpsi} \tag{12}$$

The fully-corrected endurance limit is the same as in Part I; that is, Eq. (8d). Substituting Eqs. (8a), (8d), and (12) into Eq. (7a), the new fatigue factor of safety is

$$n_f = \left(\left(\frac{13.27}{53.7}\right)^2 + \left(\frac{4.42}{90}\right)^2\right)^{-1/2} = 4.0 \tag{13}$$

Comparing Eq. (13) with Eq. (9) shows that increasing the fillet radius to r = 0.15 inches results in a 38% increase (approximately) in the fatigue factor of safety.
(b). Using a ground finish alters the Marin surface modification factor. The coefficient and exponent, for a ground finish, from Table 6-2, see page 311, are

$$a = 1.34 \qquad \text{and} \qquad b = -0.085 \tag{14a}$$

Therefore, the surface modification factor from Eq. (6-18), see page 311, is

$$k_a = aS_{ut}^b = 1.34 \times 150^{-0.085} = 0.875 \tag{14b}$$

Substituting Eqs. (8b) and (14b) and the remaining Marin factors given in Part I into Eq. (8c), the fully corrected endurance limit is

$$S_e = 0.875 \times 75 = 65.625 \text{ kpsi} \tag{15}$$

Note that the fatigue stress concentration factor has not changed, therefore, the mean and alternating components of the normal stress are as given by Eq. (7c). Substituting Eqs. (7c), (8a), and (15) into Eq. (7a), the fatigue factor of safety can be written as

$$n_f = \left(\left(\frac{18.26}{65.625}\right)^2 + \left(\frac{6.09}{90}\right)^2\right)^{-1/2} = 3.5 \tag{16}$$

Comparing Eq. (16) with Eq. (9) indicates that grinding the surface results in a 21% increase (approximately) in the fatigue factor of safety compared to the cold-drawn surface. Also, comparing Eq. (16) with Eq. (13) shows that increasing the fillet radius to r = 0.15 inches compared to the ground surface results in a 14% increase (approximately) in the fatigue factor of safety.

Solution to Problem 4.

(i) The basic dynamic load rating and the static load rating for a single row 02-Series angular contact ball bearing with a bore diameter of 50 mm, see Table 11-2, page 587, are

$$C_{10} = 37.7\,\text{kN} \qquad \text{and} \qquad C_0 = 22.8\,\text{kN} \tag{1}$$

Since the inner ring of the ball bearing is rotating then the rotation factor, see page 585, is

$$V = 1 \tag{2}$$

The radial load and the axial load acting on the ball bearing are specified as

$$F_r = 25\,\text{kN} \qquad \text{and} \qquad F_a = 12\,\text{kN} \tag{3}$$

From Eqs. (1) and (3) the ratio of the axial load to the static load rating is

$$\frac{F_a}{C_0} = \frac{12}{22.8} = 0.5263 \tag{4}$$

The corresponding limit value for the ratio given by Eq. (4), see Table 11-1, page 586, is

$$0.42 \le e \le 0.44 \tag{5}$$

The dimensionless parameter, see Eq. (11-11a), page 585, is

$$\frac{F_a}{VF_r} = \frac{12}{(1)(25)} = 0.48 \tag{6}$$

Since Eq. (6) is greater than Eq. (5) then the axial load cannot be ignored. The equivalent radial load can be written from Eq. (11-12), see page 586, as

$$F_e = X_i V F_r + Y_i F_a \tag{7a}$$

For $i = 2,$ see Table 11-1, page 586, the radial factor $X_2 = 0.56$ and from interpolation

$$\frac{Y_2 - 1.04}{1.00 - 1.04} = \frac{0.5263 - 0.42}{0.56 - 0.42} \tag{7b}$$

Rearranging this equation, the radial factor is

$$Y_2 = 1.0096 \tag{7c}$$

Substituting the given loads and Eqs. (2) and (7c) into Eq. (7a), the equivalent radial load is

$$F_e = 0.56 \times 1 \times 25 + 1.0096 \times 12 = 26.1152 \;\; \text{kN} \tag{8}$$

Note that the equivalent radial load given by Eq. (8) is greater than the applied radial load F_r.

(ii) Substituting $F_e = F_D$ into Eq. (11-9), page 584, the basic dynamic load rating is

$$C_{10} = a_f F_e \left[\frac{x_D}{x_0 + (\theta - x_0)\left\{\ln\left(\frac{1}{R_D}\right)\right\}^{1/b}} \right]^{1/a} \tag{9a}$$

Rearranging Eq. (9a), the design life as a dimensionless multiple of the rating life can be written as

$$x_D = \left[x_0 + (\theta - x_0)\left\{\ln\left(\frac{1}{R_D}\right)\right\}^{1/b} \right]\left(\frac{C_{10}}{a_f F_e}\right)^a \tag{9b}$$

The application factor for poor seals, see Table 11-5, page 589, is

$$a_f = 1.2 \tag{10a}$$

The exponent for ball bearings, see page 580, is

$$a = 3 \tag{10b}$$

The Weibull distribution parameters are specified as

$$x_0 = 0.02, \qquad \theta = 4.459 \qquad \text{and} \qquad b = 1.483 \tag{10c}$$

The desired reliability is specified as

$$R_D = 0.96 \tag{10d}$$

Substituting Eqs. (1), (8), and (10) into Eq. (9b), the design life as dimensionless multiple of the rating life is

$$x_D = \left[0.02 + (4.459 - 0.02)\left\{\ln\left(\frac{1}{0.96}\right)\right\}^{\frac{1}{1.483}} \right]\left(\frac{37.7}{(1.2)(26.1152)}\right)^3 = 0.9289 \tag{11}$$

Check. An approximation to the basic dynamic load rating can we written from Eq. (11-10), see page 584, as

$$C_{10} = a_f F_e \left[\frac{x_D}{x_0 + (\theta - x_0)\{1 - R_D\}^{1/b}} \right]^{1/a} \tag{12a}$$

Substituting Eqs. (1), (8), and (10) into Eq. (12a), the design life as a dimensionless multiple of the rating life is

$$x_D = \left[0.02 + (4.459 - 0.02)\{1 - 0.96\}^{\frac{1}{1.483}} \right] \left(\frac{37.7}{(1.2)(26.1152)} \right)^3 = 0.917 \qquad (12b)$$

Note that the answers given by Eqs. (11) and (12b) are in good agreement.

Solution to Problem 5.

(i) The nominal major diameter (or bolt shank diameter) of the UNC 5/8 in – 11 – grade 7 steel bolt, see Table 8.2, page 425, is

$$d = 0.6250 \text{ in} \tag{1a}$$

Also, the tensile stress area, see Table 8.2, page 425, is

$$A_t = 0.226 \text{ in}^2 \tag{1b}$$

Since the length of the bolt is $L = 3.25$ in then the length of the threaded portion of the bolt can be written from Table 8-7, see page 438, as

$$L_T = 2\,d + \frac{1}{4} \text{ in} \tag{2a}$$

Substituting Eq. (1a) into Eq. (2a), the length of the threaded portion of the bolt is

$$L_T = 2(0.6250) + \frac{1}{4} = 1.50 \text{ in} \tag{2b}$$

The length of the unthreaded portion of the bolt (that is, the bolt shank) can be written from Table 8-7, see page 438, as

$$l_d = L - L_T \tag{3a}$$

Substituting Eq. (2b) into Eq. (3a), the length of the unthreaded portion of the bolt is

$$l_d = 3.25 - 1.50 = 1.75 \text{ in} \tag{3b}$$

The grip is the total thickness of the two plates, that is

$$l = 2(1.25) \text{ in} = 2.5 \text{ in} \tag{4}$$

The length of the threaded portion of the bolt within the grip can be written from Table 8-7, see page 438, as

$$l_t = l - l_d \tag{5a}$$

Substituting Eqs. (3b) and (4) into Eq. (5a), the length of the threaded portion of the bolt within the grip is

$$l_t = 2.5 - 1.75 = 0.75 \text{ in} \tag{5b}$$

The cross-sectional area of the unthreaded portion of the bolt is

$$A_d = \frac{\pi d^2}{4} = \frac{\pi (0.6250)^2}{4} = 0.3068 \text{ in}^2 \tag{6}$$

The stiffness of the bolt can be written from Eq. (8-17), see page 437, is

$$k_b = \frac{A_d\, A_t\, E_b}{A_d\, l_t + A_t\, l_d} \tag{7a}$$

Substituting Eqs. (1b), (3), (5), (6) and $E_b = 28\times10^6$ lbs / in^2 into Eq. (7a), the bolt stiffness is

$$k_b = \frac{(0.3068)(0.226)\times 28\times 10^6}{(0.3068\times 0.75)+(0.226\times 1.75)} = 3.1033\times 10^6 \text{ lbs / in} \tag{7b}$$

The stiffness of the plates (using the diameter of the washer face under the bolt head $\mathrm{d_W} = 1.5\ \mathrm{d}$ and the half apex angle $\alpha = 30^0$) can be written from Eq. (8-22), see page 440, as

$$k_m = \frac{0.5774\ \pi\ E_\mathrm{p}\ d}{2\ln\left(5\dfrac{0.5774\ l + 0.5\ d}{0.5774\ l + 2.5\ d}\right)} \tag{8a}$$

Substituting the modulus of elasticity $E\mathrm{p} = 34\times10^6$ lbs / in^2, the bolt shank diameter $d = 0.625$ in, and Eq. (4), into Eq. (8a), the stiffness of the plates is

$$k_m = \frac{0.5774\times\pi\times 34\times 10^6\times 0.625}{2\ln\left(5\dfrac{0.5774\times 2.5+0.5\times 0.625}{0.5774\times 2.5+2.5\times 0.625}\right)} = 17.981\times 10^6 \text{ lbs / in} \tag{8b}$$

The stiffness constant of the joint can be written from Eq. (f), see page 448, as

$$C = \frac{k_b}{k_b + k_m} \tag{9a}$$

Substituting Eqs. (7b) and (8b) into Eq. (9a), the stiffness constant of the joint is

$$C = \frac{3.1033\times 10^6}{3.1033\times 10^6 + 17.981\times 10^6} = 0.147 \tag{9b}$$

which implies that the percentage of the external load taken by the bolt is 14.7%. This value is acceptable since the joint constant should, in general, be less than 0.20, see Table 8-12, page 448.
(ii) The factor of safety guarding against joint separation can be written from Eq. (8-30), see page 452, as

$$n_0 = \frac{F_i}{P(1-C)} \tag{10a}$$

Substituting the preload $F_i = 20$ kips, the maximum value of the external load $P = P_{\max} = 2$ kips, and Eq. (9b) into Eq. (10a), the factor of safety against joint separation is

$$n_0 = \frac{20\times 10^3}{2\times 10^3(1-0.147)} = 11.72 \tag{10b}$$

The load factor can be written from Eq. (8-29), see page 452, as

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{F_p - F_i}{CP} \tag{11}$$

The proof strength of the SAE grade 7 steel bolt from Table 8-9, see page 444, is

$$S_p = 105 \text{ kpsi} \tag{12}$$

Therefore, the proof load from Eqs. (1b) and (12) is

$$F_P = S_p A_t = 105 \text{ x } 0.226 = 23.73 \text{ kips} \tag{13a}$$

The ratio of the given preload to the proof load from Eq. (13a) is

$$\frac{F_i}{F_P} = \frac{20}{23.73} = 0.84 \tag{13b}$$

Substituting Eqs. (1b), (9b), (12), the preload $F_i = 20$ kips, and the maximum value of the external load $P = P_{\max} = 2$ kips into Eq. (11), the load factor is

$$n_L = \frac{105 \times 10^3 \times 0.226 - 20 \times 10^3}{0.147 \times 2000} = 12.7 \tag{14}$$

(iii) The fatigue factor of safety using the Goodman criterion of failure can be written from Eq. (8.38), see page 458, as

$$n_f = \frac{S_e (S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} \tag{15}$$

The ultimate tensile strength of the SAE grade 7 steel bolt from Table 8-9, see page 444, is

$$S_{ut} = 133 \text{ kpsi} \tag{16}$$

The fully corrected endurance strength of the bolt with $d = 0.625$ in from Table 8-17, see page 457, is

$$S_e = 20.6 \text{ kpsi} \tag{17}$$

The preload stress can be written from Example 8.3, see page 451, as

$$\sigma_i = \frac{F_i}{A_t} \tag{18a}$$

Substituting the preload $F_i = 20$ kips and Eq. (1b) into Eq. (18a), the preload stress is

$$\sigma_i = \frac{20 \times 10^3}{0.226} = 88.496 \text{ kpsi} \tag{18b}$$

The mean stress can be written from Eq. (8.36), see page 457, as

$$\sigma_m = \frac{C(P_{\max} + P_{\min})}{2A_t} + \frac{F_i}{A_t} \tag{19a}$$

Substituting Eqs. (1b), (9b), (18b), $P_{\max} = 2$ kips, $P_{\min} = 750$ lb, and the preload $F_i = 20$ kips, into Eq. (19a), the mean stress can be written as

$$\sigma_m = \frac{0.147(2000 + 750)}{2 \text{ x } 0.226}\text{psi} + \frac{20\text{x}10^3}{0.226}\text{ kpsi} \tag{19b}$$

Therefore, the mean stress is

$$\sigma_m = 894.36 \text{ psi} + 88495.58 \text{ psi} = 89.39 \text{ kpsi} \tag{19c}$$

The alternating component of the stress can be written from Eq. (8-35), see page 457, as

$$\sigma_a = \frac{C(P_{\max} - P_{\min})}{2A_t} \tag{20a}$$

Substituting the maximum external load $P_{\max} = 2$ kips, the minimum external load $P_{\min} = 750$ lb, and Eqs. (1b) and (9b) into Eq. (20a), the alternating component of the stress is

$$\sigma_a = \frac{0.147(2000 - 750)}{2 \text{ x } 0.226} = 406.53 \text{ psi} \tag{20b}$$

Comparing the alternating stress with the mean stress and the preload stress, that is, Eqs. (18b), (19c), and (20b), note that

$$\sigma_a \neq \sigma_m - \sigma_i \tag{21a}$$

that is

$$0.407 \text{ kpsi} \neq 89.39 \text{ kpsi} - 88.496 \text{ kpsi} \tag{21b}$$

This indicates that the slope of the load line is not 1, see Figure 8.22, page 458. In fact, the slope of the load line for this problem is

$$r = \frac{\sigma_a}{\sigma_m - \sigma_i} = \frac{0.407}{89.39 - 88.496} = 0.455 \tag{22}$$

Substituting Eqs. (1b), (16), (17), (18b), and (20b) into Eq. (15), the fatigue factor of safety using the Goodman criterion of failure can be written as

$$n_f = \frac{20.6(133 - 88.496)}{133 \text{ x } 0.407 + 20.6(89.39 - 88.496)} \tag{23a}$$

Therefore, the fatigue factor of safety using the Goodman criterion of failure is

$$n_f = \frac{916.78}{54.131 + 18.416} = 12.6 \tag{23b}$$

Solution to Problem 6.
(i) The stiffness of the spring can be written as

$$k = \frac{\Delta F}{\Delta x} = \frac{F_{\max} - F_{\min}}{\Delta x} \tag{1}$$

The maximum and the minimum forces are specified, respectively, as

$$F_{\max} = 65 \text{ lbs} \qquad \text{and} \qquad F_{\min} = 25 \text{ lbs} \tag{2}$$

The change in the length of the spring is

$$\Delta x = l_a - l_m = 3.5 - 2.25 = 1.25 \text{ inches} \tag{3}$$

Substituting Eqs. (2) and (3) into Eq. (1), the stiffness of the spring is

$$k = \frac{65 - 25}{1.25} = 32 \text{ lbs/in} \tag{4}$$

The stiffness of the spring can also be written from Eq. (10-9), see page 528, (see Example 10.1, page 534) as

$$k = \frac{d^4 G}{8 D^3 N_a} \tag{5a}$$

Rearranging Eq. (4), the number of active coils can be written as

$$N_a = \frac{d^4 G}{8 D^3 k} \tag{5b}$$

The spring index can be written from Eq. (10.1), see page 526, as

$$C = \frac{D}{d} \tag{6a}$$

Substituting the spring index $C = 10$ and the mean coil diameter $D = 1.75$ inches into Eq. (6a), and rearranging, the wire diameter is

$$d = \frac{1.75}{10} = 0.175 \text{ in} \tag{6b}$$

The modulus of elasticity and the modulus of rigidity of music wire A228 with a wire diameter $d = 0.175$ in from Table 10-5, see page 533, respectively, are

$$E = 28.0 \text{ Mpsi} \qquad \text{and} \qquad G = 11.6 \text{ Mpsi} \tag{7}$$

Substituting Eqs. (4), (6), and (7) into Eq. (5b), the number of active coils is

$$N_a = \frac{0.175^4 \times 11.6 \times 10^6}{8 \times 1.75^3 \times 32} = 7.93 \tag{8a}$$

Therefore, the number of active coils (rounded up to the nearest quarter of a coil) is

$$N_a = 8 \tag{8b}$$

For squared and grounded ends, the total number of coils can be written from Table 10-1, see page 529, as

$$N_t = N_a + 2 \tag{9a}$$

Substituting Eq. (8b) into Eq. (9a), the total number of coils (rounded up to the nearest quarter) is

$$N_t = 8 + 2 = 10 \text{ coils} \tag{9b}$$

The solid height of the spring from Table 10-1 on page 529 is

$$L_s = d\, N_t \tag{10a}$$

Substituting Eqs. (4) and (9b) into Eq. (10a), the solid height of the spring is

$$L_s = 0.175 \times 10 = 1.75 \text{ in} \tag{10b}$$

(ii) The pitch of the spring wire for squared and ground ends can be written from Table 10-1, see page 529, as

$$p = \frac{L_0 - 2d}{N_a} \tag{11}$$

The free length of the spring (see Example 10.1, page 534) can be written as

$$L_0 = l_a + x_{\min} \tag{12a}$$

where the assembled length of the spring is given as

$$l_a = 3.5 \text{ inches} \tag{12b}$$

The deflection of the spring at the preload $F_i = F_{\min} = 25\,\text{lbs}$ and deflection given by Eq. (4) is

$$x_{\min} = \frac{F_{\min}}{k} = \frac{25}{32} \text{ in} = 0.78125 \text{ in} \tag{13}$$

Substituting Eqs. (12b) and (13) into Eq. (12a), the free length of the spring is

$$L_0 = 3.5 + 0.78125 = 4.28125 \text{ in} \tag{14}$$

Substituting the wire diameter and Eqs. (8b) and (14) into Eq. (11), the pitch of the spring wire is

$$p = \frac{4.28125 - 2 \text{ x } 0.175}{8} = 0.49 \text{ in} \tag{15}$$

(iii) The alternating and the mean components of the shear stress can be written from Eqs. (10.32) and (10.33), see page 545, as

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) \qquad \text{and} \qquad \tau_m = K_B\left(\frac{8F_mD}{\pi d^3}\right) \tag{16}$$

The Bergstrasser factor can be written from Eq. (10-5), see page 527, as

$$K_B = \frac{4C+2}{4C-3} \tag{17a}$$

Substituting the given spring index C = 10 into Eq. (17a), the Bergstrasser factor is

$$K_B = \frac{4\times 10+2}{4\times 10-3} = 1.1351 \tag{17b}$$

The alternating and the mean components of the force can be written from Eqs. (10-31), see page 545, respectively, as

$$F_a = \frac{F_{\max} - F_{\min}}{2} \qquad \text{and} \qquad F_m = \frac{F_{\max} + F_{\min}}{2} \tag{18}$$

Substituting Eq. (2) into Eqs. (18), the alternating and mean components of the force, respectively, are

$$F_a = \frac{65-25}{2} = 20 \text{ lb} \qquad \text{and} \qquad F_m = \frac{65+25}{2} = 45 \text{ lb} \tag{19}$$

Substituting Eqs. (16b) and (19) into Eqs. (16), the alternating and the mean components of the shear stress are

$$\tau_a = 1.1351\left(\frac{8\times 20\times 1.75}{\pi(0.175)^3}\right) = 18.877 \text{ kpsi} \tag{20a}$$

and

$$\tau_m = 1.1351\left(\frac{8\times 45\times 1.75}{\pi(0.175)^3}\right) = 42.473 \text{ kpsi} \tag{20b}$$

(iv) The fatigue factor of safety using the Gerber-Zimmerli fatigue failure criterion can be written from Example 10-4, see page 546, as

$$n_f = \frac{S_{sa}}{\tau_a} \tag{21}$$

The ultimate shear strength of the spring material can be written from Eq. (10.30), see page 545, as

$$S_{su} = 0.67 S_{ut} \tag{22}$$

The ultimate tensile strength can be written from Eq. (10-14), see page 532, as

$$S_{ut} = \frac{A}{d^m} \tag{23a}$$

The material specific constants for A228 music wire with a wire diameter $0.004\,\text{in} < d < 0.256$ in from Table 10-4, see page 532, are

$$A = 201\,\text{kpsi.in}^{\text{m}} \qquad \text{and} \qquad m = 0.145 \tag{23b}$$

Substituting Eqs. (23b) into Eq. (23a), the ultimate tensile strength of the A228 music wire is

$$S_{ut} = \frac{201}{(0.175)^{0.145}} = 258.795\,\text{kpsi} \tag{24}$$

Substituting Eq. (24) into Eq. (22), the ultimate shear strength of the A228 music wire is

$$S_{su} = 0.67 \times 258.795 = 173.393\,\text{kpsi} \tag{25}$$

The torsional endurance strength (that is, the Gerber ordinate intercept for the Zimmerli data for peened springs) can be written from the equation on page 544 as

$$S_{se} = \frac{S_{sa}}{1 - \left(S_{sm} / S_{su}\right)^2} \tag{26}$$

The alternating and mean components of the endurance strength for a peened spring from Eq. (10-29), see page 544, are

$$S_{sa} = 57.5\,\text{kpsi} \qquad \text{and} \qquad S_{sm} = 77.5\,\text{kpsi} \tag{27}$$

Substituting Eqs. (25) and (27) into Eq. (26), the torsional endurance strength is

$$S_{se} = \frac{57.5}{1 - \left(77.5 / 173.393\right)^2} = 71.855\,\text{kpsi} \tag{28}$$

The alternating component of the endurance strength can be written from page Eq. (6.48), see page 334, or from Example 10-4, see pages 545 and 546, as

$$S_{sa} = \frac{r^2\, S_{su}^2}{2\, S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2\, S_{se}}{r\, S_{su}} \right)^2} \,\right] \tag{29}$$

where the slope of the load line is

$$r = \frac{\tau_a}{\tau_m} = \frac{18.877}{42.473} = 0.4444 \tag{30}$$

For comparison, the slope of the load line in the worked Example 10.4, see page 546, is

$$r = \frac{\tau_a}{\tau_m} = \frac{29.7}{39.6} = 0.75 \tag{31}$$

Substituting Eqs. (25), (28), and (30) into Eq. (29), the alternating component of the endurance strength is

$$S_{sa} = \frac{(0.4444)^2(173.393)^2}{2(71.855)}\left[-1+\sqrt{1+\left(\frac{2(71.855)}{(0.4444)(173.393)}\right)^2}\right] = 46.1172 \text{ kpsi} \quad (32)$$

Substituting Eqs. (20a) and (32) into Eq. (21), the fatigue factor of safety using the Gerber-Zimmerli fatigue-failure criterion is

$$n_f = \frac{46.1172}{18.877} = 2.4 \quad (33)$$