

Project 2. Upload to Gradescope before 8:00 am on Monday, October 21st. A formal report is not required, however, credit will be given for a detailed presentation and clearly drawn figures.

The model of the Watt four-bar linkage shown in Figure 1 is proposed as a possible pump or compressor. The ground link 1 is $O_2O_4 = 200$ mm and is inclined at the angle $\beta = 15^\circ$ from the horizontal X-axis. The input link 2 (i.e., the great beam) is $DC = 336$ mm, the distance $O_2C = O_2D = 168$ mm, and the distance $O_2A = 84$ mm. The coupler link 3 is $AB = 60$ mm and the output link 4 is $O_4B = 90$ mm. The location of the coupler point M is $AM = BM = 30$ mm. The piston (link 6) of the double-acting cylinder is connected to the coupler link AB at pin M by the connecting rod ML (link 5). The forward stroke of the piston occurs when pin L moves vertically downward. The length of link 5 is $LM = 200$ mm and the length of the piston is equal to the width of the piston, that is, $h = l = 48$ mm. The vertical distance between the shaft O_2 (coincident with the origin of the fixed Cartesian reference frame) and the ground at the bottom of the double-acting cylinder is 320 mm and the horizontal distance between O_2 and the center of the double-acting piston (that is, pin L) is 93.5 mm. The input link 2 is driven counterclockwise by an electric motor with a nominal (or constant) speed of 15 rad/s counterclockwise (between $156^\circ \leq \theta_2 \leq 220^\circ$).

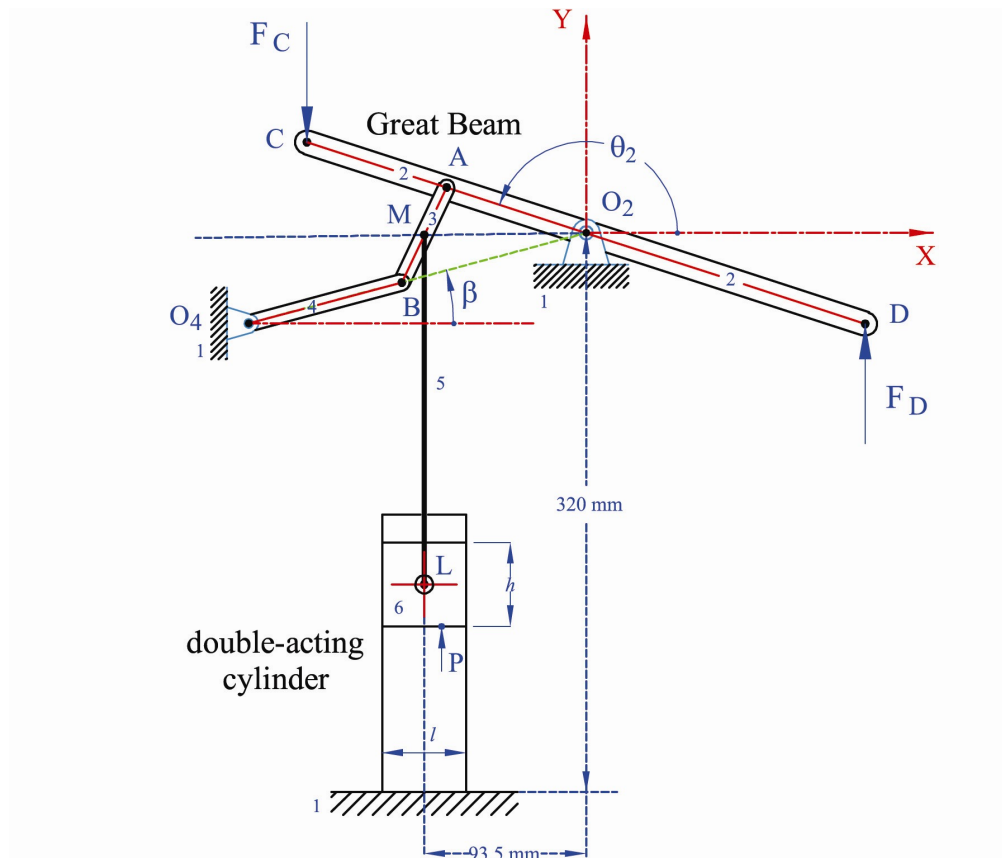


Figure 1. The Watt Four-Bar Linkage. (Not drawn to scale).

The two constant external forces $\bar{F}_C = -25 \bar{j} \text{ N}$ and $\bar{F}_D = 20 \bar{j} \text{ N}$ act at points C and D of the great beam. Also, the pressure due to the gas in the cylinder is modeled by the constant force $\bar{P} = 20 \bar{j} \text{ N}$ which acts at a particular point on the bottom face of the double acting cylinder as shown in Figure 1. The horizontal distance between pin L and the line of action of force \bar{P} is 10 mm. The inertial properties of links 2, 3, 4, 5 and 6 are as given in Table 1.

Links	Masses (kg)	Mass Moments of Inertia (Nm-s ²)	Centers of Mass
2	10	0.05	pin O ₂
3	3	0.02	pin M
4	4	0.07	pin O ₄
5	0.5	0.12	pin L
6	0.5	0.3	pin L

Table 1. Masses, Mass Moments of Inertia, and Centers of Mass for the Moving Links.

A Partial List of Some Initial Assumptions.

- (i) The acceleration of point L is the same as the Y-component of the acceleration of point M.
- (ii) Gravity is acting vertically downward (i.e., in the negative Y-direction).
- (iii) The centers of mass of links 2 and 4 (i.e., G_2 and G_4) are coincident with the ground pivots O₂ and O₄ (see Table 1), respectively.
- (iv) The center of mass of the coupler link 3 is coincident with the coupler point M.
- (v) The centers of mass of the connecting rod (link 5) and the piston (link 6) are coincident with pin L (which can only move in the vertical direction).
- (vi) The effects of friction in the linkage can be neglected.

Discuss with your TA other assumptions that you think may be necessary in order to complete a realistic dynamic force analysis of the linkage.

Part I. Dynamic Force Analysis of the Linkage.

1. Draw complete free body diagrams of links 2, 3, 4, 5, and 6. Clearly show all the forces that are acting on each free-body diagram.
2. Solve the dynamic force analysis problem by the method of inspection; i.e., solve one equation and one unknown or at most solve two equations and two unknowns. Clearly present your solution strategy. Specify the numerical values for the unknown variables, that is, the internal reaction forces, the torque acting on the shaft O₂ (henceforth referred to as the crank torque), and distance variables, for one chosen position of the input link. You can check your results for a specified input position with your TA.
3. Solve the dynamic force analysis problem by writing a computer program (that is, extend the code in Step 1 above to include the dynamic force analysis of the mechanism). Tabulate, and plot, all the unknown variables; e.g., the internal reaction forces and the crank torque, for 1 degree increments of the input link between $156^\circ \leq \theta_2 \leq 220^\circ$ (that is, the forward stroke).

Hint: A possible solution strategy is to first solve the Newton-Euler equations for link 5 and link 6 and then solve the Newton-Euler equations for links 2, 3, and 4.

4. Tabulate the X and Y components of the internal reaction forces, the crank torque, and all other unknown variables for 1 degree increments of the input link. Discuss your plots indicating important points on the plots.
5. Show, and discuss, the free body diagrams of the linkage, if the task is to investigate the internal reaction forces when the input link rotates clockwise between $220^\circ \geq \theta_2 \geq 156^\circ$, i.e., the piston moves upward (the return stroke). Comment on the differences between the internal reaction forces and the crank torque during the forward stroke and the return stroke.

Part II: Static Force Analysis of the Linkage.

1. Draw free body diagrams of links 2, 3, 4, 5, and 6. Clearly show all the forces that are acting on each free-body diagram.
2. Write the symbolic equations that are required to solve all the unknown variables (e.g., the internal reaction forces, the crank torque, and any distances). The equations should be written in terms of the fixed X-Y coordinate system shown in Figure 1.
3. Perform a static force analysis by hand for one posture of the input link. Solve the problem by inspection; i.e., one equation and one unknown, or two equations and two unknowns.
4. Perform the static force analysis by modifying the computer program used for the dynamic force analysis. Print out the X and Y components of the internal reaction forces, the crank torque, and any other unknown variables, for 1 degree increments of the input link. Show the important plots of the internal reaction forces, the crank torque, and any other unknown variables for 1 degree increments of the input link.
5. Discuss your results and important points from the plots. Compare your results from the static force analysis with your results from the dynamic force analysis. Do you observe any similarities, or major differences, in these results? Explain why.

Part III: The Equation of Motion. (see Chapter 12, Section 12.9, pages 449-459 in the text book).

1. Write the power equation for the linkage in symbolic form. Then tabulate, and plot, the power due to: (i) the time rate of change of kinetic energy; and (ii) the time rate of change of potential energy. Explain the significance of each term in the power equation (i.e., the percentage contribution).
2. Write the equation of motion for the linkage in symbolic form. Then tabulate, and plot, the equivalent mass moment of inertia of the mechanism for 1 degree increments of the input link.
3. Use the equation of motion to tabulate, and plot, the crank torque for 1 degree increments of the input link.
4. Compare your results for the crank torque obtained from the equation of motion with your results obtained from the Newton-Euler formulation in Part I. Comment on the comparison.