PROBLEM 3.6

For H\(_2\)O, determine the specified property at the indicated state. Locate the state on a sketch of the \(T\)-\(v\) diagram.

(a) \(T = 140^\circ\text{C}\), \(v = 0.5\ \text{m}^3/\text{kg}\). Find \(p\), in bar.
(b) \(p = 30\ \text{MPa}, T = 100^\circ\text{C}\). Find \(v\), in \text{m}^3/\text{kg}.
(c) \(p = 10\ \text{MPa}, T = 485^\circ\text{C}\). Find \(v\), in \text{m}^3/\text{kg}.
(d) \(T = 80^\circ\text{C}, x = 0.75\). Find \(p\), in bar, and \(v\), in \text{m}^3/\text{kg}.

(a) \(T = 140^\circ\text{C}, v = 0.5\ \text{m}^3/\text{kg}\). Find \(p\), in bar.

\[
p = p_{\text{sat}}(140^\circ\text{C}) = 3.613\ \text{bar}
\]

\[
\begin{array}{c}
\text{Table A-3: } v_f = 1.0435 \times 10^{-3} \ \text{m}^3/\text{kg}, v_g = 1.673 \ \text{m}^3/\text{kg}. \ \text{Since } v_f < v < v_g, \ \text{the state is in the two-phase liquid-vapor region, as shown.}

\text{From Table A-3, the pressure is the saturation pressure at } 140^\circ\text{C: } p = 3.613\ \text{bar}. \\
\end{array}
\]

(b) \(p = 30\ \text{MPa}, T = 100^\circ\text{C}\). Find \(v\), in \text{m}^3/\text{kg}.

\[
p_c = 220.9\ \text{bar (Table A-3)}
\]

\[
T_c = 374.14^\circ\text{C (Table A-3)}
\]

The pressure is higher than the critical pressure, as shown on the diagram. Hence, the state is in the compressed liquid region.

\[
\begin{array}{c}
\text{From Table A-5: } v = 1.0290 \ \text{m}^3/\text{kg}. \\
\end{array}
\]
Problem 3.6 (Continued)

(c) \( p = 10 \text{ MPa}, T = 485^\circ \text{C} \). Find \( v \), in \( \text{m}^3/\text{kg} \).

\[ 10 \text{ MPa} = 100 \text{ bar} \]

\[ 485^\circ \text{C} \]

\[ T_c = 374.14^\circ \text{C} \]

\( T_{\text{sat}} (100 \text{ bar}) = 311.1^\circ \text{C} \) (Table A-3)

Since the temperature is higher than \( T_{\text{sat}} \) at 100 bar, the state is superheated vapor.

Interpolating in Table A-4, we get

\[ v = 0.03160 + (485 - 480)/(520 - 480) \times (0.02343 - 0.3160) = 0.03058 \text{ m}^3/\text{kg} \]

(d) \( T = 80^\circ \text{C}, \ x = 0.75 \). Find \( p \), in bar, and \( v \), in \( \text{m}^3/\text{kg} \).

\[ p_{\text{sat}} (80^\circ \text{C}) = 0.4739 \text{ bar} \] (Table A-2)

\[ 80^\circ \text{C} \]

\[ x = 0.75 \]

Eq. 3.2: \[ v_x = v_f + x(v_g - v_f) \]

With data from Table A-2 at 80°C

\[ v = 1.0291 \times 10^{-3} + (0.75)(3.407 - 1.0291 \times 10^{-3}) \]

\[ = 2.556 \text{ m}^3/\text{kg} \]
PROBLEM 3.18

A closed, rigid tank contains a two-phase liquid-vapor mixture of Refrigerant-22 initially at -20°C with a quality of 50.36%. Energy transfer by heat into the tank occurs until the refrigerant is at a final pressure of 6 bar. Determine the final temperature, in °C. If the final state is in the superheated vapor region, at what temperature, in °C, does the tank contain only saturated vapor?

\[ v_1 = \frac{v_{fl}}{x_1} + x_1(v_{gl} - v_{fl}) = 0.7427 \times 10^{-3} + (0.5036)(0.0926 - 0.7427 \times 10^{-3}) = 0.0470 \, m^3/kg \]

Since \( v_2 = v_1 \), State 2 is in the superheated vapor region (\( v_2 > v_{g@6bar} \)). Thus, interpolating at 6 bar with \( v_2 = 0.0470 \, m^3/kg \) in Table A-9 we get

\[ T_2 \approx 43.75°C \]

Since State 2 is superheated vapor, the tank contains only saturated vapor at the condition where \( v_g = 0.0470 \, m^3/kg \). Referring to Table A-7, this occurs at \( T = 0°C \).
**PROBLEM 3.51**

**KNOWN:** Water contained in a piston-cylinder assembly, initially a two-phase liquid-vapor mixture undergoes two processes in series. State data is provided.

**FIND:** Show the two processes of the water in series on a T-V diagram. For the overall process of the water evaluate the work and heat transferred, each in KJ/kg.

**SCHEMATIC & GIVEN DATA:**

**ENGINEERING MODEL:**
1. The water is the closed system.
2. Volume change is the only work mode.
3. Kinetic and potential energy effects are negligible.

**ANALYSIS:** With assumption 2, \( W = \int p \, dV = \int_1^2 p \, dV + \int_2^3 p \, dV \) \( \Rightarrow W = \int_1^3 p \, dV = p (V_3 - V_1) \) (1)

With data from Table A-3, \( V_1 = V_f + V_i (V_g - V_f) = (1.0528) + 0.2 (1.159 - (1.0528)) = 0.2326 \) m³/kg

With \( \frac{V_f}{V_i} = \frac{A_0}{m} \), \( \frac{V_2}{V_i} = \frac{L_1 + L_2}{L_1} = \frac{0.8 m}{0.5 m} \Rightarrow \frac{V_3}{V_i} = \frac{0.8 m}{0.5 m} \)

Also, \( \frac{V_3}{V_2} = \frac{V_3}{V_2} \)

Thus, Eq.(1) gives \( W = 1.5 \text{bar} \times 10^5 \text{N/m}^2 \times (0.3722 - 0.2326) \text{m}^3 / \text{kg} \times 10 \text{kJ/kg} = 20.94 \text{kJ} \) (2)

Reducing an energy balance, \( Q_1 + \Delta KE + \Delta PE = Q_2 - W \), we get:

\( Q_1 = W + m (u_3 - u_1) \Rightarrow \frac{Q_1}{m} = \frac{W}{m} + (u_3 - u_1) \) (2)

With data from Table A-3, \( u_1 = u_f + x (u_g - u_f) = 466.94 + 0.2 (851.7 - 466.94) = 877.99 \) kJ/kg

Interpolation with Table A-3, \( u_3 = u_f = u_{f_1} \), \( u_2 = 2561.48 \) kJ/kg.

Then, Eq.(2) gives:

\( \frac{Q_1}{m} = \frac{20.94}{V_3} + (2561.48 - 877.99) \text{kJ/kg} = 1704.93 \text{kJ/kg} \) (3)
Problem 2.45

**KNOWN:** A gas contained in a piston-cylinder assembly undergoes a constant-pressure expansion while being slowly heated. State data are provided.

**FIND:** For the gas, evaluate work and heat transfer. For the piston, evaluate work and change in potential energy.

**SCHEMATIC & GIVEN DATA:**

![Diagram of a piston-cylinder assembly]

- $p_{in} = 1\, \text{bar}$
- $p = 2\, \text{bar}$
- $V_1 = 0.1\, \text{m}^3$
- $V_2 = 0.12\, \text{m}^3$
- $(U_2 - U_1) = 0.25\, \text{kJ}$

**ENGINEERING MODEL:**

1. As shown in the schematic, two closed systems are considered: the gas and the piston.
2. The gas undergoes a constant-pressure process.
3. For the gas there is no change in potential energy (see Example 2.3) and no overall change in kinetic energy.
4. For the piston, there is no heat transfer. Also, there is no change in internal energy; no overall change in kinetic energy; and no friction.

**ANALYSIS:** (a) Taking the gas as the system, the work is obtained from Eq. 2.17: $W = \int p\, dV = p[V_2 - V_1] = (2 \times 10^5 \, \text{N/m}^2)(0.12 - 0.1)\, \text{m}^3 \times \frac{1\, \text{kJ}}{10^8\, \text{N/m}^2} = 4\, \text{kJ}$

Reducing an energy balance, $\Delta U + \Delta KE + \Delta PE = Q - W \Rightarrow Q = W + \Delta U$

$Q = 4\, \text{kJ} + 0.25\, \text{kJ} = 4.25\, \text{kJ}$

(b) Taking the piston as the system, an energy transfer by work occurs on the bottom surface from the gas. At the top surface the piston does work on the atmosphere:

$W_{piston} = \int F\, d\alpha = (P_{atm} - PA)\, d\alpha = (P_{atm} - P)\, d\alpha$

$= (2\, \text{bar}) (0.12 - 0.1)\, \text{m}^3 \times \frac{1\, \text{kJ}}{10^8\, \text{N/m}^2}$

$= -2\, \text{kJ}$

An energy balance for the piston reduces as follows:

$\Delta U + \Delta KE + \Delta PE \, \text{piston} = W_{piston} - W_{piston}$

$\Delta PE \, \text{piston} = -W_{piston}$

$= -2\, \text{kJ}$

1. **Overall energy balance sheet** in terms of magnitudes:

| Input: $Q = 4.25\, \text{kJ}$ | Disposition of the energy input:
| --- | --- |
| $\Delta U$ in the gas: | 0.25\, \text{kJ}$
| $\Delta PE$ in the piston: | 2.00\, \text{kJ}$
| Transfer by work to the atmosphere | 3.00\, \text{kJ}$

Total: $4.25\, \text{kJ}$
Problem 2.64

**Known:** Power cycles A and B operate in series.

**Find:** Determine an expression for the thermal efficiency of an overall cycle consisting of A and B together in terms of $\eta_A$ and $\eta_B$.

**Schematic & Given Data:**

(a) A and B in series

(b) Overall cycle

**Engineering Model:**
1. Cycles A and B and the overall cycle are power cycles.
2. Energy transfer is positive in the directions of the arrows on the schematic.

**Analysis:**

\[ \eta_A = \frac{W_A}{Q_1} = 1 - \frac{Q_2}{Q_1} \Rightarrow Q_2 = Q_1 (1 - \eta_A) \quad (1) \]

\[ \eta_B = \frac{W_B}{Q_2} = 1 - \frac{Q_3}{Q_2} \Rightarrow Q_3 = Q_2 (1 - \eta_B) \]

\[ Q_3 = Q_2 (1 - \eta_B) = Q_1 (1 - \eta_A)(1 - \eta_B) \quad (3) \]

\[ \eta = \frac{(W_A + W_B)}{Q_1} = 1 - \frac{Q_3}{Q_1} \quad (4) \]

Introducing (3) into (4),

\[ \eta = 1 - \frac{Q_1 (1 - \eta_A)(1 - \eta_B)}{Q_1} \]

\[ = 1 - (1 - \eta_A)(1 - \eta_B) \]

\[ = 1 - (1 - \eta_A \eta_B + \eta_A \eta_B) \]

\[ \eta = \eta_A + \eta_B - \eta_A \eta_B \quad (5) \]

1. Sample calculation: $\eta_A = 0.25$, $\eta_B = 0.32$

\[ \eta = 0.25 + 0.32 - (0.25 \times 0.32) \]

\[ = 0.49 (49\%) \]
Problem 4.73

Steady-state operating data are provided for a compressor and heat exchanger in Fig. P4.73. The power input to the compressor is 50 kW. As shown on the figure, nitrogen (N\textsubscript{2}) flows through the compressor and heat exchanger with a mass flow rate of 0.25 kg/s. The nitrogen is modeled as an ideal gas. A separate cooling stream of helium, modeled as an ideal gas with \( k = 1.67 \), also flows through the heat exchanger. Stray heat transfer and kinetic and potential energy effects are negligible. Determine the mass flow rate of the helium, in kg/s.

**Known:** Steady-state data are provided for a compressor and heat exchanger in series. Two gases are involved: N\textsubscript{2} and helium.

**Find:** Determine the mass flow rate of the helium, in kg/s.

**Schematic & Given Data:**

**Engineering Model:**

1. A control volume enclosing both components is considered.
2. The control volume is at steady state.
3. For the control volume, \( \dot{Q}_{cv} \) is negligible and kinetic and potential energy effects are ignored.
4. The ideal gas model applies to each gas. For helium, \( k = 1.67 \).

**Analysis:** Applying mass and energy rate balances to a control volume enclosing both components we get,

\[
\dot{m}_4 = \dot{m}_1 \left[ \frac{h_1 - h_3}{h_4 - h_5} \right] + \frac{-W_{cv}}{h_4 - h_5}
\]

Evaluating properties, data from Table A-2.3 is used for N\textsubscript{2}:

\[
(h_1 - h_3) = \left( \frac{h_{f1} - h_{f3}}{M} \right) = \left( \frac{(8141 - 10180) \text{kJ/kmol}}{28.01 \text{kg/kmol}} \right) = -72.8 \text{ kJ/kg}
\]

For helium,

\[
(h_5 - h_4) = c_p \left( T_5 - T_4 \right) = \left( \frac{1.67}{0.67} \right) \left( \frac{8.314 \text{ KJ}}{4.003 \text{ kg K}} \right)(150 \text{ K}) = 776.5 \text{ kJ/kg}
\]

Collecting results,

\[
\dot{m}_4 = \frac{50 \text{ kW}}{1 \text{ kW}} \left[ \frac{1 \text{ kJ/s}}{2 \text{ kJ}} \right] + \frac{(0.25 \text{ kg/s})(-72.8 \text{ kJ/kg})}{776.5 \text{ kJ/kg}} = \frac{0.041 \text{ kg/s}}{}
\]
Problem 4.78

A rigid tank of volume 2 m$^3$ initially contains air at 0.21 bar, 290 K. A leak develops and air flows in slowly from the surroundings which are at 1.1 bar, 312 K. After a while, the tank and its surroundings come to equilibrium. What is the final temperature in the tank, in °C, and how much mass has entered the tank, in kg? What is the amount of energy transfer by heat for a control volume that encloses the tank, in kJ? Neglect kinetic and potential energy effects, and assume the air is an ideal gas with constant specific heats evaluated at 300 K.

KNOWN: Air leaks into a rigid tank from the surrounding atmosphere, initially at a lower pressure and temperature relative to the surroundings.

FIND: Determine the temperature in the tank when equilibrium is achieved, the amount of mass that entered the tank, and the amount of heat transfer.

SCHEMATIC AND GIVEN DATA:

ENGINEERING MODEL: (1) The inside of the tank is the one-inlet, no-exit control volume. (2) The control volume is assumed to be rigid, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible. (4) The air is modeled as an ideal gas, and since the temperature change is not very big, we can use constant specific heats evaluated at 300 K.

ANALYSIS: At the final equilibrium state, the tank contents and the surroundings are in equilibrium, so $p_2 = p_{surr}$ and $T_2 = T_{surr} = 312$ K.

The initial mass is $m_1 = \frac{p_1 V}{RT_1} = \frac{(0.21 \text{ bar})(2 \text{ m}^3)}{(8.314 \text{ kJ} / \text{mol} \cdot \text{K})(290 \text{ K})} \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 0.5046 \text{ kg}$

The final mass is $m_2 = \frac{p_2 V}{RT_2} = \frac{(1.1 \text{ bar})(2 \text{ m}^3)}{(8.314 \text{ kJ} / \text{mol} \cdot \text{K})(312 \text{ K})} \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| = 2.457 \text{ kg}$

Reducing the mass rate balance and integrating over the time of the process

$$\frac{dm_{cv}}{dt} = \dot{m}_i \rightarrow \int_{t_1}^{t_2} \left( \frac{dm_{cv}}{dt} \right) dt = \int_{t_1}^{t_2} \dot{m}_i dt \rightarrow m_2 - m_1 = \int_{t_1}^{t_2} \dot{m}_i dt \hspace{1cm} (*)$$

So, the amount of mass that enters is

$$m_2 - m_1 = 2.457 - 0.5046 = 1.952 \text{ kg}$$

To find the amount of energy transfer by heat, we use the energy rate balance:

$$\frac{dKE_{cv}}{dt} + \frac{dPE_{cv}}{dt} + \frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i (h_i + \frac{v_i^2}{2} + g z_i)$$
Integrating over the time of the process

\[ U_2 - U_1 = Q_{cv} + \int_{t_1}^{t_2} \hat{m}_i h_{\text{surr}} \, dt \quad \rightarrow \quad Q_{cv} = U_2 - U_1 - h_{\text{surr}} \int_{t_1}^{t_2} \hat{m}_i \, dt \]

With (*) and noting that \( U = mu \) and that \( \int_{t_1}^{t_2} \hat{m}_i \, dt = m_2 - m_1 \)

\[ Q_{cv} = m_2 u_2 - m_1 u_1 - h_{\text{surr}}(m_2 - m_1) \]

Since \( h = u + pv \) and for an ideal gas \( pv = RT \), and since \( T_2 = T_{\text{surr}} \)

\[ Q_{cv} = m_2 u_{\text{surr}} - m_1 u_1 - m_2(u_{\text{surr}} + RT_{\text{surr}}) + m_1(u_{\text{surr}} + RT_{\text{surr}}) \]

Collecting terms

\[ Q_{cv} = -m_2 (RT_{\text{surr}}) - m_1 [(u_1 - u_{\text{surr}}) - RT_{\text{surr}}] \]

With \( u_1 - u_{\text{surr}} = c_v(T_1 - T_{\text{surr}}) \)

\[ Q_{cv} = -m_2 (RT_{\text{surr}}) - m_1 [c_v(T_1 - T_{\text{surr}}) - RT_{\text{surr}}] \]

\[ = -(2.457 \text{ kg})(8.314 \frac{\text{kJ}}{28.97 \text{kg} \cdot \text{K}})(312 \text{ K}) - (0.5046 \text{ kg})(0.718 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(290 - 312)\text{K} - \left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg} \cdot \text{K}}\right)(312 \text{ K}) \]

\[ = -166.8 \text{ kJ (out)} \]
Problem 4.82

A rigid tank whose volume is $0.5 \text{ m}^3$, initially containing ammonia at $20^\circ\text{C}$, 1.5 bar, is connected by a valve to a large supply line carrying ammonia at 12 bar, $60^\circ\text{C}$. The valve is opened only as long as required to fill the tank with additional ammonia, bringing the total mass of ammonia in the tank to 143.36 kg. Finally, the tank holds a two-phase liquid-vapor mixture at $20^\circ\text{C}$.

Determine the heat transfer between the tank contents and the surroundings, in kJ, ignoring kinetic and potential energy effects.
Problem 4.82 (Continued)

Substituting values into Eq. (1):

\[ Q_{CV} = \left[ (1.433 \text{ kg})(286.2 \text{ kJ/kg}) - (0.533 \text{ kg})(1371.79 \text{ kJ/kg}) - (1553.07 \text{ kJ/kg}) \right] \left[ 143.36 - 0.533 \right] \text{ kg} \]

\[ = -181,522 \text{ kJ} \]

Energy transferred by heat to the surroundings.
Problem 4.86

A rigid tank having a volume of 0.1 m$^3$ initially contains water as a two-phase liquid-vapor mixture at 1 bar and a quality of 1%. The water is heated in two stages:

Stage 1: Constant-volume heating until the pressure is 20 bar.
Stage 2: Continued heating while saturated water vapor is slowly withdrawn from the tank at a constant pressure of 20 bar. Heating ceases when all the water remaining in the tank is saturated vapor at 20 bar.

For the water, evaluate the heat transfer, in kJ, for each stage of heating. Ignore kinetic and potential energy effects.
Problem 4.86 (Continued)

PROCESS 1-2:
\[ \Delta U + \Delta KE + \Delta PE = Q - W \]
\[ Q = m_1 [u_2 - u_1] \]
\[ = 5.56 \text{kg} [1194.4 - 438.25] \text{KJ} \]
\[ = 4204.2 \text{ KJ} \]

PROCESS 2-3
Mass and energy rate balances for the one-exit control volume reduce to,
\[ \frac{dQ_{cv}}{dt} = -he \] and \[ \frac{dU_{cv}}{dt} = Q'_{cv} - he \frac{dmv}{dt} \]
Combining them gives,
\[ \frac{dU_{cv}}{dt} = Q'_{cv} + he \frac{dmv}{dt} \]
Integrating with respect to time,
\[ \Delta U_{cv} = Q'_{cv} + he [m_3 - m_2] \Rightarrow (m_3 u_3 - m_2 u_2) = Q'_{cv} + he [m_3 - m_2] \]
Solving,
\[ Q'_{cv} = (m_3 u_3 - m_2 u_2) - he [m_3 - m_2], \text{ where } m_2 = m_1. \]

Inserting values,
\[ Q'_{cv} = [4 \text{ kg} \cdot (2600.3 \text{ KJ} \text{ kg}^{-1}) - (5.56 \text{ kg}) \cdot (1194.4 \text{ KJ} \text{ kg}^{-1})] - \left[ \frac{2799.1 \text{ KJ}}{\text{Kg}} \right] [1 - 5.56] \text{ KJ} \]
\[ = 8725.2 \text{ KJ} \]

1. The total heat transfer is \(4204.2 \text{ KJ} + 8725.2 \text{ KJ} = 12929.4 \text{ KJ} \)
An overall energy balance reads,
\[ Q_{cv} = (m_3 u_3 - m_1 u_1) - he (m_3 - m_1) \]
\[ = [4 \text{ kg} \cdot (2600.3 \text{ KJ} \text{ kg}^{-1}) - (5.56 \text{ kg}) \cdot (438.25 \text{ KJ} \text{ kg}^{-1})] - \left[ \frac{2799.1 \text{ KJ}}{\text{Kg}} \right] [1 - 5.56] \text{ KJ} \]
\[ = 12929.4 \text{ KJ} \]
which checks the total given previously.
Problem 4.88
The procedure to inflate a hot air balloon requires a fan to move an initial amount of air into the balloon envelope followed by heat transfer from a propane burner to complete the inflation process. After a fan operates for 10 minutes with negligible heat transfer with the surroundings, the air in an initially deflated balloon achieves a temperature of 80°F and a volume of 49,100 ft³. Next the propane burner provides heat transfer as air continues to flow into the balloon without use of the fan until the air in the balloon reaches a volume of 65,425 ft³ and a temperature of 210°F. Air at 77°F and 14.7 lbf/in.² surrounds the balloon. The net rate of heat transfer is $7 \times 10^6$ Btu/h. Ignoring effects due to kinetic and potential energy, modeling the air as an ideal gas, and assuming the pressure of the air inside the balloon remains the same as that of the surrounding air, determine (a) the power required by the fan, in hp. (b) the time required for full inflation of the balloon, in min.

**KNOWN:** A hot air balloon undergoes inflation by a fan is heated by a propane burner.

**FIND:** Determine the power required by the fan, and the time required for full inflation of the balloon.

**SCHEMATIC AND GIVEN DATA:**
Part (a)

```
State 1
(deflated balloon)
\( t = t_1 = 0 \)

Surrounding Air
\( T_{\text{surr}} = 77°F \)
\( p_{\text{surr}} = 14.7 \text{ lbf/in.}^2 \)

\( W_{\text{fan}} = ? \)
\( T_i = T_{\text{surr}} \)
\( p_i = p_{\text{surr}} \)
```

Part (b)

```
State 2
\( t = t_2 = 10 \text{ min} \)

\( V_2 = 49,100 \text{ ft}^3 \)
\( T_2 = 80°F \)
\( p_2 = p_{\text{surr}} \)

\( \dot{Q}_{\text{cv}} = 7 \times 10^6 \text{ Btu/h} \)
```

```
State 3
\( t = t_3 = ? \)

\( V_3 = 65,425 \text{ ft}^3 \)
\( T_3 = 210°F \)
\( p_3 = p_{\text{surr}} \)
```

```
Surrounding Air
\( T_{\text{surr}} = 77°F \)
\( p_{\text{surr}} = 14.7 \text{ lbf/in.}^2 \)
```

ENGINEERING MODEL:
1. The control volumes are defined by the dashed lines on the accompanying diagrams.
2. Air is modeled as an ideal gas.
3. For the control volumes, kinetic and potential energy effects can be ignored.
4. The state of the air entering the balloon remains constant and equivalent to the state of the surrounding air.
5. Air inside the balloon has the same pressure as the surrounding air during the entire process.
6. Heat transfer with the surroundings is negligible during process 1-2.

ANALYSIS:
(a) Since the control volume has a single inlet and no exit, the mass rate balance for process 1-2 reduces to
\[ \frac{dm_{cv}}{dt} = \dot{m}_i \]
The energy rate balance for process 1-2 reduces to
\[ \frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i h_i \]
Neglecting heat transfer with the surroundings and combining mass and energy rate balances result in
\[ \frac{dU_{cv}}{dt} = -\dot{W}_{cv} + \frac{dm_{cv}}{dt} h_i \]
By assumption 4 of the engineering model, the specific enthalpy at the inlet is constant. Accordingly, integration gives
\[ \Delta U_{cv} = -\dot{W}_{cv} \Delta t + \Delta m_{cv} h_i \]
\[ m_2 u_2 - m_1 u_1 = -\dot{W}_{cv} \Delta t + (m_2 - m_1) h_i \]
Since the balloon is deflated at state 1, \( m_1 = 0 \). The equation reduces to
\[ m_2 u_2 = -\dot{W}_{cv} \Delta t + m_2 h_i \]
Work associated with the control volume is due to expansion of the control volume and the work required by the fan. Thus, \( W_{cv} = W_{\text{fan}} + W_{\text{expansion}} \). The work associated with the control volume due to expansion of the air at constant pressure is
\[ W_{\text{expansion}} = \int_1^2 p \, dV = p(V_2 - V_1) = pV_2 = \left(14.7 \frac{\text{lbf}}{\text{in}^2}\right)(49,100 \text{ ft}^3) \left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right) \left(\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}\right) \]
\[ W_{\text{expansion}} = 133,592 \text{ Btu} \]
The sign is positive as expected for work due to expansion.

The work associated with the fan is \( \dot{W}_{\text{fan}} \Delta t \). Substituting values and solving for the fan power give
\[ m_2 u_2 = -(\dot{W}_{\text{fan}} \Delta t + W_{\text{expansion}}) + m_2 h_i \]
Problem 4.88 (Continued – Page 3)

\[ \dot{W}_{\text{fan}} = \frac{m_2(h_i - u_2) - W_{\text{expansion}}}{\Delta t} \]

The mass in the control volume at state 2 can be determined using the ideal gas equation of state. Converting air temperature to absolute scale, \( T_2 = 80^\circ \text{F} = 540^\circ \text{R} \) and substituting

\[ p_2V_2 = m_2RT_2 \Rightarrow m_2 = \frac{p_2V_2}{RT_2} \]

\[ m_2 = \frac{14.7 \text{ lbf/in}^2 \cdot 49,100 \text{ ft}^3}{144 \text{ in}^2 \cdot 1 \text{ ft}^2 \cdot 28.97 \text{ lbmol}^{-1} \cdot 540^\circ \text{R}} = 3609 \text{ lb} \]

The specific enthalpy of the inlet air \( (T_i = 77^\circ \text{F} = 537^\circ \text{R}) \) and the specific internal energy of the air at state 2 \( (T_2 = 540^\circ \text{R}) \) are determined from Table A-22E: \( h_i = 128.34 \text{ Btu/lb} \) and \( u_2 = 92.04 \text{ Btu/lb} \). Solving for fan power

\[ \dot{W}_{\text{fan}} = \frac{(3609 \text{ lb})(128.34 \text{ Btu/lb} - 92.04 \text{ Btu/lb}) - (133,592 \text{ Btu})}{10 \text{ min}} = -6.10 \text{ hp} \]

The sign is negative as expected since the fan requires power input to operate.

(b) During process 2-3, the mass rate balance takes the form

\[ \frac{dm_{\text{cv}}}{dt} = \dot{m}_i \]

The energy rate balance for process 2-3 reduces to

\[ \frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \dot{m}_i h_i \]

Combining mass and energy rate balances results in

\[ \frac{dU_{\text{cv}}}{dt} = \dot{Q}_{\text{cv}} - \dot{W}_{\text{cv}} + \frac{dm_{\text{cv}}}{dt} h_i \]

By assumption 4 of the engineering model, the specific enthalpy at the inlet is constant. Accordingly, integration gives

\[ \Delta U_{\text{cv}} = \dot{Q}_{\text{cv}} \Delta t - \dot{W}_{\text{cv}} \Delta t + \Delta m_{\text{cv}} h_i \]

\[ m_3u_3 - m_2u_2 = \dot{Q}_{\text{cv}} \Delta t - W_{\text{cv}} + (m_3 - m_2)h_i \]

Rearranging and solving for time associated with process 2-3 yields
Problem 4.88 (Continued – Page 4)

\[ \Delta t = \frac{m_3(u_3 - h_t) - m_2(u_2 - h_t) + W_{cv}}{\dot{Q}_{cv}} \]

The mass in the control volume at state 3 can be determined using the ideal gas equation of state. Converting air temperature to absolute scale, \( T_3 = 210^\circ F = 670^\circ R \) and substituting

\[ p_3 V_3 = m_3 RT_3 \Rightarrow m_3 = \frac{p_3 V_3}{RT_3} \]

\[ m_3 = \frac{(14.7 \text{ lbf/in}^2)(65,425 \text{ ft}^3)}{1545 \text{ ft lbf/lbmol} \cdot ^\circ R} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| = 3876 \text{ lb} \]

The specific internal energy of the air at state 3 \((T_3 = 670^\circ R)\) is determined from Table A-22E using interpolation: \( u_3 = 114.40 \text{ kJ/kg} \).

The work associated with the control volume is due to expansion of the balloon at constant pressure

\[ W_{cv} = \int_2^3 p dV = p(V_3 - V_2) = \left(14.7 \frac{\text{lbf}}{\text{in}^2}\right)(65,425 \text{ ft}^3 - 49,100 \text{ ft}^3) \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}} \right| \]

\[ W_{cv} = 44,417 \text{ Btu} \]

Substituting values and solving for time associated with process 2-3 give

\[ \Delta t = \frac{(3876 \text{ lb})(114.4 \frac{\text{Btu}}{\text{lb}} - 128.34 \frac{\text{Btu}}{\text{lb}}) - (3609 \text{ lb})(92.04 \frac{\text{Btu}}{\text{lb}} - 128.34 \frac{\text{Btu}}{\text{lb}}) + (44,417 \text{ Btu})}{1 \text{ Btu/h}} \left| \frac{60 \text{ min}}{1 \text{ h}} \right| \]

\[ \Delta t = 7 \times 10^6 \text{ Btu/h} \]

\[ \Delta t = 1.04 \text{ min} \]

Solving for time associated with the entire inflation process yields

\[ t_3 = t_2 + (\Delta t)_{23} = 10 \text{ min} + 1.04 \text{ min} = 11.04 \text{ min} \]