Given
Air in piston-cylinder device undergoing a thermodynamic cycle

Find
(a) P-v diagram
(b) Specific work (kJ/kg) and specific heat transfer (kJ/kg) during each process
(c) Thermal efficiency (%) of the cycle
(d) Specific entropy generation (kJ/kg-K) during the isothermal compression process

System

System = Air inside the cylinder and part of the surrounding

T_b = T_{surrounding} = 290 K

Assumptions
Quasi-equilibrium
Ignore KE change
Ignore PE change
Air behaves as an ideal gas
Neglect friction
No other work except boundary work
Closed \( \Rightarrow m = \text{constant} \)

Basic Equations

\[
W_{\text{boundary}} = \int PdV
\]

\[
\left. \frac{dE}{dt} \right|_{\text{system}} = \dot{Q} - W + \sum_{\text{in}} \dot{m} (h + ke + pe)_{\text{in}} - \sum_{\text{out}} \dot{m} (h + ke + pe)_{\text{out}}
\]

Integrating: \( \dot{Q} - W = \Delta U + \Delta KE + \Delta PE \)

\[
P_v = R_{air} T\quad R_{air} = \frac{R}{MW_{air}}
\]
Solution
(a) P-v diagram

(b) Moving boundary work during compression of the gas from 1 to 2:

\[ w_{12} = \int P \, dv = \int_{1}^{2} \frac{R_{air} T}{v} \, dv = R_{air} \left( T_1 = T_2 \right) \ln \frac{v_2}{v_1} = \frac{8.314}{28.97} \times 300 \times \ln \frac{28.97}{2} \times \frac{0.2}{2} \, \text{m}^3/\text{kg} \]

\[ w_{12} = -198 \, \text{kJ/kg} \quad \text{w < 0 work done on the system} \]

Considering energy balance for the gas from 1 to 2: \[ q_{12} - w_{12} = u_2 - u_1 \Rightarrow q_{12} = w_{12} \]

\[ q_{12} = -198 \, \text{kJ/kg} \quad \text{q < 0 heat transfer out of the system} \]

Moving boundary work during compression of the gas from 2 to 3: \[ w_{23} = \int_{2}^{3} P \, dv \]

\[ w_{23} = 0 \]

Considering energy balance for the gas from 2 to 3: \[ q_{23} - w_{23} = u_3 - u_2 \Rightarrow q_{23} = u_3 - u_2 \]

\[ q_{23} = +991 \, \text{kJ/kg} \quad \text{q > 0 heat transfer into the system} \]
Moving boundary work during expansion of the gas from 3 to 4:

\[ w_{34} = \int \frac{P}{v} dv = \int \frac{R_{\text{air}} T}{v} dv = R_{\text{air}} (T_3 = T_4) \ln \frac{v_4}{v_3} = \frac{8.314}{28.97} \times 1500 \times \ln \frac{2}{0.2} \text{ m}^3/\text{kg} \]

\[ w_{34} = +991 \frac{\text{kJ}}{\text{kg}} \quad w > 0 \text{ work done by the system} \]

Considering energy balance for the gas from 3 to 4:

\[ q_{34} = u_4 - u_3 \Rightarrow q_{34} = w_{34} \]

\[ q_{34} = +991 \frac{\text{kJ}}{\text{kg}} \quad q > 0 \text{ heat transfer into the system} \]

Moving boundary work during expansion of the gas from 4 to 1:

\[ w_{41} = \int \frac{P}{v} dv \]

\[ w_{41} = 0 \]

Considering energy balance for the gas from 4 to 1:

\[ q_{41} = u_1 - u_4 \Rightarrow q_{41} = u_1 - u_4 \]

\[ q_{41} = -991 \frac{\text{kJ}}{\text{kg}} \quad q < 0 \text{ heat transfer out of the system} \]

(c) Thermal efficiency of the cycle:

\[ \eta_{\text{thermal}} = \frac{w_{\text{net, out}}}{q_{\text{in}}} \]

For the given cycle:

\[ w_{\text{net, out}} = w_{34} + w_{12} = 793 \frac{\text{kJ}}{\text{kg}} \quad \text{and} \quad q_{\text{in}} = q_{23} + q_{34} = 1980 \frac{\text{kJ}}{\text{kg}} \]

\[ \Rightarrow \eta_{\text{thermal}} = \frac{793}{1980} \frac{\text{kJ}}{\text{kg}} = 0.4 \quad \Rightarrow \quad \eta_{\text{thermal}} = 40\% \]

(d) Considering entropy balance for the system and its surrounding:

\[ \sigma_{\text{generation}} = -\frac{q_{12}}{T_{\text{boundary}} = T_{\text{surrounding}}} + (s_2 - s_1) = -\frac{q_{12}}{T_{\text{boundary}} = T_{\text{surrounding}}} + \left( s_2^0 - s_1^0 - R_{\text{air}} \ln \frac{P_2}{P_1} \right) \]

\[ \sigma_{\text{generation}} = -\frac{q_{12}}{T_{\text{boundary}} = T_{\text{surrounding}}} - R_{\text{air}} \ln \frac{v_1}{v_2} = -\frac{198}{(17 + 273) \text{K}} + \left( -0.287 \frac{\text{kJ}}{\text{kg-K}} \ln \frac{2}{0.2} \text{ m}^3/\text{kg} \right) \]

\[ \sigma_{\text{generation}} = +0.02192 \frac{\text{kJ}}{\text{kg-K}} \quad \sigma > 0 \text{ irreversible process} \]
Given
A steam power plant

Find
(a) Isentropic efficiency (%) of the adiabatic steam turbine
(b) Specific heat transfer (kJ/kg) for the boiler and the condenser
(c) Specific entropy generation (kJ/kg-K) for the entire power cycle
(d) T-s diagram

System

Assumptions
Steady state, steady flow
One-dimensional, uniform flow
Ignore KE change
Ignore PE change
Boiler and condenser: $\dot{W}_{CV} = 0$
Turbine and pump: $\dot{Q}_{CV} = 0$

Basic Equations
\[
\frac{dm}{dt}_{system} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}
\]
\[
\frac{dE}{dt}_{system} = \dot{Q} - \dot{W} + \sum_{in} \dot{m}_{in} (h + ke + pe)_{in} - \sum_{out} \dot{m}_{out} (h + ke + pe)_{out}
\]
\[
\frac{dS}{dt}_{system} = \sum_{j} \frac{\dot{Q}_j}{T_{j,boundary}} + \sum_{in} \dot{m}_{in} s_{in} - \sum_{out} \dot{m}_{out} s_{out} + \dot{\sigma}_{generation}
\]
Solution
(a) Mass balance for the turbine: \( \dot{m}_3 = \dot{m}_4 = \dot{m}_{\text{steam}} \)

Energy balance for the actual turbine: \( w_{\text{actual}} = (h_3 - h_4) = (3214.5 - 2247) \frac{\text{kJ}}{\text{kg}} = 967.5 \frac{\text{kJ}}{\text{kg}} \)

For the isentropic process through the turbine: \( s_3 = s_{4s} = 6.7710 \frac{\text{kJ}}{\text{kg-K}} \)

\( \Rightarrow s_{f,45^\circ\text{C}} < s_{4s} < s_{g,45^\circ\text{C}} \Rightarrow \text{saturated liquid-vapor mixture} \)

For saturated liquid-vapor mixture: \( x_{4s} = \frac{s_{4s} - s_{f,45^\circ\text{C}}}{s_{g,45^\circ\text{C}} - s_{f,45^\circ\text{C}}} = \frac{6.7710 - 0.63861}{8.1633 - 0.63861} = 0.86 \)

\( x_{4s} = \frac{h_{4s} - h_{f,45^\circ\text{C}}}{h_{g,45^\circ\text{C}} - h_{f,45^\circ\text{C}}} \Rightarrow 0.815 = \frac{h_{4s} - 188.43}{2582.4 - 188.43} \Rightarrow h_{4s} = 2139.5 \frac{\text{kJ}}{\text{kg}} \)

Energy balance for the isentropic turbine: \( w_{\text{isentropic}} = (h_3 - h_{4s}) = (3214.5 - 2139.5) \frac{\text{kJ}}{\text{kg}} = 1075 \frac{\text{kJ}}{\text{kg}} \)

Isentropic efficiency of the turbine: \( \eta_{\text{turbine}} = \frac{w_{\text{actual}}}{w_{\text{isentropic}}} = \frac{967.5 \frac{\text{kJ}}{\text{kg}}}{1075 \frac{\text{kJ}}{\text{kg}}} \Rightarrow \eta_{\text{turbine}} = 90\% \)

(b) Mass balance for the boiler: \( \dot{m}_2 = \dot{m}_3 = \dot{m}_{\text{steam}} \)

Energy balance for the boiler: \( q_{\text{boiler}} = (h_3 - h_2) = (3214.5 - 192.8) \frac{\text{kJ}}{\text{kg}} \Rightarrow q_{\text{boiler}} = +3020 \frac{\text{kJ}}{\text{kg}} \)

q > 0 heat transfer into the system

Mass balance for the condenser: \( \dot{m}_4 = \dot{m}_1 = \dot{m}_{\text{steam}} \)

Energy balance for the condenser: \( q_{\text{condenser}} = (h_1 - h_4) = (188.4 - 2247) \frac{\text{kJ}}{\text{kg}} \Rightarrow q_{\text{condenser}} = -2060 \frac{\text{kJ}}{\text{kg}} \)

q < 0 heat transfer out of the system

(c) Entropy balance for the entire power cycle:

\[ \sigma_{\text{cycle}} = \frac{-q_{\text{boiler}}}{T_{\text{boiler}}} - \frac{q_{\text{condenser}}}{T_{\text{condenser}}} = \frac{-3020 \frac{\text{kJ}}{\text{kg}}}{(500 + 273) \text{K}} - \frac{-2060 \frac{\text{kJ}}{\text{kg}}}{(30 + 273) \text{K}} \]

\[ \sigma_{\text{cycle}} = +2.8918 \frac{\text{kJ}}{\text{kg-K}} \]
Alternatively

Considering entropy balance for the pump: \( \sigma_{\text{pump}} = (s_2 - s_1) = +0.0011 \frac{\text{kJ}}{\text{kg-K}} \)

Considering entropy balance for the boiler:

\[
\sigma_{\text{boiler}} = -\frac{q_{\text{boiler}}}{T_{\text{boiler}}} + (s_3 - s_2) = -\frac{+3020 \text{kJ}}{\text{kg}} + \left( \frac{6.7710 - 0.63971}{(500 + 273) \text{K}} \right) \frac{\text{kJ}}{\text{kg-K}} = +2.2244 \frac{\text{kJ}}{\text{kg-K}}
\]

Considering entropy balance for the turbine:

\( \sigma_{\text{turbine}} = (s_4 - s_3) = +0.3388 \frac{\text{kJ}}{\text{kg-K}} \)

Considering entropy balance for the condenser:

\[
\sigma_{\text{condenser}} = -\frac{q_{\text{condenser}}}{T_{\text{condenser}}} + (s_1 - s_4) = -\frac{-2060 \text{kJ}}{\text{kg}} + \left( \frac{0.63861 - 7.1098}{(30 + 273) \text{K}} \right) \frac{\text{kJ}}{\text{kg-K}} = +0.32751 \frac{\text{kJ}}{\text{kg-K}}
\]

Specific entropy generation for the entire heat pump cycle:

\[
\sigma_{\text{cycle}} = \sigma_{\text{pump}} + \sigma_{\text{boiler}} + \sigma_{\text{turbine}} + \sigma_{\text{condenser}} \Rightarrow \sigma_{\text{cycle}} = +2.8918 \frac{\text{kJ}}{\text{kg-K}}
\]

(e) T-s diagram
Given
A piston-cylinder device contains 100 kg of steam at 20 bar and 240° C (State 1). Steam is heated from its initial state either (a) at constant pressure until its volume doubles (State 2) or (b) at constant temperature until its volume doubles (State 3). Assume that the surrounding pressure is 1 bar and temperature is 20° C.

Find
(a) Show both processes on T-v diagram. Label states, show property values, and indicate appropriate lines of constant pressure.
(b) Calculate the change in exergy (kJ) of steam during the constant pressure heating process. Is it positive or negative? Explain why.
(c) Calculate the change in exergy (kJ) of steam during the constant temperature heating process. Is it positive or negative? Explain why.

System

Assumptions
- Quasi-equilibrium process
- Ignore KE and PE changes

Basic Equations
\[
\frac{dm_{CV}}{dt} = \sum_{i} m_{i} - \sum_{e} m_{e} \Rightarrow m_{CM} = \text{constant} = m_{1} = m_{2}
\]

Solution
(a) State 1: P₁ = 20 bar, T₁ = 240°C
Table A-3: T_{sat}(P₁) = 212.4°C ⇒ T₁ > T_{sat}(P₁) ⇒ superheated vapor (SHV)
Table A-4: v₁ = 0.1085 m³/kg, u₁ = 2659.6 kJ/kg, s₁ = 6.4952 kJ/kg-K

Constant Pressure Heating
State 2: P₂ = P₁ = 20 bar, v₂ = 2v₁ = 0.217 m³/kg
Table A-3: v₂(P₂) = 0.09963 m³/kg ⇒ v₂ > vₖ ⇒ superheated vapor (SHV)
Table A-4: Interpolating:
\[
\frac{T_2 - 640}{700 - 640} = \frac{u_2 - 3362.2}{3470.9 - 3362.2} = \frac{s_2 - 7.8035}{7.9487 - 7.8035} = \frac{0.217 - 0.2091}{0.2232 - 0.2091}
\]
\( T_2 = 673.6^\circ C, u_2 = 3423.1 \text{ kJ/kg}, s_2 = 7.8848 \text{ kJ/kg-K} \)

Constant Temperature Heating
State 3: \( T_3 = T_1 = 240^\circ C, v_3 = 2v_1 = 0.217 \text{ m}^3/\text{kg} \)

Table A-3: \( v_k(T_3) = 0.05976 \text{ m}^3/\text{kg} \Rightarrow v_3 > v_k \Rightarrow \text{superheated vapor (SHV)} \)

Table A-4: Interpolating:

\[
\begin{array}{cccccc}
33 & 3 & 15 & 2676.9 & 6.6628 & 0.217 \\
10 & 15 & 2692.9 & 2676.9 & 6.8817 & 6.6628 & 0.2275 & 0.1483 \\
\end{array}
\]

\( P_3 = 10.66 \text{ bar}, u_3 = 2690.8 \text{ kJ/kg}, s_2 = 6.8528 \text{ kJ/kg-K} \)

T-v diagram is shown below.

(b) Change in exergy during constant pressure heating:
\[
\Delta \tilde{E}_p = m \left[ (u_2 - u_1) + P_0 (v_2 - v_1) - T_0 (s_2 - s_1) \right] \\
= 100 \text{ kg} \times \left[ (3423.1 - 2659.6) \frac{\text{kJ}}{\text{kg}} + 100 \text{ kPa} \times (0.217 - 0.1085) \frac{\text{m}^3}{\text{kg}} - 293 \text{ K} \times (7.8848 - 6.4952) \frac{\text{kJ}}{\text{kg-K}} \right] \\
\Delta \tilde{E}_p = +36,720 \text{ kJ} ; \text{ temperature increases and pressure remains constant in this heating process } \Rightarrow \text{ the state of steam in the cylinder moves further away from the dead state } \Rightarrow \text{ exergy change is positive i.e. } \Delta \tilde{E}_p > 0
(c) Change in exergy during constant temperature heating:

\[
\Delta \tilde{E}_T = m \left[ (u_3 - u_1) + P_0 (v_3 - v_1) - T_0 (s_3 - s_1) \right]
\]

\[
= 100 \text{ kg} \times \left[ (2690.8 - 2659.6) \frac{\text{kJ}}{\text{kg}} + 100 \text{ kPa} \times (0.217 - 0.1085) \frac{\text{m}^3}{\text{kg}} - 293 \text{ K} \times (6.8528 - 6.4952) \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right]
\]

\[
\Delta \tilde{E}_p = -6,273 \text{ kJ}; \text{ pressure decreases and temperature remains constant in this heating process} \Rightarrow \text{ the state of steam in the cylinder moves closer to the dead state} \Rightarrow \text{ exergy change is negative i.e. } \Delta \tilde{E}_T < 0
\]