

Lab Project No. 3. A formal report is due at the beginning of lab on Monday, November 28th, or Tuesday, November 29th.

The wheel (link 2) of the mechanism shown in Figure 1 rolls, without slipping, on the ground. The wheel, which has a radius $\rho_2 = 12.5$ mm, is actuated by a hydraulic actuator connected between the ground pin O_4 and the shaft at point A. The actuator force is proportional to the velocity of point A where the constant of proportionality (that is, the viscous coefficient) is $C = 2000$ N.sec/m. The shaft at A moves from $O_4 A = R_{22} = 75$ mm to $O_4 A = R_{22} = 150$ mm and the velocity and acceleration of point A for this displacement are as shown in Figure 2. A linear spring, with free length $R_S = 110$ mm and spring rate $k = 500$ N/m, is connected between the ground pivot O_8 and the shaft at A. There is a constant torque $\bar{T}_{12} = 15 \bar{k}$ N.m acting on the shaft at A and a force \bar{F}_P (with unknown magnitude and direction) acting at the coupler point P. Given $O_6 O_4 = R_1 = 150$ mm, $O_4 O_8 = R_{11} = 185$ mm, $AB = R_3 = 75$ mm, $AC = R_{33} = 212.5$ mm, $BC = 150$ mm, $O_4 B = R_4 = 100$ mm, $DC = R_5 = 150$ mm, $O_6 D = R_6 = 62.5$ mm, and $CP = 150$ mm. The only significant friction in the mechanism is between the wheel and the ground where the coefficient of friction is $\mu = 0.25$.

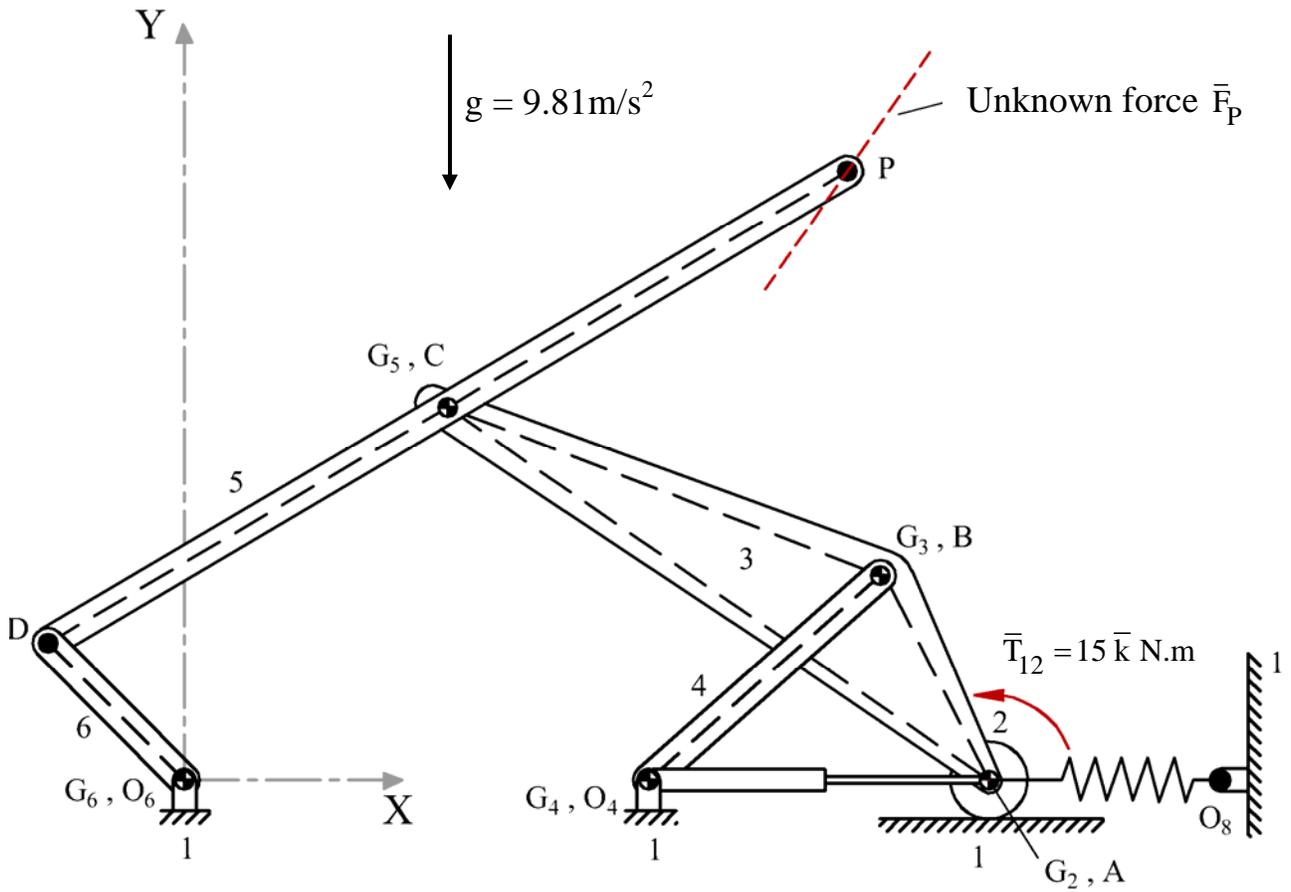


Figure 1. The Six-Bar Mechanism.

(i) For the displacement $75 \text{ mm} \leq R_{22} < 112.5 \text{ mm}$, the velocity and acceleration of point A are

$$\dot{R}_{22} = +\sqrt{2\ddot{R}_{22}(R_{22} - 0.075)} \text{ m/s} \quad (1a)$$

and

$$\ddot{R}_{22} = +0.125 \text{ m/s}^2 \quad (1b)$$

(i) For the displacement $112.5 \text{ mm} \leq R_{22} \leq 150 \text{ mm}$, the velocity and acceleration of point A are

$$\dot{R}_{22} = +\sqrt{2\ddot{R}_{22}(R_{22} - 0.15)} \text{ m/s} \quad (2a)$$

and

$$\ddot{R}_{22} = -0.125 \text{ m/s}^2 \quad (2b)$$

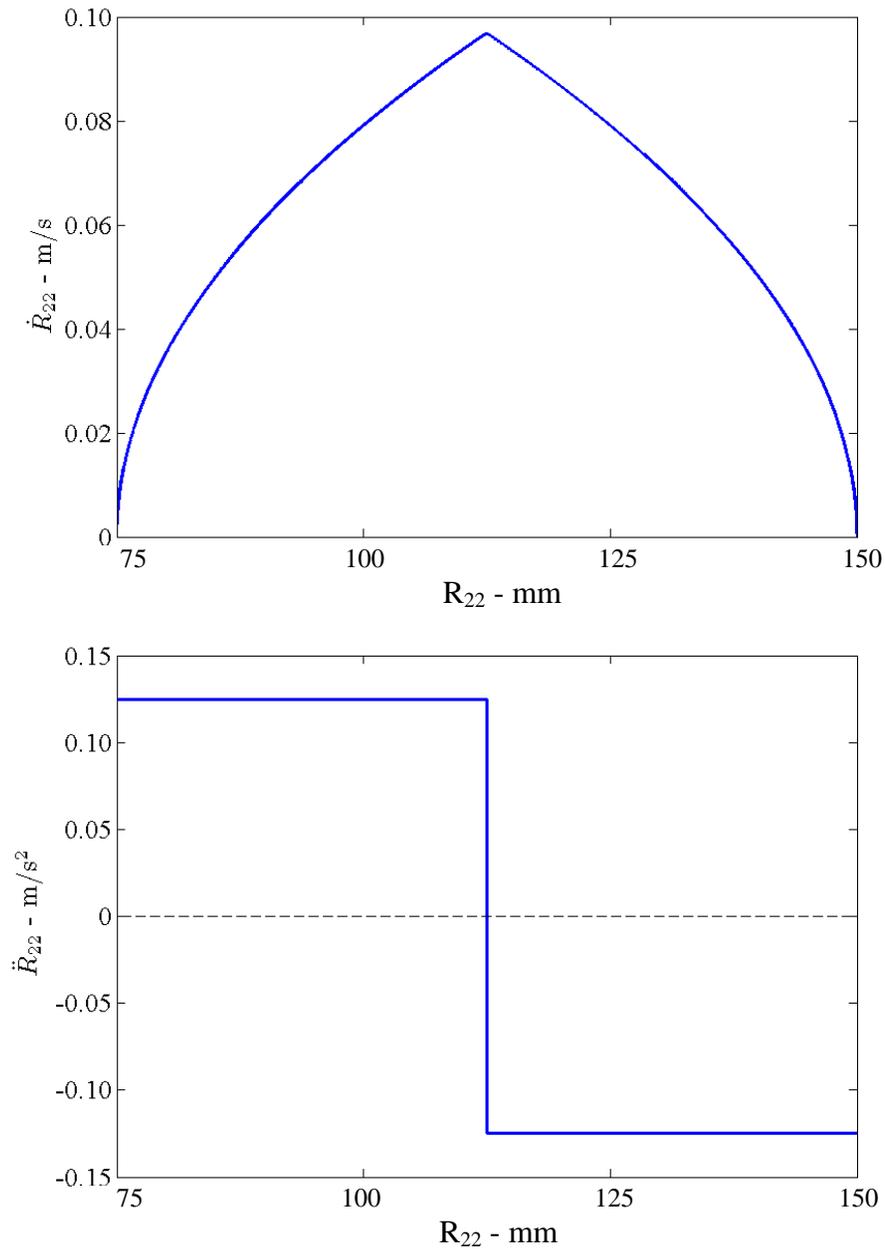


Figure 2. The velocity and acceleration of point A against the position of point A.

The masses of the moving links are $m_2 = 5 \text{ kg}$, $m_3 = 25 \text{ kg}$, $m_4 = 12 \text{ kg}$, $m_5 = 5 \text{ kg}$, and $m_6 = 15 \text{ kg}$. The mass moments of inertia (or the second moments of mass) of the moving links, about their respective centers of mass, are $I_{G2} = 30 \text{ N}\cdot\text{mm}\cdot\text{s}^2$, $I_{G3} = 450 \text{ N}\cdot\text{mm}\cdot\text{s}^2$, $I_{G4} = 75 \text{ N}\cdot\text{mm}\cdot\text{s}^2$, $I_{G5} = 250 \text{ N}\cdot\text{mm}\cdot\text{s}^2$, and $I_{G6} = 20 \text{ N}\cdot\text{mm}\cdot\text{s}^2$.

Notes:

- (i) Gravity is acting vertically downward (i.e., in the negative Y-direction) as shown in Figure 1.
- (ii) The mass of the linear spring and the mass of the hydraulic actuator can be ignored.
- (iii) The diameter of the shaft at point A can be ignored.
- (iv) The center of mass of link 2 is coincident with point A, the center of mass of the coupler link 3 is coincident with pin B, the centers of mass of links 4 and 6 are coincident with the ground pins O_4 and O_6 , respectively, and the center of mass of link 5 is coincident with pin C.
- (v) The external forces are: (a) the hydraulic actuator force; (b) the force acting at coupler point P; and (c) the gravitational force. Also, there is a constant external torque acting on the shaft at A.
- (vi) Note that the ground is only below the wheel, that is, there is no ground above the wheel. With the constraint that the wheel must always be rolling on the ground then the wheel cannot leave the ground.
- (vii) Please be careful with SI units; for example, mass is kilograms (kg), mass moment of inertia (or second moment of mass) is $\text{kg}\cdot\text{m}^2$ (or $\text{N}\cdot\text{m}\cdot\text{s}^2$), and force is Newtons (where $1 \text{ N} = 1 \text{ kg}\cdot\text{m}/\text{s}^2$).

I: Dynamic Force Analysis of the Mechanism.

- (i) Draw complete free body diagrams of the hydraulic actuator, the linear spring, the wheel, and links 3, 4, 5, and 6. For each free body diagram, list the unknown variables and write the symbolic equations that are required to solve for the unknown variables.
- (ii) Solve the dynamic force analysis problem by the method of inspection. The solution strategy should be clearly presented. Numerical values for all unknown variables should be specified for one arbitrary position of the input link. You should be able to verify these values with your TA.
- (iii) Solve the dynamic force analysis problem by writing a computer program using the method of inspection that was developed in Step (ii).
- (iv) Tabulate the hydraulic actuator force acting on the shaft at point A for 2.5 mm increments of the input position R_{22} . Plot the hydraulic actuator force for 0.1 mm increments of the input position.
- (v) Tabulate the spring force acting on the axle of the wheel for 2.5 mm increments of the input position R_{22} . Plot the spring force for 0.1 mm increments of the input position.
- (vi) Tabulate the friction force between the wheel and the ground for 2.5 mm increments of the input position R_{22} . Plot this force for 0.1 mm increments of the input position.
- (vii) Tabulate the normal force between the wheel and the ground link for 2.5 mm increments of the input position R_{22} . Plot this force for 0.1 mm increments of the input position.
- (viii) Tabulate the X and Y components of the internal reaction forces between links 3, 4, 5, and 6 for 2.5 mm increments of the input position R_{22} . Plot the X and Y components of these forces for 0.1 mm increments of the input position. Discuss the plots indicating important points on the plots.
- (ix) Tabulate the X and Y components of the external force \bar{F}_P for 2.5 mm increments of the input position R_{22} . Plot this force for 0.1 mm increments of the input position.
- (x) Tabulate the component of the external force \bar{F}_P that is tangential to the path of point P for 2.5 mm increments of the input position R_{22} . Plot this force for 0.1 mm increments of the input position.
- (xi) Search the web for the most appropriate hydraulic actuator that could be used to drive the wheel. Please consider the size, the cost, and the load capacity of the actuator when making your final decision. Include these details in your formal report.

Deliverable: Part I will be regarded as a deliverable that will receive 20% of the total grade for lab attendance and quizzes. In order to qualify for this grade, the deliverable must be submitted to your lab TA at the beginning of lab on Monday, November 7th, or Tuesday, November 8th. A formal report is not required for this deliverable and a discussion of your work/results in Part I can be postponed until the final report.

II: Static Force Analysis of the Mechanism.

(A) The Analytical Method.

- (i) Draw complete free body diagrams of the hydraulic actuator, the spring, the wheel, and links 3, 4, 5, and 6 for static equilibrium of the mechanism. Write the symbolic equations that are required to solve for all of the unknown variables. Note that the actuator force, required to hold the mechanism in static equilibrium, is proportional to the velocity of point A for each input position, see Figure 2. (Recall that the constant of proportionality is specified as $C = 2000 \text{ N}\cdot\text{sec}/\text{m}$). For all other purposes, the velocity and the acceleration of point A can be taken to be zero.
- (ii) Perform a static force analysis of the mechanism by hand for one value of the input position. Solve the problem by the method of inspection.
- (iii) Perform the static force analysis by modifying the computer program used for the dynamic force analysis of the mechanism.
- (iv) Repeat Steps (vi) through (viii) of Part I for the static force analysis.

(B) The Graphical Method.

- (i) List the two-force members, the three-force members, links with two forces and a torque, and/or four-force members in the mechanism. Then perform a static force analysis using the graphical method. Note that the force polygons must be drawn accurately and neatly to a good scale. Hand drawn figures are acceptable, however, an attempt at computer generated force polygons is encouraged.
- (ii) For the initial iteration of the graphical method you should investigate different scenarios of neglecting the external forces and/or the external torque. For example, you could try to neglect the effects of gravity on the mechanism or remove the external force acting at the coupler point P.
- (iii) For the final iteration you must include the effects of all of the external forces and the external torque on the mechanism. Compare your answers for the different scenarios and comment on the comparisons.
- (iv) Compare your results from the graphical approach with your answers from the analytical approach in Part II (A). Comment on the comparisons.

III: The Equation of Motion. (See Chapter 14, Section 14.9, page 645)

- (i) Write the Power Equation in symbolic form for this mechanism. Then tabulate the equivalent mass of the mechanism for 2.5 mm increments of the input position.
- (ii) Explain the significance of each term in the Power Equation and determine the percentage contribution of each term to the net power of the mechanism.
- (iii) Using the Equation of Motion, compute and plot the component of the external force \bar{F}_P that is tangential to the path of point P for 0.1 mm increments of the input position.
- (iv) Compare your answers for the component of the external force \bar{F}_P that is tangential to the path of point P, obtained in Step (iii), with your answers for the tangential component of the external force at point P obtained from the Newton-Euler method in Part I. Comment on the comparison of the answers.