A turbine operating at steady state with steam entering at \( p_1 = 30 \) bar, \( T_1 = 350^\circ C \) and a mass flow rate of 10 kg/s. A half of the stream is extracted at \( p_2 = 5 \) bar, \( T_2 = 200^\circ C \). The remaining steam exits at \( p_3 = 0.15 \) bar, \( x_3 = 0.90 \). Neglect the heat loss from the turbine to the environment at \( T_0 = 25^\circ C \) and \( p_o = 1 \) bar. Determine:

(a) The total work produced by the turbine, in kW.
(b) The exergy destruction, in kW.
(c) The second-law effectiveness (or exergetic efficiency) for the turbine.

<table>
<thead>
<tr>
<th>State</th>
<th>( p ) (bar)</th>
<th>( T ) (°C)</th>
<th>( h ) (kJ/kg)</th>
<th>( s ) (kJ/kg·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>350</td>
<td>3115.3</td>
<td>6.7428</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>200</td>
<td>2855.4</td>
<td>7.0592</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>53</td>
<td>2361.7</td>
<td>7.2831</td>
</tr>
</tbody>
</table>

**Given:**

A steam turbine operates at steady state

State 1: \( p_1 = 30 \) bar, \( T_1 = 350^\circ C \)

State 2: a half of the stream is extracted at \( p_2 = 5 \) bar, \( T_2 = 200^\circ C \)

State 3: a half of the stream is extracted at \( p_3 = 0.15 \) bar, \( x_3 = 0.90 \)

A mass flow rate of 10 kg/s.

Neglect the heat loss from the turbine.

The environment is at \( T_0 = 25^\circ C \) and \( p_o = 1 \) bar.

**Find:**

(a) the total work produced by the turbine, in kW.
(b) the exergy destruction, in kW.
(c) the second-law effectiveness (or exergetic efficiency) for the turbine.
System sketch:

\[ p_1 = 30 \text{ bar} \]
\[ T_1 = 350^\circ \text{C} \]
\[ \dot{m}_1 = 10 \text{ kg/s} \]
\[ p_2 = 5 \text{ bar} \]
\[ T_2 = 200^\circ \text{C} \]
\[ \dot{m}_2 = 5 \text{ kg/s} \]
\[ p_3 = 0.15 \text{ bar} \]
\[ x_3 = 0.90 \]
\[ \dot{m}_3 = 5 \text{ kg/s} \]

Assumptions:
1. Steady state steady flow
2. Neglect heat loss
3. Neglect KE and PE

Basic equations:

\[
\frac{dm_{cv}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e
\]
\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right)
\]
\[
\frac{dS_{cv}}{dt} = \frac{\dot{Q}_{cv}}{T_b} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{\sigma}_{cv}
\]

Solution:

\[ m_1 = 10 \text{ kg/s} \]
\[ m_2 = 5 \text{ kg/s} \]
\[ m_3 = 5 \text{ kg/s} \]

(a)

Energy balance:

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{v_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{v_e^2}{2} + gz_e \right)
\]
\[
\dot{W}_{cv} = \dot{m}_i h_i - \sum \dot{m}_e h_e
\]
\[ \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 10 \frac{\text{kg}}{\text{s}} \times 3115.3 \frac{\text{kJ}}{\text{kg}} - 5 \frac{\text{kg}}{\text{s}} \times 2855.4 \frac{\text{kJ}}{\text{kg}} - 5 \frac{\text{kg}}{\text{s}} \times 2361.7 \frac{\text{kJ}}{\text{kg}} \]
\[ = 5067.5 \text{ kW} \]

(b)

Entropy balance:

\[ \frac{dS_{CV}}{dt} = \sum \frac{\dot{Q}_{in}}{T_b} + \sum \dot{m}_l s_l - \sum \dot{m}_e s_e + \dot{\sigma}_{CV} \]
\[ \dot{\sigma}_{CV} = \sum \dot{m}_e s_e - \dot{m}_l s_l \]
\[ = \dot{m}_3 s_3 + \dot{m}_2 s_2 - \dot{m}_1 s_1 \]
\[ = 5 \frac{\text{kg}}{\text{s}} \times 7.2831 \frac{\text{kJ}}{\text{kgK}} + 5 \frac{\text{kg}}{\text{s}} \times 7.0592 \frac{\text{kJ}}{\text{kgK}} - 10 \frac{\text{kg}}{\text{s}} \times 6.7428 \frac{\text{kJ}}{\text{kgK}} \]
\[ = 4.2835 \text{ kW/K} \]
\[ \dot{E}_d = T_0 \dot{\sigma}_{CV} = (25 + 273)K \times 4.2835 \frac{\text{kW}}{\text{K}} \]
\[ = 1276.48 \text{ kW} \]

(c)

\[ \varepsilon = \frac{\dot{W}_{CV}}{\dot{W}_{CV} + \dot{E}_d} = \frac{5067.5 \text{ kW}}{5067.5 + 1276.48 \text{ kW}} = 0.7987 \]

Alternatively,

\[ \varepsilon = \frac{\dot{W}_{CV}}{\dot{m}_1 \dot{e}_{f,1} - \dot{m}_2 \dot{e}_{f,2} - \dot{m}_3 \dot{e}_{f,3}} \]
SP-11

An insulated, rigid tank initially contains 2.2 lb-mol of argon (Ar) at 80°F, 14.5 psia. The tank is connected by a valve to a large vessel containing nitrogen (N₂) at 440°F, 58 psia. A quantity of nitrogen flows into the tank, forming an Ar-N₂ mixture at temperature T and pressure p. Plot T, in °F, and p, in psia, versus the amount of N₂ within the tank, in 0 to 20 lb-mol. Also, find the pressure when 0.2 lb-mol of N₂ entered the tank.

Note: Please include your EES code, parametric tables, and plots. Assumptions, basic equation(s), system sketch can be either submitted separately or included within the EES code. EES code should contain variable definitions, comments, etc.

**Given:**

An insulated rigid tank contains

State 1: 2.2 lb-mol of argon (Ar) at \( T_1 = 80°F \), \( p_1 = 14.5 \text{ psia} \)

Nitrogen (N₂) at 440°F, 58 psia is supplied to the tank.

State 2: an Ar-N₂ mixture

**Find:**

(a) Plot T, in °F, and p, in psia, versus the amount of N₂ within the tank, in lb-mol.

(b) Find the pressure when 0.2 lb-mol of N₂ entered the tank.

**System sketch:**

![System Sketch](image-url)
Assumptions:

1. While mixing, gases inside the tank are in equilibrium.
2. Both gases are ideal gases.
3. Both gases follow Dalton’s model
4. Neglect ∆KE and ∆PE
5. No work
6. No heat transfer

Basic equations:

\[
\frac{dm_{cv}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e \\
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i \left( h_i + \frac{v_i^2}{2} + g z_i \right) - \sum \dot{m}_e \left( h_e + \frac{v_e^2}{2} + g z_e \right) 
\]

Solution:

Because the volume of the tank is constant, an ideal gas law becomes

\[
V = \left[ \frac{N R T_1}{P_1} \right]_{Ar} = \left[ \frac{(N_{Ar} + N_{N_2}) R T}{P} \right] \rightarrow p = p_1 \left[ 1 + \frac{N_{N_2}}{N_{Ar}} \right] * \frac{T}{T_1} \quad ------ \text{Eqn (1)}
\]

Mass balance:

Argon: \( m_{Ar} = \text{constant} \rightarrow N_{Ar} = \text{constant} \)

Nitrogen: \( \Delta m_{N_2} = m_{N_2,i} \rightarrow \Delta N_{N_2} = N_{N_2,i} \)

Energy balance:

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{m}_i \left( h_i + \frac{v_i^2}{2} + g z_i \right) - \sum \dot{m}_e \left( h_e + \frac{v_e^2}{2} + g z_e \right) \\
\frac{dE_{cv}}{dt} = \dot{m}_i h_i
\]

Integrate w.r.t time,

\[
\Delta E = \Delta U + \Delta KE + \Delta PE \\
= U_2 - U_1 = m_i h_i
\]
\[ m_2 u_2 - m_2 u_2 = m_1 h_i \]

\[ (N_{Ar} \cdot \bar{u}_{Ar,2} + N_{N_2} \cdot \bar{u}_{N_2,2}) - N_{Ar} \cdot \bar{u}_{Ar,1} = N_{N_2} \cdot \bar{h}_{N_2,in} \]  \quad ------ Eqn (2)

EES code to find pressure and temperature when 0.2 lb-mol of N\textsubscript{2} entered the tank
EES code to plot pressure and temperature

```
"given"
N_Ar=2.2  "lb-mol of Ar"
T_1=80    "[F] initial temperature"
p_1=14.5  "[psia] initial pressure"
T_in=440  "[F] inlet temperature of N2"
p_in=58   "[psia] inlet pressure of N2"

"Energy balance"
((N_Ar*u_Ar2) + (N_N2*u_N2)) - (N_Ar*u_Ar1) = (N_N2*h_N2)

u_Ar1=intenergy(Ar, T=T_1)  "[Btu/lb-mol] internal energy of Ar at initial temperature"
h_N2=enthalpy(N2, T=T_in)    "[Btu/lb-mol] enthalpy of N2 at inlet temperature"
u_Ar2=intenergy(Ar, T=T_2)   "[Btu/lb-mol] internal energy of Ar at final temperature"
u_N2=intenergy(N2, T=T_2)    "[Btu/lb-mol] internal energy of N2 at final temperature"

"Ideal gas law"
p_2=p_1*(1+(N_N2/N_Ar)*(T_2+459.67)/(T_1+459.67))
```

![Parametric Table](image)

![Graph](image)