PROBLEM 7.72

Water vapor at 6 MPa, 600°C enters a turbine operating at steady state and expands adiabatically to 10 kPa. The mass flow rate is 2 kg/s and the isentropic turbine efficiency is 94.7%. Kinetic and potential energy effects are negligible. Determine

(a) the power developed by the turbine, in kW.
(b) the rate at which exergy is destroyed within the turbine, in kW.
(c) the exergetic turbine efficiency.

Let \( T_i = 298 \text{ K}, p_i = 1 \text{ atm}. \)

**Analysis:**

(a) Reducing mass and energy rate balances, we get

\[
W_t = m (h_i - h_f) . \quad \text{Also,} \quad \eta_e = \frac{h_i - h_f}{h_i - h_{i,s}}
\]

Collecting results,

\[
W_t = m \eta_e (h_i - h_f)
\]

Table A-9 gives \( h_i = 3638.4 \text{ kJ/kg}, S_i = 7.1677 \text{ kJ/kg K}. \) Using data from Table A-3 with \( T_i = 373.15 \text{ K}, x_{25} = 7.0473 - 0.649 = 0.864. \) Then \( h_{25} = 191.83 + 0.864(357.28) = 218.12 \text{ kJ/kg}. \)

Inserting values, Eq. (1) gives

\[
W_t = (2 \times 2.257)(3645.84 - 218.12) \text{ kJ} = 262.74 \text{ kW}
\]

(b) \( \dot{E}_d = T_0 \dot{S} . \) With \( \dot{S} = m (S_f - S_i) \) from an entropy rate balance,

\[
\dot{E}_d = T_0 \dot{S} = T_0 m (S_f - S_i)
\]

To find \( S_f, \) fix the exit state using the expression for power from Part (a):

\[
\frac{W_t}{m} = h_i - h_f \Rightarrow h_f = h_i - \frac{W_t}{m} = \left[3658.4 \text{ kJ/kg} - \frac{262.74 \text{ kJ}}{2.257} \right] = 2344.7 \text{ kJ/kg}
\]

Then,

\[
x_f = \frac{7.0473 - 0.9723}{2344.7} = 0.7 \Rightarrow S_f = 0.6473 + 0.7 (8.1502 - 0.6493) = 7.4001 \text{ kJ/kgK}
\]

Inserting values, Eq. (2) gives

\[
\dot{E}_d = (298 \times 2.257) (7.4001 - 7.1677) \text{ kJ} = 138.5 \text{ kW}
\]

(c) An energy rate balance gives,

\[
\dot{E}_i = \frac{m}{\dot{W}_t} \left[ (1 - \eta_e) \dot{O} - \dot{W}_t + \dot{W}_t (\dot{c}_f - \dot{c}_i) - \dot{E}_d \right]
\]

Thus,

\[
\dot{W}_t = \frac{m}{\dot{W}_t} \left( \dot{W}_t + \dot{E}_d \right) = \left( \frac{262.74}{262.74 + 138.5} \right) = 0.95 (95\%)
\]
Saturated water vapor at 1 bar enters a direct-contact heat exchanger operating at steady state and mixes with a stream of liquid water entering at 25°C, 1 bar. A two-phase liquid-vapor mixture exits at 1 bar. The entering streams have equal mass flow rates. Neglecting heat transfer with the surroundings and effects of motion and gravity, determine for the heat exchanger
(a) the rate of exergy destruction, in kJ per kg of mixture exiting.
(b) the exergetic efficiency given by Eq. 7.29.
Let \( T_0 = 20°C \) and \( p_0 = 1 \) bar.

**KNOWN:** Steady-state operating data are provided for a direct-contact heat exchanger.

**FIND:** Determine (a) the rate of exergy destruction per unit mass exiting and (b) the exergetic efficiency given by Eq. 7.29.

**SCHEMATIC AND GIVEN DATA:**

\[
\begin{align*}
T_0 &= 20°C \\
p_0 &= 1 \text{ bar} \\
p_1 &= 1 \text{ bar} \\
\text{Saturated vapor} \\
m_1 &= m_2 \\
p_2 &= 1 \text{ bar} \\
T_2 &= 25°C \\
p_3 &= 1 \text{ bar}
\end{align*}
\]
<table>
<thead>
<tr>
<th>State</th>
<th>$T$ (°C)</th>
<th>$p$ (bar)</th>
<th>$h$ (kJ/kg)</th>
<th>$s$ (kJ/kg·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>1</td>
<td>2675.5</td>
<td>7.3594</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>1</td>
<td>104.89</td>
<td>0.3674</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>1</td>
<td>See solution</td>
<td>See solution</td>
</tr>
</tbody>
</table>

ENGINEERING MODEL:
1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the effects of motion and gravity can be ignored.
3. The exergy reference environment is $T_0 = 20^\circ\text{C}$, $p_0 = 1$ bar.

ANALYSIS:
(a) Reducing the energy rate balance based on assumptions:

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

where mass balance reduces to

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Then with $\dot{m} = \dot{m}_1 = \dot{m}_2$

$$\dot{m} = \frac{\dot{m}_3}{2}$$

Substituting the above into Eq. (1) and rearranging yield

$$0 = m(h_1 + h_2) - \dot{m}_3 h_3 = \dot{m}_3 \left[ \frac{(h_1 + h_2)}{2} - h_3 \right]$$

Rearrange and substitute data values listed in the accompanying table:

$$h_3 = \frac{h_1 + h_2}{2} = \left( \frac{2675.5 \text{ kJ/kg}}{2} - 104.89 \text{ kJ/kg} \right) = 1390.195 \text{ kJ/kg}$$

Since $h_T < h_3 < h_g$ at $p_3$, state 3 is a two-phase liquid-vapor mixture and $x_3$ follows:
\[ x_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{\left(1390.195 \frac{\text{kJ}}{\text{kg}} - 417.46 \frac{\text{kJ}}{\text{kg}}\right)}{2258.0 \frac{\text{kJ}}{\text{kg}}} = 0.431 = 43.1\% \]

The corresponding entropy, \( s_3 \), is

\[ s_3 = s_f + x_3(s_g - s_f) = 1.3026 \text{ kJ/kg} \cdot \text{K} + (0.431)(7.3594 - 1.3026) \text{ kJ/kg} \cdot \text{K} = 3.9131 \text{ kJ/kg} \cdot \text{K} \]

The rate of exergy destruction can be obtained by using an exergy rate balance or using \( \dot{E}_d = T_0 \dot{\sigma}_{cv} \) where \( \dot{\sigma}_{cv} \) is the rate of entropy production obtained from an entropy balance. Using an entropy balance, simplified based on assumptions

\[ \dot{\sigma}_{cv} = \dot{m}_3 s_3 - \dot{m}_3 (s_1 + s_2) = \dot{m}_3 (s_3 - \frac{\dot{m}_3}{2}(s_1 + s_2)) = \dot{m}_3 \left(s_3 - \frac{(s_1 + s_2)}{2}\right) \]

\[ \frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_3 - \frac{(s_1 + s_2)}{2} = 3.9131 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} - \frac{\left(7.3594 + 0.3674\right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}}{2} = 0.0497 \text{ kJ/kg} \cdot \text{K} \]

The exergy destruction rate per unit mass exiting can be determine as follows:

\[ \frac{\dot{E}_d}{\dot{m}_3} = \frac{\dot{\sigma}_{cv}}{\dot{m}_3} T_0 = \left(0.0497 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(293 \text{ K}) = 14.562 \text{ kJ/kg} \]

(b) The exergetic efficiency given by Eq. 7.29

\[ \varepsilon = \frac{\dot{m}(e_{f3} - e_{f2})}{\dot{m}(e_{f1} - e_{f3})} = \frac{h_3 - h_2 - T_0(s_3 - s_2)}{h_1 - h_3 - T_0(s_1 - s_3)} \]

\[ \varepsilon = \frac{(1390.195 - 104.89) \frac{\text{kJ}}{\text{kg}} - (293 \text{ K})(3.9131 - 0.3674) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}}{(2675.5 - 1390.195) \frac{\text{kJ}}{\text{kg}} - (293 \text{ K})(7.3594 - 3.9131) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}} = 0.894 = 89.4\% \]
Problem 12.16

KNOWN: A mixture with a specified molar analysis enters a compressor operating at steady state at a specified mass flow rate and state and exits at a specified temperature and pressure.

FIND: Determine the power required, the isentropic compressor efficiency, and the rate of exergy destruction.

SCHEMATIC & GIVEN DATA

\[ P_1 = 10 \text{ bar} \]
\[ T_1 = 30^\circ \text{C} \]
\[ N_1 = 1 \text{ kg/s} \]
\[ T_0 = 300 \text{ K} \]

\[ P_2 = 25 \text{ bar} \]
\[ T_2 = 147^\circ \text{C} \]

<table>
<thead>
<tr>
<th>( Y_i )</th>
<th>( M_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>28.01</td>
</tr>
<tr>
<td>0.4</td>
<td>44.01</td>
</tr>
</tbody>
</table>

ENGINEERING MODEL: (1) The compressor is well insulated and operates at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant. (4) For the environment, \( T_0 = 300 \text{ K} \).

ANALYSIS: Reduction of the mass and energy rate balances at steady state gives

\[ \dot{W}_C = m \left[ h_1 - h_2 \right] = m \left[ \frac{Y_1}{M_i} \left( Y_1 \alpha_1 T_1 h_1 + Y_2 \alpha_2 T_1 h_2 \right) \right] \]

where \( M \) is the mixture molecular weight: \( M = Y_1 M_{C_1} + Y_2 M_{C_2} = 0.6 \times 28.01 + 0.4 \times 44.01 = 34.41 \text{ kg/mol} \). Then, with data from Table A-23

\[ \dot{W}_C = (1 \text{ kg/s}) \left[ 0.6 \left( 981.03 - 1222.5 \right) + 0.4 \left( 9543.8 - 1420.6 \right) \right] \]

\[ = -113.7 \text{ kW} \]

To determine the isentropic compressor efficiency requires the power for an isentropic compression from state 1 to the pressure \( P_2 \) — that is, from 1 to state 2s. For this process, \( s_{2s} - s_1 = 0 \). Or

\[ Y_1 \left( s_{2s} - s_1 \right) h_1 + Y_2 \left( s_{2s} - s_1 \right) h_2 = 0 \Rightarrow Y_1 \left( s_{2s} - s_1 \right) h_1 = Y_2 \left( s_{2s} - s_1 \right) h_2 \]

Then, with data from Table A-23

\[ Y_1 \left( s_{2s} - s_1 \right) h_1 = (0.6)(981.94) + (0.4)(214.25) = 210.03 \text{ kJ/mole-K} \]

(1) Solving this equation by iteration with table data: \( T_{2s} = 39.9 \text{ K} \). Accordingly

\[ \left( \dot{W}_C \right)_2 = m \left[ h_1 - h_2 \right] = m \left[ \frac{Y_1}{M_i} \left( Y_1 \alpha_1 T_1 h_1 + Y_2 \alpha_2 T_1 h_2 \right) \right] \]

\[ = \frac{1}{34.41} \left[ 0.6 \left( 981.03 - 1110.7 \right) + 0.4 \left( 9543.8 - 13330.7 \right) \right] = -92.85 \text{ kW} \]

The isentropic compressor efficiency is then

\[ \eta_C = \frac{\left( \dot{W}_C \right)_2}{\dot{W}_C} = \frac{-92.85}{-113.7} = 0.817 \times 100\% = 81.7\% \]

The rate of exergy destruction can be found using \( \dot{E}_D = k_0 T_0 \dot{S}_{CV} \), where \( \dot{S}_{CV} \) is the rate of entropy production obtained from an entropy rate balance, which reduces at steady state as follows:

\[ 0 = \sum_{i=1}^{n} \dot{m}_i \left( s_i - s_0 \right) + \dot{S}_{CV} \Rightarrow \dot{S}_{CV} = \dot{m}_i \left( s_2 - s_1 \right) \]

\[ \dot{S}_{CV} = m \left[ \frac{Y_1}{M_i} \left( s_2 - s_1 \right) h_1 + Y_2 \left( s_2 - s_1 \right) h_2 \right] \]
The terms \((\bar{v}_z - \bar{v}_i) \mu_L\) and \((\bar{v}_z - \bar{v}_i) \mu_C\) are evaluated using Eq. 12.36, so

\[
\dot{c}_{cv} = m \left[ y_{\text{H}_{2}} \left( \bar{v}_{\text{H}_{2}}(T_{\text{b}}) - \bar{v}_{\text{H}_{2}}(T_{i}) - \frac{R \ln \frac{P_{\text{b}}}{P_{i}}}{P_{i}} \right) + y_{\text{H}_{2}O} \left( \bar{v}_{\text{H}_{2}O}(T_{\text{b}}) - \bar{v}_{\text{H}_{2}O}(T_{i}) - \frac{R \ln \frac{P_{\text{b}}}{P_{i}}}{P_{i}} \right) \right] \frac{1}{M}
\]

\[
= m \left[ y_{\text{H}_{2}} \left( \bar{v}_{\text{H}_{2}}(T_{\text{b}}) - \bar{v}_{\text{H}_{2}}(T_{i}) \right) - y_{\text{H}_{2}O} \left( \bar{v}_{\text{H}_{2}O}(T_{\text{b}}) - \bar{v}_{\text{H}_{2}O}(T_{i}) \right) - \frac{R \ln \frac{P_{\text{b}}}{P_{i}}}{P_{i}} \right] \frac{1}{M}
\]

Then, with \(\bar{v}_{\text{in}}\) data from Table A.2.3

\[
\dot{c}_{cv} = (1 \text{ kg}) \left[ \begin{array}{c}
0.6(201.459 - 191.969) + 0.4(227.258 - 214.284) - 8.34 \text{ kcal} \\
34.1
\end{array} \right] \frac{\text{kcal}}{\text{mol K}} \frac{1}{1 \text{ kJ/kg}}
\]

\[
= 0.0516 \text{ kW/K}
\]

Multiplying this by \(T_0\)

\(\dot{E_d} = T_0 \dot{c}_{cv} = (300 \text{ K})(0.0516 \text{ kW/K}) = 15.48 \text{ kW} \)

\(\dot{E_d}\)

1. An iterative solution using table data can be avoided by using IT. The results for \(\dot{W}_C, T_{2s}, \dot{h}_C\), and \(\dot{E_d}\) obtained using IT agree with the values given here.
2. When expressed as a percentage of the power input to the compressor

\[
\% = \left[ \frac{\dot{E_d}}{C \dot{W}_{cv}} \right] \times 100 = \left[ \frac{15.48}{113.7} \right] \times 100 = 13.6 \%
\]
Problem 12.24, revised 9/2017

Argon (Ar), at 300 K, 1 bar with a mass flow rate of 1 kg/s enters the insulated mixing chamber shown in Fig. P12.36 and mixes with carbon dioxide (CO$_2$) entering as a separate stream at 575 K, 1 bar with a mass flow rate of 0.5 kg/s. The mixture exits at 1 bar. Assume ideal gas behavior with $k = 1.67$ for Ar and $k = 1.25$ for CO$_2$. For steady-state operation, determine

(a) the molar analysis of the exiting mixture.
(b) the temperature of the exiting mixture, in K.
(c) the rate of entropy production, in kW/K.

Solution:

Known:
Argon and carbon dioxide enter an insulated mixing chamber operating at steady state. Data are known at the inlets and exit.

Find:
Determine (a) the molar analysis of the exiting mixture, (b) the exit temperature, (c) the rate of entropy production.

Schematic and Known Data:

Engineering Model:
(1) The control volume is at steady state, with $\dot{W}_{CV} = \dot{Q}_{CV} = 0$.
(2) Kinetic and potential energy effects are negligible.
(3) The ideal gas model applies, and the specific heats are constant.
Analysis:

(a) Based on the entering mass flow rates, the gravimetric analysis of the exiting mixture is

\[ m_{f,\text{Ar}} = \frac{\dot{m}_1}{\dot{m}_3} = \frac{1}{1.5} = 0.6667, \quad m_{f,\text{CO}_2} = \frac{\dot{m}_2}{\dot{m}_3} = \frac{0.5}{1.5} = 0.3333 \]

Following the method of Example 12.2 for 1 kg/s of mixture

<table>
<thead>
<tr>
<th>Component</th>
<th>( \frac{m_i}{M_i} )</th>
<th>( n_i )</th>
<th>( y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>0.6667 ÷ 39.94</td>
<td>0.01669</td>
<td>0.6880</td>
</tr>
<tr>
<td>CO\textsubscript{2}</td>
<td>0.3333 ÷ 44.01</td>
<td>0.00757</td>
<td>0.3120</td>
</tr>
</tbody>
</table>

Molar analysis

(b) The mass and energy rate balances to

\[ 0 = \dot{m}_1 h_{\text{Ar},1} + \dot{m}_2 h_{\text{CO}_2,2} - \dot{m}_3 h_{\text{max}} \]

With \( \dot{m}_3 = \dot{m}_1 + \dot{m}_2 \) and \( h_{\text{max}} = m_{f,\text{Ar}} h_{\text{Ar},3} + m_{f,\text{CO}_2} h_{\text{CO}_2,3} \) we get

\[ 0 = \dot{m}_1 h_{\text{Ar},1} + \dot{m}_2 h_{\text{CO}_2,2} - \frac{m_{f,\text{Ar}} h_{\text{Ar},3}}{\dot{m}_1} - \frac{m_{f,\text{CO}_2} h_{\text{CO}_2,3}}{\dot{m}_2} \]

\[ 0 = \dot{m}_1 \left( h_{\text{Ar},1} - h_{\text{Ar},3} \right) + \dot{m}_2 \left( h_{\text{CO}_2,2} - h_{\text{CO}_2,3} \right) \]

Solving \( T_3 \) and inserting values

\( T_3 = \frac{m_1 c_{p,\text{Ar}} T_1 + m_2 c_{p,\text{CO}_2} T_1}{m_1 c_{p,\text{Ar}} + m_2 c_{p,\text{CO}_2}} = \frac{(1)(0.5189)(300) + (0.5)(0.9446)(575)}{(1)(0.5189) + (0.5)(0.9446)} = 431 \text{ K} \)

(c) Reducing the entropy balance

\[ 0 = \sum_{i=0}^{\infty} \left( \frac{Q}{T} \right) + \dot{m}_1 s_{\text{Ar},1} + \dot{m}_2 s_{\text{CO}_2,2} - m_{f,\text{Ar}} \dot{m}_3 s_{\text{Ar},3} - m_{f,\text{CO}_2} \dot{m}_3 s_{\text{CO}_2,3} + \dot{\sigma}_{CV} \]

\[ = \dot{m}_1 \left( s_{\text{Ar},1} - s_{\text{Ar},3} \right) + \dot{m}_2 \left( s_{\text{CO}_2,2} - s_{\text{CO}_2,3} \right) + \dot{\sigma}_{CV} \]
Solving for \( \dot{\sigma}_{CV} \), and introducing \( p_{Ar,3} = y_{Ar}p_3 \) and \( p_{CO_2,3} = y_{CO_2}p_3 \)

\[
\dot{\sigma}_{CV} = \dot{m}_1 c_{pAr} \frac{T_1}{T_3} - R_{Ar} \ln y_{Ar} + \dot{m}_2 c_{pCO_2} \frac{T_2}{T_3} - R_{CO_2} \ln y_{CO_2}
\]

\[
= (1 \text{ kg/s}) \left[ \left( 0.5189 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \ln \left( \frac{431}{300} \right) - \frac{8.314}{39.94} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln(0.6888) \right] + (0.5 \text{ kg/s}) \left[ \left( 0.9446 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) \ln \left( \frac{431}{575} \right) - \frac{8.314}{44.01} \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \ln(0.3120) \right]
\]

\[
= 0.2397 \frac{\text{kJ} \cdot \text{s}}{\text{K}} \cdot \frac{1 \text{ kW}}{1 \frac{\text{kJ}}{\text{s}}} = 0.2397 \frac{\text{kW}}{\text{K}}
\]