HEAT AND MASS TRANSFER
Area Exam
EQUATION SHEET

Conservation Laws

Control Volume Energy Balance: $\dot{E}_{in} - \dot{E}_{out} - \dot{E}_{gen} = \Delta\dot{E}_{st}$

$$\Delta\dot{E}_{st} = mC_p\frac{dT}{dt} \text{ and } \dot{E}_{gen} = q'''V$$

Surface Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$

Conduction

Fourier's Law: $q''_{cond,x} = -k\frac{\partial T}{\partial x}$; $\mathrm{q_{cond,x}} = \mathrm{q''_{cond,x}\,A_c}$

Heat Flux Vector: $\overrightarrow{q''} = q''_x\vec{i} + q''_y\vec{j} + q''_z\vec{k} = -k\left[\frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}\right]$

Heat Diffusion Equation:

Rectangular Coordinates: $\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + q''' = \rho C_p\frac{\partial T}{\partial t}$

Cylindrical Coordinates: $\frac{1}{r}\frac{\partial}{\partial r}\left(k_r r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k_\phi\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right) + q''' = \rho C_p\frac{\partial T}{\partial t}$

Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(k_r r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k_\phi\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k_\theta\sin\theta\frac{\partial T}{\partial \theta}\right) + q''' = \rho C_p\frac{\partial T}{\partial t}$$

Thermal Diffusivity: $\alpha = \frac{k}{\rho C_p}$

Thermal Resistance

Conduction Resistance:

Plane Wall: $R_{cond} = \frac{L}{kA_c}$ **Cylinder:** $R_{cond} = \frac{ln(r_o/r_i)}{2\pi Lk}$

Sphere: $R_{cond} = \frac{(1/r_i)-(1/r_o)}{4\pi k}$

Convection Resistance: $R_{conv} = \frac{1}{hA_s}$

Radiation Resistance: $R_{rad} = \frac{1}{h_r A_s}$ where $h_r = \epsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$

Resistance of Surface with Convection & Radiation: $\frac{1}{R_{tot}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$

Two-dimensional Conduction Resistance: $R_{cond,2D} = \frac{1}{Sk}$

Extended Surfaces (Fins)

Fin Effectiveness: $\epsilon_f = \frac{q_f}{hA_{c,b}\theta_b}$

Single Fin Efficiency: $\eta_f = \frac{q_f}{hA_f\theta_b}$

Fin Array Efficiency: $\eta_o = \frac{q_{total}}{hA_{total}\theta_b} = 1 - \frac{NA_f}{A_{total}}(1 - \eta_f)$

Single Fin Resistance: $R_{cond,fin} = \frac{1}{\eta_f hA_f}$

Fin Array Resistance: $R_{cond,array} = \frac{1}{\eta_o hA_{total}}$

Transient Conduction

Lumped Capacitance Analysis:

Biot Number: $\boldsymbol{B}i = \frac{R_{cond}}{R_{conv}} = \frac{hL_c}{k_{solid}}$, $L_c = \frac{V}{A_s}$

Fourier Number: $Fo = \frac{\alpha t}{L_c^2}$

Time Constant: $\tau_t = \frac{\rho VC_p}{hA_s} = R_tC_t$

Temperature: $\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = exp\left(-\frac{t}{\tau_t}\right) = exp\left[-\left(\frac{hL_c}{k}\right)\left(\frac{\alpha\, t}{L_c^2}\right)\right] = exp[-(Bi)(Fo)]$

Heat Rate: $\frac{Q}{Q_o} = \left[1 - exp\left(-\frac{t}{\tau_t}\right)\right]$

$$Q_o = mC_p(T_i - T_\infty) = \rho VC_p(T_i - T_\infty)$$

Dimensionless Parameters

Reynolds Number: $Re = \frac{\rho V_\infty L_c}{\mu} = \frac{V_\infty L_c}{\nu}$

Prandtl Number: $Pr = \frac{\nu}{\alpha}$

Nusselt Number: $Nu = \frac{hL_c}{k_f}$

Schmidt Number: $Sc = \frac{\nu}{D_{AB}}$

Lewis Number: $Le = \frac{\alpha}{D_{AB}}$

Sherwood Number: $Sh = \frac{h_m L_c}{D_{AB}}$

Grashoff Number: $Gr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$, $\beta = 1/T$ for ideal gas

Rayleigh Number: $\boldsymbol{Ra} = GrPr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha}$

Convection

Newton's Law of Cooling: $q''_{conv} = h(T_s - T_\infty)$; $q_{conv} = q''_{conv} A_s$

Average Heat Transfer Coefficient: $\bar{h} = \frac{1}{A_s}\int_{As} h\, dA_s$

Boundary Layer Thickness: $\frac{\delta}{\delta_t} \approx Pr^n$ where n =1/3

Turbulence:

External Flat-Plate Flow: $Re_{x,c} \geq 5x10^5$

Internal Flow: $Re_D \geq 2{,}300$

Internal Flow:

Mass Flow Rate: $\dot{m} = \rho u_m A_c$

Mean (Bulk) Velocity: $u_m = \frac{1}{\rho A_c}\int_{A_c} \rho u\, dA_c$

Mean (Bulk) Temperature: $T_m = \frac{1}{\dot{m}C_p}\int_{A_c} \rho u\, C_p T dA_c$

Heat Transfer Rate: $q''_{conv} = h(T_s - T_m)$

Constant Heat Flux: $T_m(x) = T_{m,i} + \frac{q''_{conv}P}{\dot{m}C_p}x$

Constant Surface Temperature: $\frac{T_s - T_{m,x}}{T_s - T_{m,i}} = exp\left(-\frac{P\bar{h}}{\dot{m}C_p}x\right)$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,x}}{T_s - T_{m,i}} = exp\left(-\frac{PL\bar{h}}{\dot{m}C_p}\right)$$

$$\Delta T_{LMTD} = \frac{\Delta T_o - \Delta T_i}{ln\left(\frac{\Delta T_o}{\Delta T_i}\right)}$$

$$q_{conv} = \bar{h}A_s\Delta T_{LMTD} = \dot{m}C_p\left(T_{m,o} - T_{m,i}\right)$$

Entry Length (Laminar):

Hydrodynamic: $\frac{x_{fd,h}}{D} = 0.05Re_D$

Thermal: $\frac{x_{fd,T}}{D} = 0.05Re_D Pr$

Entry Length (Turbulent): $\frac{x_{fd,all}}{D} > 10$

Mass Transfer

Mass Transfer: $n''_A = h_m(\rho_{A,s} - \rho_{A,\infty})$; $q''_{evap} = n''_A h_{fg}$

Average Mass Transfer Coefficient: $\bar{h} = \frac{1}{A_s}\int_{As} h_m \, dA_s$

Boundary Layer Thickness: $\frac{\delta}{\delta_c} \approx Sc^n$ **;** $\frac{\delta_t}{\delta_c} \approx Le^n$ where n =1/3

Heat and Mass Analogy: $\frac{Nu}{Sh} = \frac{Pr^n}{Sc^n}$ **;** $\frac{h}{h_m} = \frac{k}{D_{AB}Le^n} = \rho C_p Le^{1-n}$ were n=1/3

Heat Exchangers

Heat Transfer Rate: $q = q_h = C_h(T_{h,i} - T_{h,o}) = q_c = C_c(T_{c,i} - T_{c,o}) = UA\Delta T_{LMTD}$

Overall Heat Transfer Coefficient: $\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R''_{f,i}}{A_i} + \frac{ln(D_o/D_i)}{2\pi kL} + \frac{R''_{f,o}}{A_i} + \frac{1}{h_o A_o}$

Log-Mean Temperature Difference: $\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$

Parallel Flow:
$\Delta T_1 = T_{h,i} - T_{c,i}$
$\Delta T_2 = T_{h,o} - T_{c,o}$

Counter Flow:
$\Delta T_1 = T_{h,i} - T_{c,o}$
$\Delta T_2 = T_{h,o} - T_{c,i}$

Heat Capacity Rate: $C_h = \dot{m}_h c_{p,h}$ and $C_c = \dot{m}_h c_{p,h}$; $C_r = \frac{C_{min}}{C_{max}}$

Effectiveness: $\epsilon_{HEX} = \frac{q}{q_{max}} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})}$

Number of Transfer Units: $NTU = \frac{UA}{C_{min}}$

Radiation

Radiation Intensity: $I_{\lambda,e} = \frac{dq}{dA_1 \cos\theta d\omega d\lambda}$

Total Hemispherical Emissive Power:

$E = \int_0^\infty \int_0^{2\pi} \int_0^\pi I_{\lambda,e}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta d\phi d\lambda$

Diffuse Surface: $E = \pi I_e$ and $J = I_{e+r}$

Diffuse Irradiation: $G = \pi I_i$

Emissive Power: $E = \epsilon E_b = \epsilon \sigma T_s^4$

Radiosity: $J = \epsilon E_b + \rho G$

Net Radiation Heat Transfer: $q''_{rad} = E - \alpha G$; $q_{rad} = q''_{rad} A_s$

Stefan-Boltzmann Constant: $\sigma = 5.67x10^{-8}$ W/m^{2}K^4

Wien's Law: $\lambda_{max} T = 2898\ \mu m\ K$

Blackbody Power Fraction: $F_{0\to\lambda} = \frac{\int_0^\lambda E_{\lambda,b}(\lambda,T)d\lambda}{\int_0^\infty E_{\lambda,b}(\lambda,T)d\lambda}$

Table 12.2 will be provided as needed on exam.

Emissivity and Absorptivity

Spectral-Directional Emissivity: $\epsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) \equiv \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{BB}(\lambda,T)}$

Spectral-Directional Absorptivity: $\alpha_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,abs}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$

Total Emissivity (Diffuse Surface): $\epsilon(T) = \frac{\int_0^\infty \epsilon_\lambda(\lambda,T) E_{BB}(\lambda,T)d\lambda}{\int_0^\infty E_{BB}(\lambda,T)d\lambda}$

Total Absorptivity (Diffuse Surface): $\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)d\lambda}{\int_0^\infty G_\lambda(\lambda)d\lambda}$

View Factor: $F_{ij} \equiv \frac{\text{Total Radiosity leaving } A_i \text{ and arriving at } A_j}{\text{Total Radiosity leaving } A_i} = \frac{q_{ij}}{J_i A_i}$

View Factor Summation Rule: $\sum_{j=1}^N F_{ij} = 1$ where N is number of surfaces

View Factor Reciprocity Relation: $A_i F_{ij} = A_j F_{ji}$

View Factor Composite Receiver: $F_{i(j)} = \sum_{k=1}^n F_{ik}$

View Factor Composite Emitter: $F_{(j)i} = \frac{\sum_{k=1}^n A_k F_{ki}}{\sum_{k=1}^n A_k}$

Diffuse-Gray Surface Exchange:

Surface Radiation Resistance: $R_{rad,surf} = \frac{1-\epsilon}{A\epsilon}$

Geometrical Radiation Resistance: $R_{rad,geo} = \frac{1}{A_i F_{ij}}$