Assumption:

No heat loss (Q) in the shower water supply system.

Water in the supply is an incompressible liquid

1D, uniform flow, uniform state

SSSF

Neglect ke and pe

Basic Equation:

\[
\frac{dm_{sys}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}
\]

\[
\frac{dE_{sys}}{dt} = \dot{Q} - W + \sum_{in} \dot{m}_{in} (h + ke + pe)_{in} - \sum_{out} \dot{m}_{out} (h + ke + pe)_{out}
\]

\[
h_{\text{compressed liquid}}(P, T) = h_{\text{sat liq}}(T) + v_{\text{sat liq}}(T)[P - P_{\text{sat}}(T)]
\]

Solution:

\[
\frac{dm_{sys}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}
\]

\[
\dot{m}_{\text{hot}} + \dot{m}_{\text{cold}} = \dot{m}_{\text{mixer}}
\]
\[ h_f(water, 15\,^\circ C) = 62.981 \frac{kJ}{kg} \]
\[ h_f(water, 55\,^\circ C) = 230.26 \frac{kJ}{kg} \]
\[ v_f(water, 15\,^\circ C) = 0.0010009 \frac{m^3}{kg} \]
\[ v_f(water, 55\,^\circ C) = 0.0010146 \frac{m^3}{kg} \]
\[ P_{sat}(15\,^\circ C) = 0.017058 \text{ bar} \]
\[ P_{sat}(55\,^\circ C) = 0.15762 \text{ bar} \]
\[ h_{cold}(water, 15\,^\circ C, 550\,kPa) = 62.981 \frac{kJ}{kg} + \left(0.0010009 \frac{m^3}{kg}\right)(550 - 1.7058\,kPa) = 63.5 \frac{kJ}{kg} \]
\[ h_{hot}(water, 55\,^\circ C, 550\,kPa) = 230.26 \frac{kJ}{kg} + 0.0010146 \frac{m^3}{kg}(550 - 15.762\,kPa) = 231 \frac{kJ}{kg} \]
\[
\frac{dE_{sys}}{dt} = \dot{Q} - \dot{W} + \sum_{in} \dot{m}_{in}(h + ke + pe)_{in} - \sum_{out} \dot{m}_{out}(h + ke + pe)_{out}
\]
\[
\dot{m}_{mixer} h_{mixer} = \dot{m}_{hot} h_{hot} + \dot{m}_{cold} h_{cold}
\]
\[
h_{mixer} = \frac{\dot{m}_{hot}}{\dot{m}_{mixer}} h_{hot} + \frac{\dot{m}_{cold}}{\dot{m}_{mixer}} h_{cold} = 0.75 \times \frac{231\,kJ}{kg} + 0.25 \times 63.5 \frac{kJ}{kg} = 189 \frac{kJ}{kg}
\]
From the tables, Mixer Temperature is about 45\,^\circ C because \( h(\text{compressed liquid}, 45\,^\circ C, 550\,kPa) = 188.43\,kJ/kg + 0.09595\times(550 - 9.6) = 189\,kJ/kg \)
\[
A_{shower} = \#\text{holes} \times A_{hole} = 34\text{holes} \times \frac{\pi}{4}(0.0025m)^2 = 0.0001668971 m^2
\]
\[
A_{mixer} = \frac{\pi}{4}(0.0189m)^2 = 0.000281m^2
\]
Find the shower velocity
Shower flows 9.6 liters/min
\[
\dot{V} = \dot{V}_{shower} A_{shower}
\]
\[ V_{\text{shower}} = \frac{V}{A_{\text{shower}}} = \frac{9.6 \text{ liters} \left( \frac{1\text{ min}}{60\text{ sec}} \right) \left( \frac{m^3}{1000\text{ liters}} \right)}{0.0001668971m^2} = 0.959 \frac{m}{s} \]

**Find the mixer velocity**

The water is an incompressible liquid so density change is negligible so the mass conservation equation looks like

System 2 mass conservation equation:

\[ \bar{V}_{\text{shower}}A_{\text{shower}} = \bar{V}_{\text{mixer}}A_{\text{mixer}} \]

\[ \bar{V}_{\text{mixer}} = \frac{\bar{V}_{\text{shower}}A_{\text{shower}}}{A_{\text{mixer}}} = \frac{9.6 \text{ liters} \left( \frac{1\text{ min}}{60\text{ sec}} \right) \left( \frac{m^3}{1000\text{ liters}} \right)}{0.000281m^2} = 0.569 \frac{m}{s} \]

**Find the hot and cold water velocities**

System 1 mass conservation equation:

\[ \bar{V}_{\text{mixer}}A_{\text{mixer}} = \bar{V}_{\text{hot}}A_{\text{hot}} + \bar{V}_{\text{cold}}A_{\text{cold}} \]

All three areas are the same

\[ \bar{V}_{\text{mixer}} = \bar{V}_{\text{hot}} + \bar{V}_{\text{cold}} \]

\[ \bar{V}_{\text{cold}} = \bar{V}_{\text{mixer}} \left( 1 - \frac{\bar{V}_{\text{hot}}}{\bar{V}_{\text{mixer}}} \right) = 0.569 \frac{m}{s} (1 - 0.75) = 0.142 \frac{m}{s} \]

\[ \bar{V}_{\text{hot}} = \bar{V}_{\text{mixer}} \left( 1 - \frac{\bar{V}_{\text{cold}}}{\bar{V}_{\text{mixer}}} \right) = 0.569 \frac{m}{s} (1 - 0.25) = 0.427 \frac{m}{s} \]

**New water temp with heat transfer**

\[ \frac{dE_{\text{sys}}}{dt} = Q - W + \sum_{in} \dot{m}_{in} (h + ke + pe)_{in} - \sum_{out} \dot{m}_{out} (h + ke + pe)_{out} \]

\[ \dot{m}_{\text{mixer}} h_{\text{mixer}} = Q + \dot{m}_{\text{hot}} h_{\text{hot}} + \dot{m}_{\text{cold}} h_{\text{cold}} \]

\[ h_{\text{mixer}} = \frac{Q}{\dot{m}_{\text{mixer}}} + \frac{\dot{m}_{\text{hot}}}{\dot{m}_{\text{mixer}}} h_{\text{hot}} + \frac{\dot{m}_{\text{cold}}}{\dot{m}_{\text{mixer}}} h_{\text{cold}} \]

\[ = \frac{-100W \times \left( \frac{1kW}{1000W} \right)}{1000 \frac{kg}{m^2} \left( 0.569 \frac{m}{s} \right) (0.000281m^2)} + 0.75 \times \frac{231kJ}{kg} + 0.25 \times 63.5 \frac{kJ}{kg} \]

\[ = 189 \frac{kJ}{kg} \]

From the tables, Mixer Temperature is about 45 C.
Find: From the example above, you are to compute the flow rate (litres/min) and power (transform to kW) for oil being pumped at 900 rpm while supplying 10 bar pressure. Also compute the pump efficiency, defined as the increase in oil energy divided by the required power.

Given: for oil being pumped at 900 rpm while supplying 10 bar pressure

EFD: System = oil in pump

Assumptions:
steady state, steady flow
1-D uniform flow, uniform state
neglect ke and pe
adiabatic pump
incompressible fluid

Basic Equation:

\[
\frac{dm_{sys}}{dt} = \sum_{in} m_{in} - \sum_{out} m_{out}
\]

\[
\frac{dE_{sys}}{dt} = Q - W + \sum_{in} m_{in} (h + ke + pe)_{in} - \sum_{out} m_{out} (h + ke + pe)_{out}
\]

Solution:
From the oil graph, you can read that at a differential pressure of 10 bar and an rpm of 900 the required power is 0.9 HP and the flow rate was about 16 LPM.

From 100 cP Fluid (Oil) graph:

\[
\text{flow rate} = 16 \, \text{liters/min}
\]

\[
\text{Required Power} = 0.9 \, \text{hp}
\]

Convert power from hp to kW

\[
\text{Required Power} = 0.9 \, \text{hp} \times \frac{1 \, \text{kW}}{1.34 \, \text{hp}} = 0.672 \, \text{kW}
\]

\[
\frac{dm_{\text{sys}}}{dt} = \sum_{\text{in}} m_{\text{in}} - \sum_{\text{out}} m_{\text{out}}
\]

\[
\frac{dE_{\text{sys}}}{dt} = Q - W + \sum_{\text{in}} m_{\text{in}} (h + ke + pe)_{\text{in}} - \sum_{\text{out}} m_{\text{out}} (h + ke + pe)_{\text{out}}
\]

\[
m (h_{\text{out}} - h_{\text{in}}) = Q - W
\]

Energy Balance Review and Derivation of why \( \dot{W} = -\dot{m}(vdP) = VdP \)

\[dU = dQ - dW \quad \text{(for any process, neglecting DKE and DPE)}\]

\[dU = dQ - pdV \quad \text{(for any quasi-static process, no DKE or DPE)}\]
\[ H = U + PV \] therefore \[ dH = dU + PdV + VdP \] (remember to use calculus product rule on \( d(PV) \))

so \[ dH = dQ - dW + pdV + Vdp \] (any process)

\[ dH = dQ + Vdp \] (for any quasi-static process)

Take time derivative and express \( V \) in terms of \( v \) and \( m \)

\[
\frac{dH}{dt} = \dot{Q} + m (vdP)
\]

\[
\dot{m}(h_{out} - h_{in}) = \dot{Q} + \dot{m} (vdP)
\]

\[
\dot{m}v = \dot{V}
\]

so \( W = -\dot{V}dP \)

Increase in oil energy = \( \dot{m}(h_{out} - h_{in}) = -W = -(\dot{V}dP) = \dot{V}\Delta P = \)

\[
\left(16 \frac{1}{m}\right) \left(\frac{1\text{min}}{60 \text{ sec}}\right) \left(\frac{m^3}{1000 \text{ l}}\right) \left(10\text{bar}\right) \left(\frac{100\text{kPa}}{\text{bar}}\right) = 0.267 \text{ kW}
\]

Pump efficiency = \[
\frac{\text{increase in oil energy}}{\text{Required Power}} = \frac{0.267\text{kW}}{0.672\text{ kW}} = 0.397 = 39.7%\]
Find: new air (and water) temperature

EFD:

Assumption:

Steady State, steady flow
no work done by duct
adiabatic duct
neglect difference in KE and PE
water and air exit at the same temperature

Basic Equation:

\[
\frac{dm_{sys}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}
\]

\[
\frac{dE_{sys}}{dt} = \dot{Q} - \dot{W} + \sum_{in} \dot{m}_{in} (h + ke + pe)_{in} - \sum_{out} \dot{m}_{out} (h + ke + pe)_{out}
\]

Solution:

\[
\frac{dm_{sys}}{dt} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}
\]

\[
\frac{dE_{sys}}{dt} = \dot{Q} - \dot{W} + \sum_{in} \dot{m}_{in} (h + ke + pe)_{in} - \sum_{out} \dot{m}_{out} (h + ke + pe)_{out}
\]

\[
\dot{m}_{air, out} h_{air, out} + \dot{m}_{water, out} h_{water, out} = \dot{m}_{air, in} h_{air, in} + \dot{m}_{water, in} h_{water, in}
\]

\[
h_{air, in} (\text{air, 35 C (308K)}) = 308.2 \frac{kJ}{kg}
\]

\[
h_{water, in} (\text{sat liq, 20 C}) = 83.914 \frac{kJ}{kg}
\]
\[ h_{\text{air, out}} (\text{air, 10 C (283K)}) = 283.1 \frac{kJ}{kg} \]

\[ h_{\text{water, out}} (\text{sat vap, 10 C}) = 2519.2 \frac{kJ}{kg} \]

\[
\frac{1}{s} kg \left( 283.1 \frac{kJ}{kg} \right) + 0.01 \frac{kg}{s} \left( 2519.2 \frac{kJ}{kg} \right) = 1 \frac{kg}{s} \left( 308.2 \frac{kJ}{kg} \right) + 0.01 \frac{kg}{s} \left( 83.914 \frac{kJ}{kg} \right)
\]

\[ 308.29 \text{ kJ} = 309.03914 \text{ kJ} \]

These numbers differ by less than 1%. Final air and water temperature is about 10 C.