A piston-cylinder device contains a mixture of 0.2 kg of H₂ and 1.6 kg of N₂ at 100 kPa and 300 K. Heat is now transferred to the mixture at constant pressure until the volume is doubled. Assuming constant specific heats evaluated at the average of the initial and final mixture temperatures, determine (a) the heat transfer and (b) the entropy change of the mixture.

**Given:**
A piston-cylinder,

State 1: a mixture of 0.2 kg H₂ and 1.6 kg N₂

\[ P_1 = 100 \text{ kPa}, \quad T_1 = 300 \text{ K} \]

State 2: the same mixture

\[ V_2 = 2V_1, \quad P_2 = P_1 \]

Heat transfer occurs at constant pressure.

Constant specific heat at the average temperature.

**Find:**

(a) the heat transfer

(b) the entropy change of the mixture

**System sketch:**

![System sketch diagram]
**Assumptions:**

1. Quasi-equilibrium process, uniform state over the tank (Not steady state!)
2. Neglect $\Delta KE$ and $\Delta PE$
3. No shaft work
4. Both gases and a mixture are treated as an ideal gas
5. Heat transfer at constant pressure
6. Constant specific heats at the average temperature of state1 and state2

**Basic equations:**

\[
m = \text{constant} \\
\Delta E = \Delta U + \Delta KE + \Delta PE = Q - W
\]

Boundary Work: \( W_b = \int PdV \)

Ideal Gas Constant Specific Heat
\[
\Delta u = C_v(T_2 - T_1) \; ; \; \Delta h = C_p(T_2 - T_1) \\
\Delta s = C_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right) = C_p \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right)
\]

**Solution:**

(a)

Because \( V_2 = 2V_1 \), \( P_2 = P_1 \) with an ideal gas assumption,
\[
m_2 = \frac{P_2 V_2}{RT_2} = \frac{P_1 V_1}{RT_1} \rightarrow \frac{P_1 2V_1}{RT_2} = \frac{P_1 V_1}{RT_1} \rightarrow \frac{2}{T_2} = \frac{1}{T_1}
\]

Substitute known values,
\[
T_2 = 2T_1 = 600 \, K
\]

The control volume is a closed system. The energy balance equation for this system is
\[
\Delta E = \Delta U + \Delta KE + \Delta PE = Q - W
\]

Rewrite,
\[ E_2 - E_1 = U_2 - U_1 = Q - W_{\text{boundary}} \]

Rearrange,

\[ Q = U_2 - U_1 + W_{\text{boundary}} = U_2 - U_1 + P_1(V_2 - V_1) = H_2 - H_1 \]

\[ Q = H_2 - H_1 = \Delta H|_{H_2} + \Delta H|_{N_2} \]
\[ = [mC_p(T_2 - T_1)]_{H_2} + [mC_p(T_2 - T_1)]_{N_2} \]
\[ = (0.2\,kg)\left(14.501\frac{kJ}{kgK}\right)(600 - 300\,K) + (1.6\,kg)\left(1.049\frac{kJ}{kgK}\right)(600 - 300\,K) \]
\[ = 1374\,kJ \]

The specific heat for each gas at 450K (average of 300K and 600K) can be found from Table A-20. The constant pressure specific heats for H\(_2\) and N\(_2\) are 14.501 \(\frac{kJ}{kgK}\) and 1.049 \(\frac{kJ}{kgK}\).

**Therefore, the amount of heat transfer is** 1374 kJ, **and its direction is into the control volume.**

(b)

Entropy changes for each gas with an ideal gas assumption and constant pressure process can be written as,

\[ \Delta S|_{H_2} = [m(s_2 - s_1)]_{H_2} = m_{H_2} \left(C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right)_{H_2} \]
\[ = (0.2\,kg)(14.501\frac{kJ}{kgK})(\ln \frac{600K}{300K}) \]
\[ = 2.01\,kJ/K \]

\[ \Delta S|_{N_2} = [m(s_2 - s_1)]_{N_2} = m_{N_2} \left(C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right)_{N_2} \]
\[ = (1.6\,kg)(1.049\frac{kJ}{kgK})(\ln \frac{600K}{300K}) \]
\[ = 1.16\,kJ/K \]
\[ \Delta S|_{\text{mixture}} = \Delta S|_{H_2} + \Delta S|_{N_2} = 3.17 \text{ kJ/}K \]

Therefore, the entropy change of the mixture during this process is 3.17 kJ/K.
(1) Is it possible to obtain saturated air from unsaturated air without adding any moisture?

(Yes/No) It is possible by decreasing the temperature of unsaturated air, so that pressure of the water vapor, $P_v$, becomes the saturated vapor pressure, $P_g$, at the final temperature. For example, unsaturated air with relative humidity 32% at 40 °C can have saturated air (i.e. relative humidity 100%) at 20 °C just by decreasing the temperature.

(2) Is the relative humidity of saturated air necessarily 100 percent?

(Yes/No) Saturated air means 100 percent relative humidity.

(3) Can the water vapor in air be treated as an ideal gas?

(Yes/No) The water vapor in air is treated as an ideal gas. In moist air, the overall mixture and each mixture component behave as ideal gases.
(4) In ME Room 2061, the dry-bulb temperature is 25°C, the wet-bulb temperature 20°C, and the air pressure is 1 atm. Determine (a) humidity ratio, (b) relative humidity, (c) dew point temperature, and (d) specific enthalpy of the moist air.

The humidity ratio, relative humidity, dew point temperature can be determined from psychrometric chart, Figure A-9.

(a) Humidity ratio, \( \omega = 0.0126 \frac{kg\ water}{kg\ air} \)

(b) Relative humidity, \( \phi = 63\% \)

(c) Dew point temperature, \( T_{dew} = 17.5\ \degree C \)

(d) Specific enthalpy of the moist air, \( h = 57.5 \frac{kJ}{kg\ air} \)
(SP 14)

A moist air stream enters a heated channel with $\dot{m}_{a1} = 0.4\ kg/s$, $T_1 = 15^\circ C$, and $\phi_1 = 80\%$. The moist air stream exits with $T_2 = 47^\circ C$ and $\phi_2 = 30\%$. The moist air stream flows over a pan of liquid water at a temperature $T_3 = 60^\circ C$ as it moves through the channel. The level of liquid water is kept at a constant level by either supplying or draining water from the pan at a rate $\dot{m}_{w3}$.

(a) Find the mass flow rate of the liquid water, $\dot{m}_{w3}$, in kg/s. Is the flow at section 3 into or out of the channel?

(b) Calculate the rate of heat transfer between the walls of the channel and the moist air stream. Is the heat transfer from the walls to the moist air, or from the moist air to the walls?

**Given:**

In a heated channel,

State 1: a moist air enters with $\dot{m}_{a1} = 0.4\ kg/s$, $T_1 = 15^\circ C$, and $\phi_1 = 80\%$

State 2: a moist air exits with $T_2 = 47^\circ C$ and $\phi_2 = 30\%$

State 3: a moist air flows over a pan of liquid water at $T_3 = 60^\circ C$

The level of liquid water in a pan is constant level.

All the process under 1atm.

**Find:**

(a) The mass flow rate of the liquid water, $\dot{m}_{w3}$, in kg/s & its direction

(b) the rate of heat transfer between the walls of the channel and the moist air stream & its direction.

**System sketch:**
Assumptions:
1. air and water vapor are treated as Ideal gases
2. Steady state
3. Uniform flow at inlets, exits
4. \( \dot{W}_{CV} = 0 \)
5. Neglect KE, PE
6. No heat transfer to or from surroundings

Basic equations:

Conservation of mass:
\[ \dot{m}_a = \dot{m}_{a1} = \dot{m}_{a2} \]
\[ \dot{m}_{v1} + \dot{m}_{v3} = \dot{m}_{v2} \text{ (assume water flows into CV at section 3)} \]
\[ \dot{m}_v = \omega \dot{m}_a \]

Conservation of energy:
\[ \frac{dE_{CV}}{dt} = 0 = \dot{Q}_{CV} - \dot{W}_{CV} + \dot{m}_a h_1 - \dot{m}_a h_2 + \dot{m}_{w3} h_{f3} \]
\[ h = h_a + \omega h_s \]

Solution:
(a)
Using the psychrometric chart,
\[ \omega_1 = 0.0086 \quad \omega_2 = 0.0203 \]
\[ \dot{m}_{w3} = \dot{m}_a (\omega_2 - \omega_1) = \left( 0.4 \frac{kg}{s} \right) (0.0203 - 0.0086) \]
\[ \dot{m}_{w3} = +0.00468 \frac{kg}{s} \text{ into channel} \]
Because the sign of the mass flow is positive, our initial assumption regarding the direction of the mass flow is correct.

(b)

Using the psychrometric chart,

\[ h_1 = h_{a1} + \omega_1 h_{g1} = 36.5 \frac{kJ}{kg_a} \]

\[ h_2 = h_{a2} + \omega_2 h_{g2} = 99.5 \frac{kJ}{kg_a} \]

\[ h_{f3} = 251.13 \frac{kJ}{kg} \text{ from Table A-2} \]

\[ \dot{Q}_{CV} = \dot{m}_a (h_2 - h_1) - \dot{m}_{w3} h_{f3} = \left(0.4 \frac{kg_a}{s}\right) \left(99.5 - 36.5 \frac{kJ}{kg_a}\right) - \left(0.00468 \frac{kg}{s}\right) \left(251.13 \frac{kJ}{kg}\right) \]

\[ \dot{Q}_{CV} = 25.2 - 1.18 = +24.0 \frac{kJ}{s} \text{ from the channel walls to the moist air stream} \]