(SP 15)
Utah Center Building uses outdoor air (state 1) at 35°C and 20% relative humidity to cool water by direct evaporation to state 5. The relative humidity at state 5 is 100% and energy added from the water spray is negligible. Then the cooled water runs through a cooling coil to cool the outdoor air in the air conditioning system to state 2. With a perfect cooling coil, $T_2 = T_5$. The air is then further cooled down to 15°C at state 3 by another cooling coil with chilled water. Finally, the air in the air conditioning system is cooled down by direct evaporative cooling to a relative humidity of 100% at state 4 before it is sent to a room. The moisture generated in the room is negligible and the heat generation rate in the room is 2 kW. The room air temperature in the summer is maintained at 25°C.

**Given:**
A moist air flows,

- State 1: $P_1 = 1$ atm, $T_1 = 35$ °C, $\phi_1 = 20$
- State 2: $P_2 = 1$ atm, $T_2 = T_5$
- State 3: $P_3 = 1$ atm, $T_3 = 15$ °C
- State 4: $P_4 = 1$ atm, $\phi_4 = 100$
- State 5: $P_5 = 1$ atm, $\phi_5 = 100$

Heat generation in the room is 2 kW.

The room air temperature is maintained at 25 °C.

**Find:**
(a) Label states 1, 2, 3, 4, 5, and room air and processes 1-2-3-4-room air on the psychrometric chart

(b) Complete the following table by using the psychrometric chart, excluding the shaded cells

(c) Determine the required supply airflow rate for the stream 1-4, in kg/s

(d) Calculate the heat transfer rate in the heat exchanger between stream 1-5 and stream 1-2, in kJ/s

(e) Find if water is added to the air or condensed out of the air in process 3-4
System sketch:

Assumptions:

1. Steady state, steady uniform flow
2. Neglect $\Delta KE$ and $\Delta PE$
3. No work
4. Ideal gases
5. No heat transfer from a water spray
6. No heat loss along the duct

Basic equations:

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e$$

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right)$$

Solution:

Mass balance:

$$\dot{m}_{a1,upper} = \dot{m}_{a5} \quad for \ CV1$$
\[
\dot{m}_{a1,\text{lower}} = \dot{m}_{a2} = \dot{m}_{a3} = \dot{m}_{a4} = \dot{m}_{\text{room}} \quad \text{for CV2,3,4, and 5}
\]
\[
\dot{m}_{a4}\omega_4 = \dot{m}_{\text{room}}\omega_{\text{room}} \quad \text{for CV5}
\]
\[
\rightarrow \omega_4 = \omega_{\text{room}}
\]
\[
\dot{m}_{a1,\text{lower}}\omega_1 = \dot{m}_{a2}\omega_2 = \dot{m}_{a3}\omega_3 \quad \text{for CV2 and 3}
\]
\[
\rightarrow \omega_1 = \omega_2 = \omega_3
\]

Energy balance:
\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right) \quad \text{for CV1 and 4}
\]

Rewrite for each control volume,
\[
0 = \dot{m}_{a1,\text{upper}} h_1 - \dot{m}_{a5} h_5 \pm \dot{m}_{\text{water}} h_{\text{water}} \quad \text{for CV1}
\]
\[
\therefore h_1 = h_5 \quad \therefore \dot{m}_{a1,\text{upper}} = \dot{m}_{a5}
\]
\[
0 = \dot{m}_{a3} h_3 - \dot{m}_{a4} h_4 \pm \dot{m}_{\text{water}} h_{\text{water}} \quad \text{for CV4}
\]
\[
\therefore h_3 = h_4 \quad \therefore \dot{m}_{a3} = \dot{m}_{a4}
\]

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right) \quad \text{for CV5}
\]
\[
0 = \dot{Q}_{cv} + \dot{m}_{a4} h_4 - \dot{m}_{\text{room}} h_{\text{room}}
\]

Using the psychrometric chart,
\[
T_1 = 35 \degree \text{C}, \phi_1 = 20\% \quad \rightarrow \omega_1 = 0.007 \frac{\text{kg water}}{\text{kg air}}, \quad h_1 = 53 \frac{\text{kJ}}{\text{kg air}}
\]
\( h_5 = h_1 = 53 \ \text{kJ/kg air}, \ \phi_5 = 100\% \quad \Rightarrow \quad T_5 = 18.5 \degree C \)

\[
\omega_2 = \omega_1 = 0.007 \ \frac{\text{kg water}}{\text{kg air}}, \quad T_2 = T_3 = 18.5 \degree C \quad \Rightarrow \quad h_2 = 36 \ \frac{\text{kJ}}{\text{kg air}}
\]

\[
\omega_3 = \omega_2 = 0.007 \ \frac{\text{kg water}}{\text{kg air}}, \quad T_3 = 15 \degree C \quad \Rightarrow \quad h_3 = 33 \ \frac{\text{kJ}}{\text{kg air}}
\]

\[
\omega_4 = \omega_3 = 0.007 \ \frac{\text{kg water}}{\text{kg air}}, \quad T_4 = 11.5 \degree C \quad \Rightarrow \quad h_4 = h_3 = 33 \ \frac{\text{kJ}}{\text{kg air}}, \ \phi_4 = 100\%
\]

\[
\omega_{\text{room}} = \omega_4 = 0.0085 \ \frac{\text{kg water}}{\text{kg air}}, \quad T_{\text{room}} = 25 \degree C \quad \Rightarrow \quad h_{\text{room}} = 46 \ \frac{\text{kJ}}{\text{kg air}}, \ \phi_{\text{room}} = 43\%
\]

(a)

Attached at the end of this solution

(b)

<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Room</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature [\degree C]</td>
<td>35</td>
<td>18.5</td>
<td>15</td>
<td>11.5</td>
<td>25</td>
</tr>
<tr>
<td>Relative humidity [%]</td>
<td>20</td>
<td></td>
<td>100</td>
<td></td>
<td>43</td>
</tr>
<tr>
<td>Humidity ratio [kgw/kga]</td>
<td>0.007</td>
<td>0.007</td>
<td></td>
<td>100</td>
<td>43</td>
</tr>
<tr>
<td>Enthalpy [kJ/kg]</td>
<td>53</td>
<td>36</td>
<td>33</td>
<td>46</td>
<td></td>
</tr>
</tbody>
</table>

(c)

Recall the energy balance equation for CV5,

\[
0 = \dot{Q}_{\text{cv}} + \dot{m}_a h_4 - \dot{m}_{\text{room}} h_{\text{room}} = \dot{Q}_{\text{cv}} + \dot{m}_a h_4 - \dot{m}_a h_{\text{room}}
\]

Rearrange,
\[
\dot{m}_{4a} = \frac{Q_{cv}}{h_{\text{room}} - h_4} = \frac{2kJ/s}{46 - 33 \text{ kJ/kg}} = 0.154 \text{ kg/s}
\]

Therefore, the required airflow rate is 0.154 kg/s.

(d)

Energy balance:

\[
\frac{d E_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i (h_i + \frac{v_i^2}{2} + gz_i) - \sum_e \dot{m}_e (h_e + \frac{v_e^2}{2} + gz_e)
\]

\[\text{for CV2}\]

\[0 = \dot{Q}_{cv2} + \dot{m}_{a1,\text{lower}} h_1 - \dot{m}_{a2} h_2\]

Rearrange,

\[\dot{Q}_{cv2} = \dot{m}_{a2} h_2 - \dot{m}_{a1,\text{lower}} h_1 = \dot{m}_{a2} (h_2 - h_1) = \dot{m}_{a4} (h_2 - h_1)\]

\[= 0.154 \frac{kg}{s} (36 - 53) \frac{kJ}{kg}\]

\[= -2.618 \text{ kJ/s}\]

Therefore, the heat transfer rate for the stream 1\(\rightarrow\)2 is \(-2.618 \text{ kJ/s}\) and the direction is out of the stream 1\(\rightarrow\)2. Because there is no heat loss along the duct, the amount of heat rejection from the stream 1\(\rightarrow\)2 is equal to the amount of heat addition into the stream 1\(\rightarrow\)5. Thus, the heat transfer rate for the stream 1\(\rightarrow\)5 is \(2.618 \text{ kJ/s}\) and its direction is into the stream 1\(\rightarrow\)5. However, this heat addition by the water spray is neglected when we solve an energy balance in previous parts as described in the problem statement.

(e)

Recall

\[\omega_3 = 0.007 \frac{\text{kg water}}{\text{kg air}}\]

\[\omega_4 = 0.0085 \frac{\text{kg water}}{\text{kg air}}\]

\[\dot{m}_{a3} = \dot{m}_{a4} = 0.154 \text{ kg/s}\]
Mass balance for CV4:

\[ m_{a3} \omega_3 + \dot{m}_{\text{water}}_{\text{CV4}} = m_{a4} \omega_4 \]

\[ \dot{m}_{\text{water}}_{\text{CV4}} = m_{a4} \omega_4 - m_{a3} \omega_3 = m_{a3} (\omega_4 - \omega_3) \]

\[ = 0.154 \frac{kg \text{ air}}{s} (0.0085 - 0.007) \frac{kg \text{ water}}{kg \text{ air}} \]

\[ = 0.000231 \frac{kg \text{ water}}{s} \]

The sign we assigned for the mass flow rate of water in CV4 is correct. Therefore, the water is added to the air in process 3-4 with the amount of 0.000231 kg/s.
A rocket engine using liquid-methane and gaseous oxygen operates with gaseous oxygen flow tube that is at $P = 202 \text{ bar}$ and $T = 170 \text{ K}$ with a volume flow rate of 1 L/s.

**Given:**

liquid-methane and gaseous oxygen operates with gaseous oxygen flow tube

$P = 202 \text{ bar} = 20200 \text{ kPa}$

$T = 170 \text{ K}$

a volume flow rate of 1 L/s = 0.001 m$^3$/s

**Find:**

the mass flow rate of gaseous oxygen in the flow tube in kg/s using:

(a) The ideal gas equation of state
(b) The generalized compressibility chart
(c) The Van der Waals equation of state
(d) The Redlich-Kwong equation of state
(e) Discuss what might happen to the rocket engine if the development engineer had the wrong mass flow rate of oxygen.

**System sketch:**

none

**Assumptions:**

none

**Basic equations:**

$$PV = mRT$$

$$PV = ZmRT$$

$$P = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2}$$

$$P = \frac{\bar{R}T}{\bar{v} - b} - \frac{aT^{-\frac{1}{2}}}{\bar{v}^2 + \bar{v}b}$$
Solution:

(a)

The ideal gas equation of state,

\[ PV = mRT \rightarrow P\dot{V} = mRT \rightarrow \dot{m} = \frac{P\dot{V}}{RT} \]

\[
\dot{m} = \frac{P\dot{V}}{R_{\text{oxygen}}T} = \frac{(20200 \text{ kPa})(0.001 \text{ m}^3/\text{s})}{(0.2598 \frac{\text{kJ}}{\text{kgK}})(170 \text{K})} = 0.457 \text{ kg/s}
\]

where \( R_{\text{oxygen}} = 0.2598 \text{ kJ/kgK} \) is from Table 3.1 in Chapter 3.

(b)

From Table A-1, critical properties of oxygen are,

\[ P_c = 50.5 \text{ bar} \]
\[ T_c = 154 \text{ K} \]

The reduced properties are,

\[ P_R = \frac{P}{P_c} = \frac{202 \text{ bar}}{50.5 \text{ bar}} = 4 \]
\[ T_R = \frac{T}{T_c} = \frac{170 \text{ K}}{154 \text{ K}} = 1.10 \]

From Figure A-2, the generalized compressibility chart, \( P_R \leq 10.0 \),

\[ P_R = 4 \text{ and } T_R = 1.10 \quad \rightarrow \quad Z = 0.575 \]

Thus,

\[ PV = ZmRT \rightarrow P\dot{V} = Z\dot{m}RT \rightarrow \dot{m} = \frac{P\dot{V}}{ZRT} \]

\[
\dot{m} = \frac{P\dot{V}}{ZR_{\text{oxygen}}T} = \frac{(20200 \text{ kPa})(0.001 \text{ m}^3/\text{s})}{(0.575)(0.2598 \frac{\text{kJ}}{\text{kgK}})(170 \text{K})} = 0.795 \text{ kg/s}
\]

(c)
The van der Waals equation of state,

\[ P = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2} \]

From Table A-24, constants for the van der Waals equations of state,

\[ a_{oxygen} = 1.369 \text{ bar} \frac{m^6}{\text{kmol}^2} = 136900 \text{ Pa} \frac{m^6}{\text{kmol}^2} \]
\[ b_{oxygen} = 0.0317 \frac{m^3}{\text{kmol}} \]

Plug into the van der Waals equation of state,

\[ 202 \times 10^5 \text{ Pa} = \left( \frac{8314 \frac{J}{\text{kmol} \cdot \text{K}}}{} \right) (170\text{K}) - \frac{136900 \text{ Pa} \frac{m^6}{\text{kmol}^2}}{\bar{v} - \left(0.0317 \frac{m^3}{\text{kmol}}\right)} - \frac{a}{\bar{v}^2} \]

Solve for \( \bar{v} \) by iterating with a proper initial guess which is the answer from part (b),

\[ v = \frac{\dot{v}}{m} \]
\[ v = \frac{0.001 \text{ m}^3/\text{s}}{0.795 \text{ kg/s}} = 0.00126 \text{ m}^3/\text{kg} \]
\[ \bar{v} = v \times MW = \left(0.00126 \frac{m^3}{kg}\right) \left(32 \frac{kg}{\text{kmol}}\right) = 0.04025 \frac{m^3}{\text{kmol}} \]

With this value as an initial guess, iterate and find when RHS = LHS of the equation.

<table>
<thead>
<tr>
<th>( \bar{v} \frac{m^3}{\text{kmol}} )</th>
<th>RHS [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04025</td>
<td>807.8 \times 10^5</td>
</tr>
<tr>
<td>0.055</td>
<td>153.9 \times 10^5</td>
</tr>
<tr>
<td>0.050</td>
<td>224.6 \times 10^5</td>
</tr>
<tr>
<td>0.05123</td>
<td>202 \times 10^5</td>
</tr>
</tbody>
</table>

Thus,

\[ \dot{m} = \frac{\dot{v}}{\bar{v}/MW} = \frac{0.001 \text{ m}^3/\text{s}}{(0.05123 \frac{m^3}{\text{kmol}})/(32 \frac{kg}{\text{kmol}})} = 0.625 \text{ kg/s} \]

Comment: You can use a solver as well to solve the equation.
(d)

The Redlich-Kwong equation of state,
\[ P = \frac{RT}{\bar{v} - b} - \frac{aT^{\frac{1}{2}}}{\bar{v}^2 + \bar{v}b} \]

From Table A-24, constants for the Redlich–Kwong equations of state,
\[ a_{\text{oxygen}} = 17.22 \text{ bar} \frac{m^6}{\text{kmol}^2 K^{1/2}} = 1722000 \text{ Pa} \frac{m^6}{\text{kmol}^2 K^{1/2}} \]
\[ b_{\text{oxygen}} = 0.02197 \frac{m^3}{\text{kmol}} \]

Plug into the Redlich–Kwong equations of state,
\[ 202 * 10^5 \text{ Pa} = \frac{\left( 8314 \frac{\text{J}}{\text{kmol} K} \right) \left( 170K \right)}{\bar{v} - \left( 0.02197 \frac{m^3}{\text{kmol}} \right)} - \frac{\left( 1722000 \text{ Pa} \frac{m^6}{\text{kmol}^2 K^{1/2}} \right)}{\bar{v} \left( \bar{v} + \left( 0.02197 \frac{m^3}{\text{kmol}} \right) \right) \left( 170K \right)^{1/2}} \]

Solve for \( \bar{v} \) by iterating from the answer of part (c),

<table>
<thead>
<tr>
<th>( \bar{v} ) [( \frac{m^3}{\text{kmol}} )]</th>
<th>RHS [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.051</td>
<td>131.98 * 10^5</td>
</tr>
<tr>
<td>0.045</td>
<td>175.47 * 10^5</td>
</tr>
<tr>
<td>0.042</td>
<td>214.06 * 10^5</td>
</tr>
<tr>
<td>0.0428</td>
<td>202.11 * 10^5</td>
</tr>
</tbody>
</table>

Thus,
\[ \dot{m} = \frac{\dot{V}}{\bar{v}/MW} = \frac{0.001 \text{ m}^3/\text{s}}{\left( 0.0428 \frac{m^3}{\text{kmol}} \right) / \left( 32 \frac{\text{kg}}{\text{kmol}} \right)} = 0.748 \text{ kg/s} \]

Comment: You can use a solver as well to solve the equation.

(e)

If the mass flow of oxygen is miscalculated, then this would affect the amount of methane fuel used, potentially reducing performance or damaging the engine. If oxygen is used for cooling, then this could also affect the performance or survival of the rocket engine.
(SP18)

Given the same liquid-methane and gaseous oxygen rocket engine as in SP17, the oxygen flow increases from $T_1 = 170$ K to $T_2 = 172$ K and the specific volume on a molar basis increases from $\tilde{v}_1 = 0.055 \text{ m}^3/\text{kmol}$ to $\tilde{v}_2 = 0.057 \text{ m}^3/\text{kmol}$.

**Given:**

liquid-methane and gaseous oxygen

$P = 202 \text{ bar} = 20200 \text{ kPa}$

$T_1 = 170 \text{ K}$ to $T_2 = 172 \text{ K}$

$\tilde{v}_1 = 0.055 \text{ m}^3/\text{kmol}$ to $\tilde{v}_2 = 0.057 \text{ m}^3/\text{kmol}$

**Find:**

(a) Find the change in pressure (bar) using the Van der Waals equation of state and Eqn. 11.13a, which states that for $z = function (x,y)$, then $dz = (\partial z/\partial x)_y \, dx + (\partial z/\partial y)_x \, dy$. In this case treat pressure as $z$, temperature as $x$, and specific volume on a molar basis as $y$.

(b) Compare this with the change in pressure (bar) that you would compute directly from $p_2 - p_1$ using the Van der Waals equation of state.

(c) Now the oxygen goes through a constant temperature expansion process from $\tilde{v}_2 = 0.057 \text{ m}^3/\text{kmol}$ to $\tilde{v}_3 = 0.059 \text{ m}^3/\text{kmol}$. Again using the Van der Waals equation of state, derive an expression for $(\partial P/\partial T)_\tilde{v} = function (\tilde{v})$. Then using the third Maxwell equation, $(\partial s/\partial \tilde{v})_T = (\partial P/\partial T)_\tilde{v}$, find $s_3 - s_2$ in kJ/\text{kmol-K}$ (Hint: at constant T, $(\partial s/\partial \tilde{v})_T$ becomes $d\tilde{s}/d\tilde{v}$ such that you can integrate the function $(\tilde{v})$ you found earlier.

**System sketch:**

none

**Assumptions:**

none

**Basic equations:**

\[ P = \frac{\tilde{R} T}{\tilde{v} - b} - \frac{a}{\tilde{v}^2} \]

**Solution:**
(a)

Using the equation \( dz = (\partial z/\partial x) \, dx + (\partial z/\partial y) \, dy \) and \( P = \frac{\bar{R} T}{\bar{v} - b} - \frac{a}{\bar{v}^2} \), establish an equation.

\[
dp = \left( \frac{\partial p}{\partial T} \right)_\bar{v} \, dT + \left( \frac{\partial p}{\partial \bar{v}} \right)_T \, d\bar{v}
\]

\[
\left( \frac{\partial p}{\partial T} \right)_\bar{v} = \frac{\bar{R}}{\bar{v} - b}
\]

\[
\left( \frac{\partial p}{\partial \bar{v}} \right)_T = -\frac{\bar{R} T}{(\bar{v} - b)^2} + \frac{2 a}{\bar{v}^3}
\]

\[
dp = \left( \frac{\bar{R}}{\bar{v} - b} \right) \, dT + \left( -\frac{\bar{R} T}{(\bar{v} - b)^2} + \frac{2 a}{\bar{v}^3} \right) \, d\bar{v}
\]

Find each term,

\( dT = T_2 - T_1 = 2 \, \text{K} \)

\( d\bar{v} = \bar{v}_2 - \bar{v}_1 = 0.057 - 0.055 = 0.002 \, \text{m}^3/\text{kmol} \)

\( T \approx \frac{T_1 + T_2}{2} = 171 \, \text{K} \)

\( \bar{v} \approx \frac{\bar{v}_1 + \bar{v}_2}{2} = 0.056 \, \text{m}^3/\text{kmol} \)

\( a_{\text{oxygen}} = 1.369 \, \text{bar} \frac{m^6}{\text{kmol}^2} = 136900 \, \text{Pa} \frac{m^6}{\text{kmol}^2} \)

\( b_{\text{oxygen}} = 0.0317 \, \text{m}^3/\text{kmol} \)

Then,

\[
dp = \left( \frac{\bar{R}}{\bar{v} - b} \right) \, dT + \left( -\frac{\bar{R} T}{(\bar{v} - b)^2} + \frac{2 a}{\bar{v}^3} \right) \, d\bar{v}
\]
\[
\begin{align*}
\frac{8314 J}{kmol K} & \frac{m^3}{(2K)} \\
\frac{(0.056 - 0.0317) m^3}{kmol} & + \left( - \frac{8314 J}{kmol K} (171K) \right) \frac{m^6}{(0.056 - 0.0317)^2} \frac{m^6}{kmol^2} + 2 \left( \frac{136900 Pa m^6}{kmol^2} \right) \left( \frac{0.056}{kmol} \right)^3 \frac{m^3}{kmol} \\
= & -1012860 Pa = -10.13 \text{ bar}
\end{align*}
\]

(b)

\[
\begin{align*}
p_2 & = \frac{8314 J}{kmol K} (172K) \frac{m^3}{(0.057 - 0.0317) m^3} \frac{m^3}{kmol} - \frac{136900 Pa}{kmol^2} \frac{m^6}{(0.057 m^3)^2} = 143.86 \text{ bar} \\

p_1 & = \frac{8314 J}{kmol K} (170K) \frac{m^3}{(0.055 - 0.0317) m^3} \frac{m^3}{kmol} - \frac{136900 Pa}{kmol^2} \frac{m^6}{(0.055 m^3)^2} = 154.04 \text{ bar} \\
p_2 - p_1 & = 143.86 - 154.04 \text{ bar} \\
= & -10.18 \text{ bar}
\end{align*}
\]

Comment: Change in pressure is within 0.5%

(c)

From 3rd Maxwell equation,

\[
\left( \frac{\partial \bar{s}}{\partial \bar{v}} \right)_T = \left( \frac{\partial p}{\partial T} \right)_v = \frac{\bar{R}}{\bar{v} - b}
\]

Hence, at constant T, P is a function of \( \bar{v} \) only and

\[
\frac{\partial \bar{s}}{\partial \bar{v}} = \left( \frac{\partial p}{\partial T} \right)_v
\]

Then,
\[ d\bar{s} = \left( \frac{\partial p}{\partial T} \right)_v \, d\bar{v} \]

Integrate,

\[ \bar{s}_3 - \bar{s}_2 = \int_{v_2}^{v_3} \left( \frac{\partial p}{\partial T} \right)_v \, d\bar{v} = \int_{v_2}^{v_3} \frac{\bar{R}}{\bar{v} - b} \, d\bar{v} \]

\[ = \frac{\bar{R} \ln(\bar{v} - b)}{v_2} = \bar{R} \ln \left( \frac{v_3 - b}{v_2 - b} \right) = 8314 \frac{J}{kmol \, K} \ln \left( \frac{0.059 - 0.0317}{0.057 - 0.0317} \right) \]

\[ = 632.55 \, \frac{kJ}{kmol \, K} = 0.63255 \, \frac{kJ}{kmol \, K} \]
A piston cylinder device contains saturated liquid water at 50 °C that is heated reversibly at constant pressure to a saturated vapor. Use the Clapeyron Equation and data from the steam tables.

**Given:**
In a piston cylinder, constant pressure reversible process from state 1 to state 2
- State 1: a saturated liquid water at 50 °C
- State 2: a saturated vapor

**Find:**
(a) Using the Clapeyron equation and T, P, and v data from the steam tables, find the change in enthalpy (hint: do not use the h data from the steam tables directly).

(b) From the definition of enthalpy \( h = u + pv \), the p and v data from the steam tables, and h from part (a), find the change in internal energy (again not using the u data from the steam tables directly).

(c) From the definition of entropy \( ds = (\delta Q/T)_{rev} \) and the results of part (a), find the change in entropy (again not using the s data from the steam tables directly).

(d) Compare these results using the h, u, and s data obtained directly from the steam tables.

**System sketch:**
None

**Assumptions:**
None

**Basic equation:**
\[
h_g - h_f = T(v_g - v_f) \left( \frac{dp}{dT} \right)_{sat} \]
Solution:

(a)

\[ h_g - h_f = T(v_g - v_f) \left( \frac{dp}{dT} \right)_{sat} \]

To obtain \( \left( \frac{dp}{dT} \right)_{sat} \) at 50 °C, prepare the plot shown below using data from Table A-2. (any method is okay to find \( \left( \frac{dp}{dT} \right)_{sat} \) at 50 °C)

![Plot of P vs T](image)

Also, from Table A-2

\[ v_g = 12.032 \frac{m^3}{kg} \quad \text{and} \quad v_f = 1.0121 \times 10^{-3} \frac{m^3}{kg} \]

Substitute into the Clapeyron equation,

\[ h_g - h_f = (50 + 273K) \left( 12.032 \frac{m^3}{kg} - 1.0121 \times 10^{-3} \frac{m^3}{kg} \right) (0.604 \text{ kPa/K}) \]

\[ = 2347 \frac{kJ}{kg} \]

(b)
\[ u_g - u_f = h_g - h_f - p(v_g - v_f) = 2347 \frac{kJ}{kg} - 12.35 \text{kPa} \left( 12.032 \frac{m^3}{kg} - 1.0121 \times 10^{-3} \frac{m^3}{kg} \right) = 2198 \frac{kJ}{kg} \]

(c)

\[ s_g - s_f = \frac{h_g - h_f}{T} = \frac{2347 \frac{kJ}{kg}}{50 + 273K} = 7.266 \frac{kJ}{kgK} \]

(d)

The values from Table A-2 are \( h_g - h_f = 2382.7 \), \( u_g - u_f = 2234.2 \), and \( s_g - s_f = 7.3725 \). The percent difference between calculated and table values is about 1.5% for all.