N-butane ($C_4H_{10}$) is burned with 85 percent theoretical air, and the products of combustion, an equilibrium mixture containing only $O_2$, $CO$, $CO_2$, $H_2$, $H_2O$, $N_2$, and NO, exit from a combustion chamber at 2000 K, 0.2 MPa. The average specific heat at constant pressure for butane is $c_p = 1.71 \text{ kJ / (kg K)}$ in the temperature range 300 K – 600 K.

(a) Determine the equilibrium composition in the product stream neglecting $O_2$ and NO (this will require 4 atom balances and 1 equilibrium equation).

(b) Now determine the mole fractions of $O_2$ and NO in the product stream using two additional equilibrium equations and the mole fractions you determined in part (a).

(c) Assuming the reactants are premixed and enter the combustion chamber at 600 K, 0.2 MPa, determine the heat transfer to or from the combustion chamber in kJ per kmol of n-butane. (Hint: the concentrations of $O_2$ and NO will be much lower than the other concentrations and do not need to be included in the thermochemistry calculations).

**Given:**

In a constant pressure combustor

$C_4H_{10}$ and 85% theoretical air burns at $P_1 = 0.2$ MPa, $T_1 = 600K$

Products exit at $P_2 = 0.2$ MPa, $T_2 = 2000K$

**Find:**

(a) $y_{2,N_2}$, $y_{2,CO_2}$, $y_{2,H_2O}$, $y_{2,CO}$, $y_{2,H_2}$

(b) $y_{2,O_2}$, $y_{2,NO}$

(c) $\dot{Q}_{CV}$

**System sketch:**

![System sketch](image)
Assumptions:
(1) Ideal gases
(2) Steady state, uniform flow
(3) Constant pressure combustion
(4) Neglect KE, PE
(5) \( \dot{W}_{CV} = 0 \)

Basic equations:
\[
\frac{dE_{CV}}{dt} = 0 = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_{k} \dot{n}_{1,k} \bar{h}_{1,k} - \sum_{k} \dot{n}_{2,k} \bar{h}_{2,k}
\]
\[
\Rightarrow \dot{Q}_{CV} = \sum_{k} \dot{n}_{2,k} \bar{h}_{2,k} - \sum_{k} \dot{n}_{1,k} \bar{h}_{1,k} = H_2 - H_1 \\
\bar{h}_{i} (T) = \bar{h}_{i,298}^0 + \Delta \bar{h}_{i,i} (T - 298)
\]
\[
K(T) = \frac{y_{C}^{v_{C}} y_{D}^{v_{D}}}{y_{A}^{v_{A}} y_{B}^{v_{B}}} \left( \frac{p}{p_{ref}} \right)^{y_{C}^{v_{D}-v_{C}}+y_{D}^{v_{C}-v_{D}}}
\]

Solution:
(a) Process Equation
Stoichiometric equation \( (a_{stoich} = x + \frac{y}{4}) \):
\[
C_{4}H_{10} + 6.5(O_{2} + 3.76N_{2}) \rightarrow 5H_{2}O + 4CO_{2} + 24.44N_{2}
\]
Reaction equation with 85% theoretical air \( (a = 0.85a_{stoich} = 5.525) \):
\[
C_{4}H_{10} + 5.525(O_{2} + 3.76N_{2}) \rightarrow bCO_{2} + cCO + dH_{2}O + eH_{2} + fN_{2} + gO_{2} + hNO
\]
Neglecting NO and O
2, the reaction equation becomes
\[
C_{4}H_{10} + 5.525(O_{2} + 3.76N_{2}) \rightarrow bCO_{2} + cCO + dH_{2}O + eH_{2} + 5.525 \times 3.76N_{2}
\]
Atom balance:
C: \( 4=b+c \) \( \Rightarrow c=4-b \)
H: \( 10=2d+2e \) \( \Rightarrow e=5-d \)
O: \( 2 \times 5.525=2b+c+d=4+b+d \) \( \Rightarrow d=7.05-b \)
Chemical equilibrium

Equilibrium equation and constant from Table A-27:

\[ \text{CO}_2 + \text{H}_2 \rightleftharpoons \text{H}_2\text{O} + \text{CO} \]

\[ K_{eq} = 10^{0.656} = 4.529 \quad \text{at } T_2 = 2000\text{K} \]

From our reaction equation,

\[ K_{eq} = \frac{y_{\text{CO}}y_{\text{H}_2\text{O}}}{y_{\text{CO}_2}y_{\text{H}_2}} \left( \frac{P}{P_{\text{ref}}} \right)^{1+1-1-1} = \frac{cd}{be} = \frac{(4 - b)(7.05 - b)}{b(5 - 7.05 + b)} \]

Solve the quadratic equation,

\[ \frac{(4 - b)(7.05 - b)}{b(5 - 7.05 + b)} = 4.529 \]

It gives,

\[ b = 2.58772 \quad \text{and} \quad -3.08802 \]

Choose the positive value. Then, \( c = 1.41228, d = 4.46228, \) and \( e = 0.53772. \) Thus, the reaction equation is

\[ \text{CH}_4 + 5.525(\text{O}_2 + 3.76\text{N}_2) \]
\[ \quad \rightarrow 2.58772 \text{CO}_2 + 1.41228 \text{CO} + 4.46228 \text{H}_2\text{O} + 0.53772 \text{H}_2 + 20.774\text{N}_2 \]

The equilibrium composition is

\[ 2.58772 \text{CO}_2 + 1.41228 \text{CO} + 4.46228 \text{H}_2\text{O} + 0.53772 \text{H}_2 + 20.774\text{N}_2 \]

At state 2, \( \dot{n}_{2,\text{tot}} \approx \dot{n}_{2,\text{CO}_2} + \dot{n}_{2,\text{CO}} + \dot{n}_{2,\text{H}_2\text{O}} + \dot{n}_{2,\text{H}_2} + \dot{n}_{2,\text{N}_2} \)

\[ = 2.58772 + 1.41228 + 4.46228 + 0.53772 + 20.774 = 29.774 \text{ kmol/s} \]

Therefore, mole fractions are

\[ y_{\text{CO}_2} = \frac{\dot{n}_{2,\text{CO}_2}}{\dot{n}_{2,\text{tot}}} = \frac{2.58772}{29.774} = 0.08691 \]

\[ y_{\text{CO}} = \frac{\dot{n}_{2,\text{CO}}}{\dot{n}_{2,\text{tot}}} = \frac{1.41228}{29.774} = 0.04743 \]

\[ y_{\text{N}_2} = \frac{\dot{n}_{2,\text{N}_2}}{\dot{n}_{2,\text{tot}}} = \frac{20.774}{29.774} = 0.69672 \]

N: 5.525×2×3.76=5.525×2×3.76
\[ y_{H_2} = \frac{\hat{n}_{2,H_2}}{\hat{n}_{2,tot}} = \frac{0.53772}{29.774} = 0.01806 \]
\[ y_{H_2O} = \frac{\hat{n}_{2,H_2O}}{\hat{n}_{2,tot}} = \frac{4.46228}{29.774} = 0.149872 \]

\[ y_{CO_2} = 0.08691 \quad y_{CO} = 0.04743 \]
\[ y_{N_2} = 0.69672 \quad y_{H_2} = 0.01806 \]
\[ y_{H_2O} = 0.149872 \]

(b) In order to solve equilibrium decompositions for NO and O\(_2\) simultaneously by adding two additional equilibrium equations, we need to establish a reaction equation as described below,
\[
20.774N_2 + 2.58772CO_2 + 1.41228CO + 4.46228H_2O + 0.53772H_2 \rightarrow xCO_2 + yCO + zO_2 + pN_2 + qNO + 4.46228H_2O + 0.53772H_2
\]

**Note:** For NO, \( \frac{1}{2}O_2 + \frac{1}{2}N_2 \Leftrightarrow NO \) is required. For O\(_2\), there are two options: \( CO_2 \Leftrightarrow CO + \frac{1}{2}O_2 \) and \( H_2O \Leftrightarrow H_2 + \frac{1}{2}O_2 \). But, we only need one more equilibrium equation as the problem statement. Then, we have to choose \( CO_2 \Leftrightarrow CO + \frac{1}{2}O_2 \) based on a higher equilibrium constant:
\[
K_{CO_2=CO+\frac{1}{2}O_2} = 10^{-2.884} > K_{H_2O=H_2+\frac{1}{2}O_2} = 10^{-3.540}.\]
Herein, it is assumed that the effects of H\(_2\)O and H\(_2\) is negligible and the gases exist as an inert gas.

Atom balance:
\[
C: \quad 2.58772 + 1.41228 = x + y \quad \Rightarrow y = 4 - x
\]
\[
O: \quad 2 \times 2.58772 + 1.41228 = 2x + y + 2z + q \quad \Rightarrow z = (6.58772 - 2x - y - q)/2
\]
\[
N: \quad 20.774 \times 2 = 2p + q \quad \Rightarrow p = 20.774 - 0.5q
\]

With equilibrium \( CO_2 \Leftrightarrow CO + \frac{1}{2}O_2 \)
\[
K_{eq}(T_2) = \frac{yCO \times \frac{1}{2}O_2}{yCO_2 \times \frac{1}{2}P_{ref}}^{1.5} = 10^{-2.884} = 0.001306 \quad at \quad T = 2000K
\]
\[
\Rightarrow \frac{yz^{0.5}}{x} \left( \frac{200 \text{ kPa}}{(x + y + z + p + q + 4.46228 + 0.53772)101.325 \text{ kPa}} \right)^{0.5} = 0.001306
\]
\[
\Rightarrow \frac{yz^{0.5}}{x(x + y + z + p + q + 4.46228 + 0.53772)^{0.5}} = 0.001306 \times \sqrt{\frac{101.325}{200}} = 0.0009296
\]
With equilibrium \( \frac{1}{2} \text{O}_2 + \frac{1}{2} \text{N}_2 \rightleftharpoons \text{NO} \)

\[
K_{eq}(T_2) = \frac{y_{\text{NO}}}{y_{\text{O}_2}^{1/2}y_{\text{N}_2}^{1/2}} \left( \frac{P_2}{P_{\text{ref}}} \right)^{1/2} = 10^{-1.699} = 0.0200 \quad \text{at } T = 2000K
\]

\[
\Rightarrow \frac{q}{z^{0.5}p^{0.5}} = 0.0200
\]

Solve 5 equations and 5 unknowns (EES code for this is attached). Then,

\[
x = 2.587 \quad y = 1.413 \quad z = 0.00008619
\]

\[
p = 20.77 \quad q = 0.0008463
\]

Therefore, the equilibrium composition is

\[
2.587 \text{CO}_2 + 1.413 \text{CO} + 0.00008619 \text{O}_2 + 20.77 \text{N}_2 + 0.0008463 \text{NO} + 4.46228 \text{H}_2\text{O} + 0.53772 \text{H}_2
\]

Consequently, the reaction becomes

\[
\text{C}_4\text{H}_{10} + 5.525(\text{O}_2 + 3.76\text{N}_2) \to 2.587 \text{CO}_2 + 1.413 \text{CO} + 0.00008619 \text{O}_2 + 20.77 \text{N}_2
\]

\[
+ 0.0008463 \text{NO} + 4.46228 \text{H}_2\text{O} + 0.53772 \text{H}_2
\]

The mole fractions for are

\[
y_{\text{CO}_2} = 0.08690 \quad y_{\text{CO}} = 0.04746
\]

\[
y_{\text{N}_2} = 0.6977 \quad y_{\text{H}_2} = 0.01806
\]

\[
y_{\text{H}_2\text{O}} = 0.1499 \quad y_{\text{NO}} = 2.8428 \times 10^{-5} \quad y_{\text{O}_2} = 2.8952 \times 10^{-6}
\]

**EES code**

```
"5 equations"
y=4-x
z=(6.58772-2x-y-q)/2
p=20.774-0.5*q
(y*z^(0.5))/x/((x+y+z+p+q+4.46228+0.53772)^(0.5))=0.0009296
q/(z^(0.5))/(p^(0.5))=0.02
```
(c)

Using Table A-23 and A-25,

Reactants

\[ T_1 = 600 \, K, \quad p_1 = 2 \, \text{bar} \]

\[ \overline{H}^0_{f,298.\,C_4H_{10}} = -125,510 \, \frac{kJ}{kmol} \]

\[ c_{\,p,\,C_4H_{10}} = 1.71 \, \frac{kJ}{kg \, K} \quad \overline{c}_{\,p,\,C_4H_{10}} = \left( 58.12 \, \frac{kg}{kmol} \right) \left( 1.71 \, \frac{kJ}{kg \, K} \right) = 99.4 \, \frac{kJ}{kmol \, K} \]

\[ \overline{h}_{s,\,C_4H_{10}}(600 \, K) - \overline{h}_{s,\,C_4H_{10}}(298 \, K) = \overline{c}_{\,p,\,C_4H_{10}}(T - 298) = \left( 99.4 \, \frac{kJ}{kmol \, K} \right)(600 - 298 \, K) \]

\[ \overline{h}_{s,\,C_4H_{10}}(600 \, K) - \overline{h}_{s,\,C_4H_{10}}(298 \, K) = 30,020 \, \frac{kJ}{kmol} \]

\[ \overline{h}_{C_4H_{10}}(600 \, K) = \overline{h}^0_{f,298.\,C_4H_{10}} + \overline{h}_{s,\,C_4H_{10}}(600 \, K) - \overline{h}_{s,\,C_4H_{10}}(298 \, K) = -95,490 \, \frac{kJ}{kmol} \]

\[ \overline{h}_{O_2}(600 \, K) = \overline{h}^0_{f,298.\,O_2} + \overline{h}_{s,\,O_2}(600 \, K) - \overline{h}_{s,\,O_2}(298 \, K) = 17,929 - 8,682 = 9,247 \, \frac{kJ}{kmol} \]

\[ \overline{h}_{N_2}(600 \, K) = \overline{h}^0_{f,298.\,N_2} + \overline{h}_{s,\,N_2}(600 \, K) - \overline{h}_{s,\,N_2}(298 \, K) = 17,563 - 8,669 = 8,894 \, \frac{kJ}{kmol} \]

\[ \dot{H}_1 = \sum_k n_{1,k} \overline{h}_{1,k} = \left( 1 \, \frac{kmol}{s} \right) \left( -95,490 \, \frac{kJ}{kmol} \right)_{C_4H_{10}} + \left( 5.525 \, \frac{kmol}{s} \right) \left( 9,247 \, \frac{kJ}{kmol} \right)_{O_2} + \left( 20.774 \, \frac{kmol}{s} \right) \left( 8,894 \, \frac{kJ}{kmol} \right)_{N_2} = 140,360 \, \frac{kJ}{s} \]
Products

\[ T_2 = 2000 \, K, \; p_1 = 2 \text{ bar} \]

\[
\bar{h}_{\text{CO}_2} (2000 \, K) = \bar{h}^0_{j, \text{CO}_2} + \bar{h}_{s, \text{CO}_2} (2000 \, K) - \bar{h}_{s, \text{CO}_2} (298 \, K) = -393,520 + 100,804 - 9,364 = -302,080 \frac{kJ}{kmol}
\]

\[
\bar{h}_{\text{CO}} (2000 \, K) = \bar{h}^0_{j, \text{CO}} + \bar{h}_{s, \text{CO}} (2000 \, K) - \bar{h}_{s, \text{CO}} (298 \, K) = -110,530 + 65,408 - 8,669 = -53790 \frac{kJ}{kmol}
\]

\[
\bar{h}_{\text{H}_2\text{O}} (2000 \, K) = \bar{h}^0_{j, \text{H}_2\text{O}} + \bar{h}_{s, \text{H}_2\text{O}} (2000 \, K) - \bar{h}_{s, \text{H}_2\text{O}} (298 \, K) = -241,820 + 82,593 - 9,904 = -169130 \frac{kJ}{kmol}
\]

\[
\bar{h}_{\text{H}_2} (2000 \, K) = \bar{h}^0_{j, \text{H}_2} + \bar{h}_{s, \text{H}_2} (2000 \, K) - \bar{h}_{s, \text{H}_2} (298 \, K) = 52,970 \frac{kJ}{kmol}
\]

\[
\bar{h}_{\text{N}_2} (2000 \, K) = \bar{h}^0_{j, \text{N}_2} + \bar{h}_{s, \text{N}_2} (2000 \, K) - \bar{h}_{s, \text{N}_2} (298 \, K) = 64,810 - 8,669 = 56,140 \frac{kJ}{kmol}
\]

\[
\dot{H}_2 = \sum_k \dot{n}_{2,k} \bar{h}_{2,k}
\]

\[
= (2.58772 \frac{kmol}{s})(-302,080 \frac{kJ}{kmol})_{\text{CO}_2}
+ (1.41228 \frac{kmol}{s})(-53790 \frac{kJ}{kmol})_{\text{CO}}
+ (4.46228 \frac{kmol}{s})(-169130 \frac{kJ}{kmol})_{\text{H}_2\text{O}}
+ (0.53772 \frac{kmol}{s})(52,970 \frac{kJ}{kmol})_{\text{H}_2}
+ (20.774 \frac{kmol}{s})(56,140 \frac{kJ}{kmol})_{\text{N}_2}
= -417,635 \frac{kJ}{s}
\]

\[
\dot{Q}_{CV} = \dot{H}_2 - \dot{H}_1 = -417,635 - 140,360 = -557,995 \frac{kJ}{s}
\]

Thus,

\[
\frac{\dot{Q}_{CV}}{1 \, kmol \, of \, n - \text{butane per second in the reaction}} = \frac{-557,995 \frac{kJ}{s}}{1 \, kmol \, of \, n - \text{butane}}
\]

\[
= -557,995 \frac{kJ}{kmol \, of \, n - \text{butane}}
\]
A simple ideal Rankine cycle with water as the working fluid operates between the pressure limits of 15 MPa in the boiler and 100 kPa in the condenser. Saturated steam enters the turbine.

a. Determine the work produced by the turbine per unit mass, the heat transferred in the boiler per unit mass, and the thermal efficiency of the cycle.

b. Now consider that irreversibilities in the turbine cause the steam quality at the outlet of the turbine to be 0.7. Determine the isentropic efficiency of the turbine and the thermal efficiency of the cycle.

**Given:**

A Rankine cycle with water operates at

\[ P_{\text{cond}} = 100 \text{ kPa}, \ P_{\text{boiler}} = 15 \text{ MPa} \]

the steam quality at the turbine inlet is 1.0

**Find:**

(a) \( \dot{W}_{\text{turbine}}/m \), \( \dot{Q}_{\text{boiler}}/m \), and \( \eta_{\text{th}} \)

(b) \( \eta_{\text{isen}}, \eta_{\text{th,irrev}} \)

**System sketch:**

None

**Assumptions:**

1. Steady state, steady flow
2. KE and PE negligible
3. Internally reversible pump
4. No work at the boiler and the condenser
5. Adiabatic at the turbine and the pump
6. Water enters the pump as a saturated liquid
7. Negligible change of specific volume in the pump
Basic equations:

\[
\frac{dE_{cv}}{dt} = 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right)
\]

Solution:

(a)

T-s diagram for the ideal Rankine cycle is

For boiler and condenser: \(\dot{Q}_{cv}/\dot{m} + h_{in} - h_{out} = 0\)

For turbine and pump: \(-\dot{W}_{cv}/\dot{m} + h_{in} - h_{out} = 0\)

Steam properties can be found from the steam tables, Table A-2 and A-3

\[
h_1 = h_{f,100kPa} = 417.46 \frac{kJ}{kg} \quad v_1 = v_{f,100kPa} = 0.0010432 \frac{m^3}{kg}
\]

Work done by an internally reversible adiabatic pump (refer Chapter 8.2.2):

\[
\frac{\dot{W}_{pump}}{\dot{m}} = v_1 (P_2 - P_1) = 0.0010432 \frac{m^3}{kg} (15,000 - 100)kPa = 15.54 \frac{kJ}{kg}
\]

\[
h_2 = h_1 + \frac{\dot{W}_{pump}}{\dot{m}} = 417.46 \frac{kJ}{kg} + 15.54 \frac{kJ}{kg} = 433.00 \frac{kJ}{kg}
\]
$P_3 = 15,000 \text{ kPa}, x_3 = 1 \quad \Rightarrow \quad h_3 = 2610.5 \frac{\text{kJ}}{\text{kg}} \quad s_3 = 5.3098 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$

$P_4 = 100 \text{ kPa}, s_4 = s_4 \quad \Rightarrow \quad x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{5.3098 - 1.3026}{7.3594 - 1.3026} = 0.6616$

$h_4 = h_f + x_4 h_{fg} = 417.46 + 0.6616 \times 2258.0 = 1911.35 \text{ kJ/kg}$

Thus,

$\dot{W}_{\text{turbine}}/\dot{m} = h_3 - h_4 = 2610.5 - 1911.35 = 699.15 \text{ kJ/kg}$

$\dot{Q}_{\text{boiler}}/\dot{m} = h_3 - h_2 = 2610.5 - 433.00 = 2177.5 \text{ kJ/kg}$

$\dot{Q}_{\text{condenser}}/\dot{m} = h_4 - h_1 = 1911.35 - 417.46 = 1493.89 \text{ kJ/kg}$

The thermal efficiency of the cycle is

$\eta_{th} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{\dot{Q}_{\text{condenser}}/\dot{m}}{\dot{Q}_{\text{boiler}}/\dot{m}} = 1 - \frac{1493.89}{2177.5} = 0.3139$

(b)

$T$-$s$ diagram for the Rankine cycle with irreversibilities at the turbine is

Except properties at state 4, all properties are identical with part (a).
\[ h_1 = 417.46 \frac{kJ}{kg} \quad h_2 = 433.00 \frac{kJ}{kg} \quad h_3 = 2610.5 \frac{kJ}{kg} \quad h_{4s} = 1911.35 \frac{kJ}{kg} \]

Because the steam quality at the outlet of the turbine is 0.7
\[ P_4 = 100 \text{ kPa}, x_4 = 1 \quad \Rightarrow \quad h_4 = h_f + x_4 h_{fg} = 417.46 + 0.7 \times 2258.0 = 1998.06 \frac{kJ}{kg} \]

The isentropic efficiency of the turbine is
\[ \eta_{isen} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{2610.5 - 1998.06}{2610.5 - 1911.35} = 0.8760 \]

Newly calculated \( \dot{Q}_{\text{condensor}}/\dot{m} \).
\[ \dot{Q}_{\text{condensor}}/\dot{m} = h_4 - h_1 = 1998.06 - 417.46 = 1580.6 \frac{kJ}{kg} \]

The thermal efficiency of the cycle with irrversibilities at the turbine is
\[ \eta_{th,irrev} = 1 - \frac{Q_{out}}{Q_{in}} = \frac{\dot{Q}_{\text{condensor}}/\dot{m}}{\dot{Q}_{\text{boiler}}/\dot{m}} = 1 - \frac{1580.6}{2177.5} = 0.2741 \]
A steam power plant operates on an ideal reheat Rankine cycle between the pressure limits of 15 MPa and 10 kPa. The mass flow rate of steam through the cycle is 12 kg/s. Steam enters both stages of the turbine at 500°C. If the moisture content of the steam at the exit of the low-pressure turbine is not to exceed 10 percent,

- a. Determine the pressure at which reheating takes place.
- b. Determine the total rate of heat input in the boiler.
- c. Determine the thermal efficiency of the cycle.
- d. Plot the cycle on a T-s diagram with respect to saturation lines.

**Given:**

An ideal reheat Rankine cycle with water operates at

\[ P_{\text{min}} = 10 \text{ kPa}, \quad P_{\text{max}} = 15 \text{ MPa} \]

\[ T_{\text{turb1,inlet}} = T_{\text{turb2,inlet}} = 500\degree C \]

\[ m = 12 \text{ kg/s} \]

The quality at the exit of the low-pressure turbine is targeted \( \geq 0.9 \)

**Find:**

- (a) \( P_{\text{reheat}} \)
- (b) \( Q_{\text{in}} \)
- (c) \( \eta_{\text{th}} \)
- (d) T-s diagram

**System sketch:**

None

**Assumptions:**

1. Steady state, steady flow
2. KE and PE negligible
3. Internally reversible pump
4. No work at the boiler and the condenser
5. Adiabatic at the turbines and the pump
6. Water enters the pump as a saturated liquid
7. Negligible change of specific volume in the pump

**Basic equations:**

\[
\frac{dE_{cv}}{dt} = 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right)
\]

**Solution:**

\(T\)-\(s\) diagram for the ideal reheat Rankine cycle is

State 1-2: pump
State 2-3: boiler
State 3-4: 1\(^{st}\) turbine
State 4-5: reheat
State 5-6: 2\(^{nd}\) turbine
State 6-1: condenser

(a)

Steam properties can be found from the steam tables, Table A-2, A-3, and A-4

\[h_1 = h_{f,10kPa} = 191.83 \frac{kJ}{kg}\]

\[v_1 = v_{f,10kPa} = 0.0010102 \frac{m^3}{kg}\]

Work done by an internally reversible adiabatic pump (refer Chapter 8.2.2):

\[
\frac{W_{pump}}{m} = v_1 (P_2 - P_1) = 0.0010102 \frac{m^3}{kg} (15,000 - 10) kPa = 15.14 \frac{kJ}{kg}
\]

\[h_2 = h_1 + \frac{W_{pump}}{m} = 191.83 \frac{kJ}{kg} + 15.14 \frac{kJ}{kg} = 206.97 \frac{kJ}{kg}\]
\[ P_3 = 15,000 \text{ kPa}, \quad T_3 = 500^\circ\text{C} \quad \Rightarrow \quad h_3 = 3307.5 \text{ kJ/kg} \quad s_3 = 6.3430 \text{ kJ/kgK} \]

(need double interpolation from the table)

From the condition, the quality at the exit of the low-pressure turbine is targeted \( \geq 0.9 \), we set \( x_6 = 0.9 \)

\[ P_6 = 10 \text{ kPa} \quad \Rightarrow \quad h_6 = h_f + x_6 h_{fg} = 191.83 + 0.9 \times 2392.8 = 2345.35 \text{ kJ/kg} \]

\[ s_6 = s_f + x_6 s_{fg} = 0.6493 + 0.9 \times (8.1502 - 0.6493) = 7.4001 \text{ kJ/kgK} \]

\[ T_5 = 500^\circ\text{C}, \quad s_5 = s_6 \quad \Rightarrow \quad \frac{s_{2MPa, 500^\circ\text{C}} - s_{P_5, 500^\circ\text{C}}}{s_{2MPa, 500^\circ\text{C}} - s_{3MPa, 500^\circ\text{C}}} = \frac{7.4317 - 7.4001}{7.4317 - 7.2338} = \frac{2MPa - P_5}{2MPa - 3MPa} \]

\[ P_5 = 2.16 \text{ MPa (the reheat pressure)} \]

\[ \frac{h_{3MPa, 500^\circ\text{C}} - h_{P_5, 500^\circ\text{C}}}{h_{3MPa, 500^\circ\text{C}} - h_{2MPa, 500^\circ\text{C}}} = \frac{3456.5 - h_5}{3456.5 - 3467.6} = \frac{3 - 2.16}{3 - 2} \quad \Rightarrow \quad h_5 = 3465.82 \text{ kJ/kg} \]

\[ P_4 = 2.16 \text{ MPa, } s_4 = s_3 \quad \Rightarrow \quad \frac{h_{2MPa, s=6.3430} - h_4}{h_{2MPa, s=6.3430} - h_{3MPa, s=6.3430}} = \frac{2800.55 - h_4}{2800.55 - 2886.34} = \frac{2 - 2.16}{2 - 3} \]

(need double interpolation from the table)

\[ \Rightarrow \quad h_4 = 2814.28 \text{ kJ/kg} \]

(b)

The rate of heat input is

\[ \dot{Q}_{in} = \dot{m}[(h_3 - h_2) + (h_5 - h_4)] = 12 \text{ kg/s} \cdot \left[ (3307.5 - 206.97) + (3465.82 - 2814.28) \right] \frac{\text{kJ}}{\text{kg}} \]

\[ = 45,024.8 \frac{\text{kJ}}{\text{s}} \]

(c)
The thermal efficiency is determined from $Q_{\text{in}}$ and $Q_{\text{out}}$.

$$
\dot{Q}_{\text{out}} = \dot{m}(h_6 - h_1) = 12 \frac{kg}{s} (2345.35 - 191.83) \frac{kJ}{kg} = 25,842.2 \frac{kJ}{s}
$$

Thus,

$$
\eta_{th} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{25,842.2}{45,024.8} = 0.4260 \text{ or } 42.60\%
$$

(d) It is drawn at the initial part of the solution.