Stony Brook University

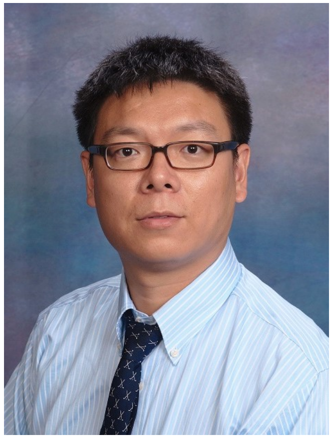
SUNY

2019 NSF EDSE Workshop

October 7-8, Purdue University

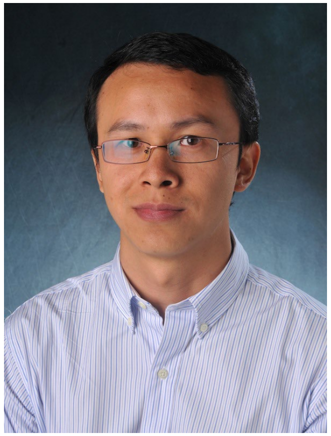
From Euclidian Space to Riemann Manifold: Topology Optimization of Conformal Structures Using Extended Level Set Methods

Award #1762287



Shikui Chen
(Stony Brook)

David Gu
(Stony Brook)

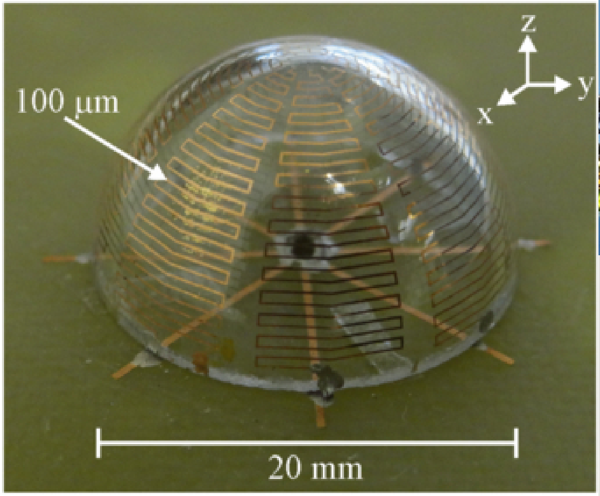


Jianliang Xiao
(UC Boulder)

Topology Optimization on Surfaces

Architecture

Retrieved from https://www.ioshmagazine.com/article/asbestos-found-sydney-opera-house-renovation



Flexible Electronics

Retrieved from https://iopscience.iop.org/article/10.1088/0960-1317/23/5/055010

Aircraft

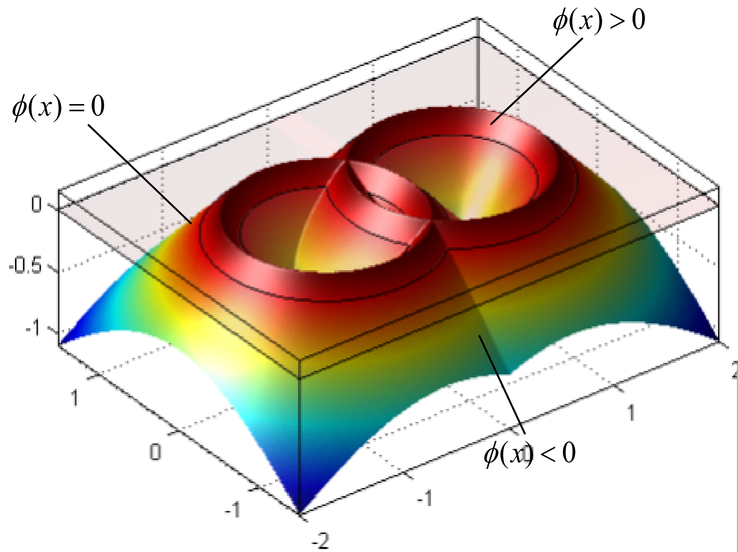
Retrieved from https://altairhyperworks.com/ResourceLibrary.aspx?discipline=Optimization&category=Case%20Studies&industry=All&product_service=All

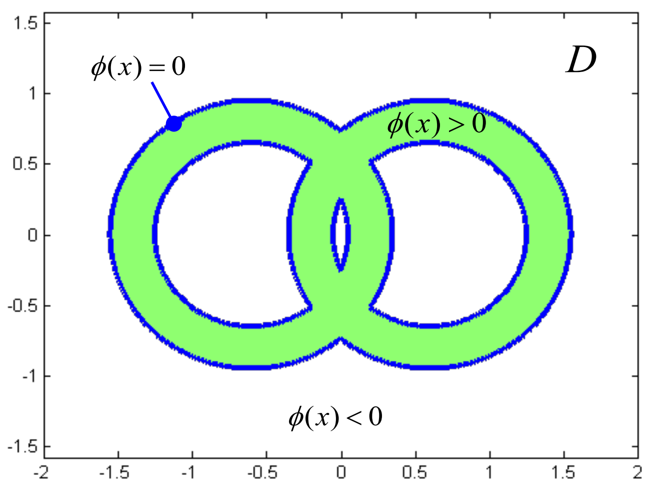
Daily Life

Elbo Chair- Generated in Project Dreamcatcher

Retrieved from: https://gallery.autodesk.com/fusion360/projects/elbo-chair--generated-in-project-dreamcatcher-made-with-fusion-360

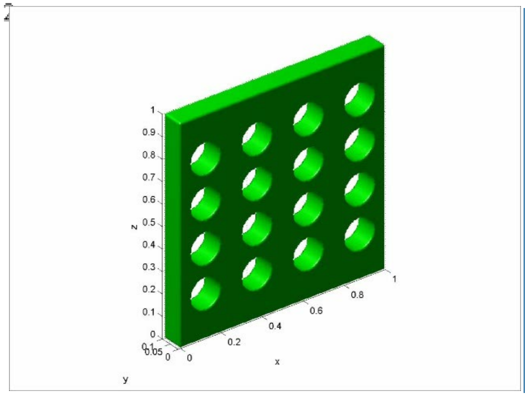
Dynamic Geometric Model: Level Set Methods





Hamilton-Jacobi Equation

$$\frac{\partial \phi}{\partial t} + V_n(x)\left|\nabla \phi\right| = 0$$



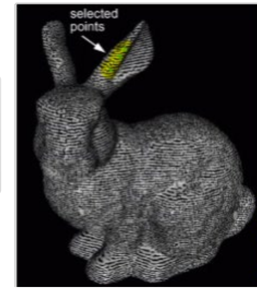
Only works in Euclidian Space

Introduction

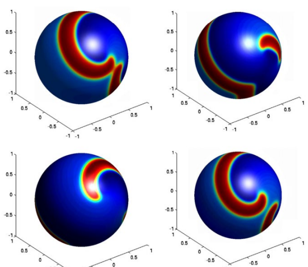
Different Methods of Solving PDEs on Manifold

Numerical Approximation

Discretized the manifold and approximate the PDEs locally



(UlrichClarenz et al., 2004)



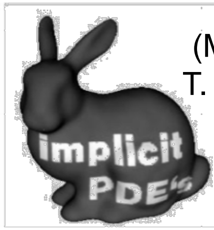
A spiral wave evolving on a sphere
(Ruuth, Steven J, and Barry Merriman, 2008)

Explicit Embedding method

Construct a space explicitly and solving the PDEs on the constructed space

Implicit Embedding method

Embedding the manifold onto one higher dimension by using level set representations, and the PDEs are solved in the Cartesian coordinate system



(M. Bertalm´ıo, L. T. Cheng, S. Osher and G. Sapiro, 2001)

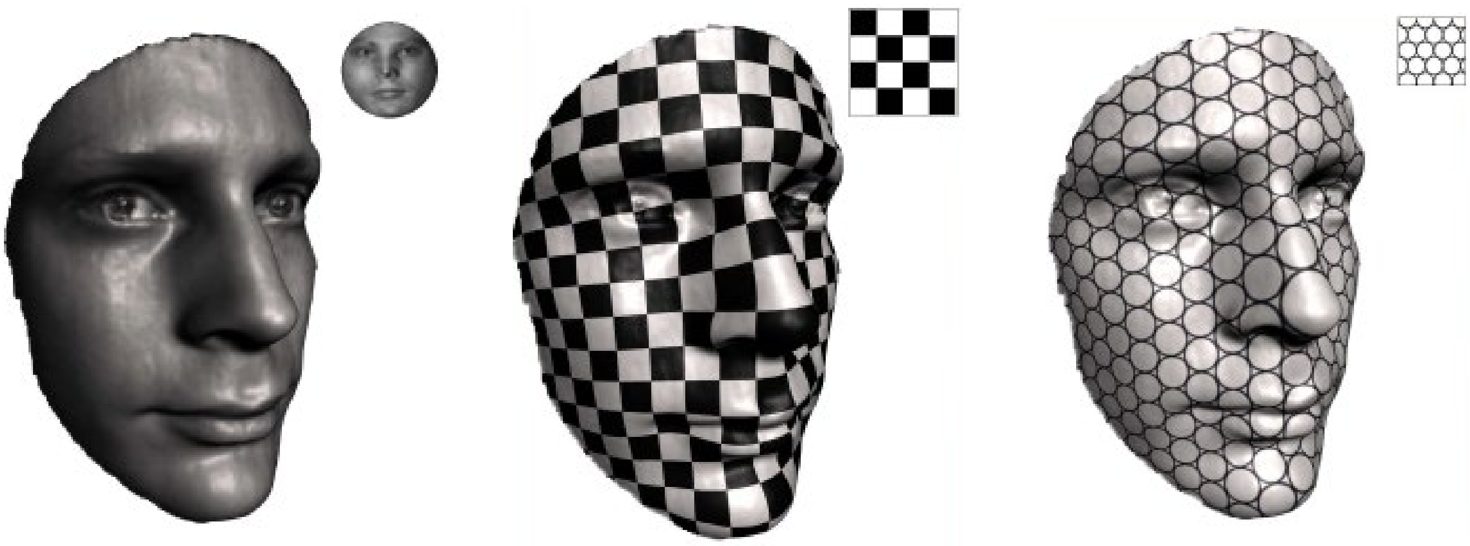
(X. Gu and Y. Wang et al., 2001)

Conformal mapping

Map the manifold conformally to a 2D domain, transfer the PDEs on a surface to its equivalent representation on the 2D domain.

Conformal Mapping Theory

Conformal Mapping: Angle-Preserving



Infinitesimal circles are mapped to Infinitesimal circles
(Source: Professor David Gu, Stony Brook University)

Conformal Mapping Theory

Conformal Mapping is a **local scaling transformation** governed by a scalar function $\lambda : S \rightarrow \mathbb{R}$ (surface to Euclidean space).

Advantages and Characteristics:

I. The **angles** are **preserved**. (maps infinitesimal circles to infinitesimal circles)

II. Transferring the **PDEs on a surface** to **an equivalent representation** on the **2D** domain with **the simplest form**. (the scalar function λ).

By solving the **Yamabe equation**:

$$\frac{d\lambda(t)}{dt} = 2(\overline{K} - K)g_{ij}(t)$$

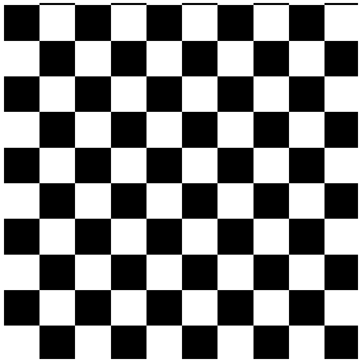
Conformal Mapping: φ

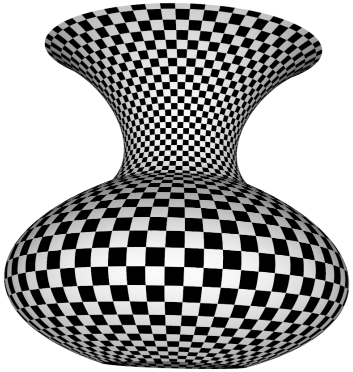
Angle Preserved!

Solving PDES on Manifold using Extended Level Set Method (X-LSM)

Merits:

1. Converting PDES on manifolds to PDES on Euclidean domains;
2. Using conformal mapping, the differential equation has the simplest form.





H-J equation

$$\frac{\partial \phi}{\partial t} + \frac{1}{\sqrt{\lambda}} V_n(x)|\nabla \phi| = 0 \quad \longleftarrow \quad \frac{\partial \phi}{\partial t} + V_n(x)|\nabla_{\mathrm{m}} \phi| = 0$$

Modification on the curvature flow

$$k = \frac{1}{\lambda} \nabla \cdot \left(\sqrt{\lambda} \frac{\nabla \phi}{|\nabla \phi|} \right) \quad \longleftarrow \quad k = \nabla_{\mathrm{m}} \cdot \vec{N} = \nabla_{\mathrm{m}} \cdot \frac{\nabla_{\mathrm{m}} \phi}{|\nabla_{\mathrm{m}} \phi|}$$

Numerical Experiment Implementation

Part 1. Topology Optimization of Load Carrying Conformal Structure

Structure topology optimization of a vase

Minimizing Compliance Problem of linear elastic material

$$\text{Min:} \quad J(u) = \int_\Omega SED \, d\Omega$$
$$\text{Subject to:} \; a(u, v, \phi) = L(v, \phi)$$
$$V(\Omega) = V^*$$

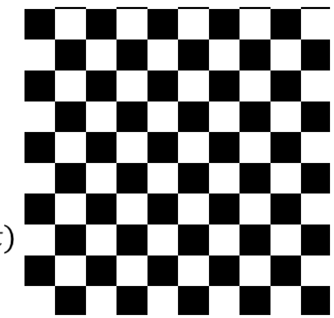
where SE-strain energy density

$$V(\Omega) = \int_\Omega d\Omega$$
$$a(u, v) = \int_\Omega \check{\varepsilon}(u) \mathbb{C} \check{\varepsilon}(v) d\Omega$$
$$L(v) = \int_\Omega f v d\Omega + \int_{\overline{\Omega}} t v d\overline{\Omega}$$

Conformal mapping

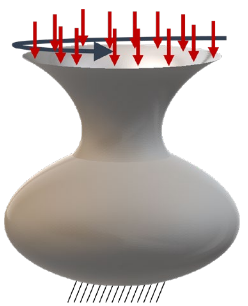
$$\frac{d\lambda(t)}{dt} = 2(\overline{K} - K) g_{ij}(t)$$

Yamabe equation



Local Shape Preserved!

Vase model

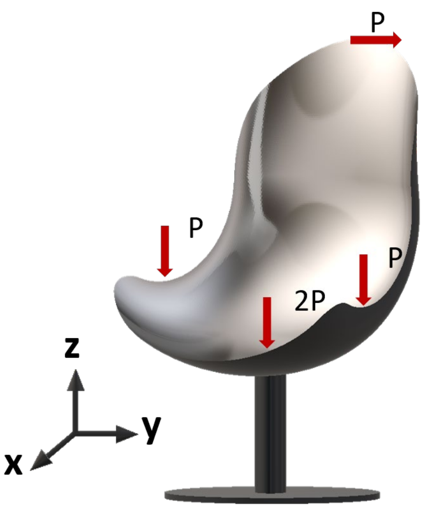


Boundary Conditions

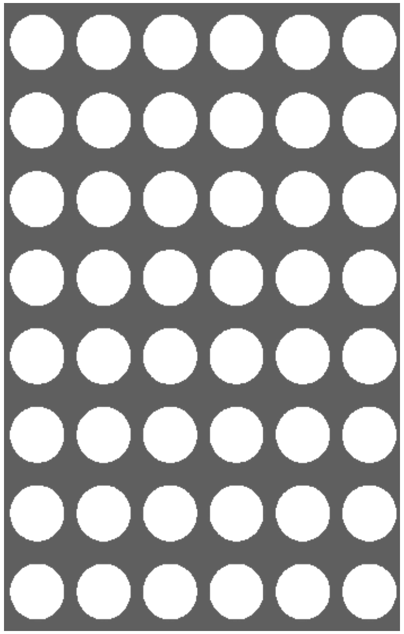
Design evolution in 3D

Design evolution in 2D

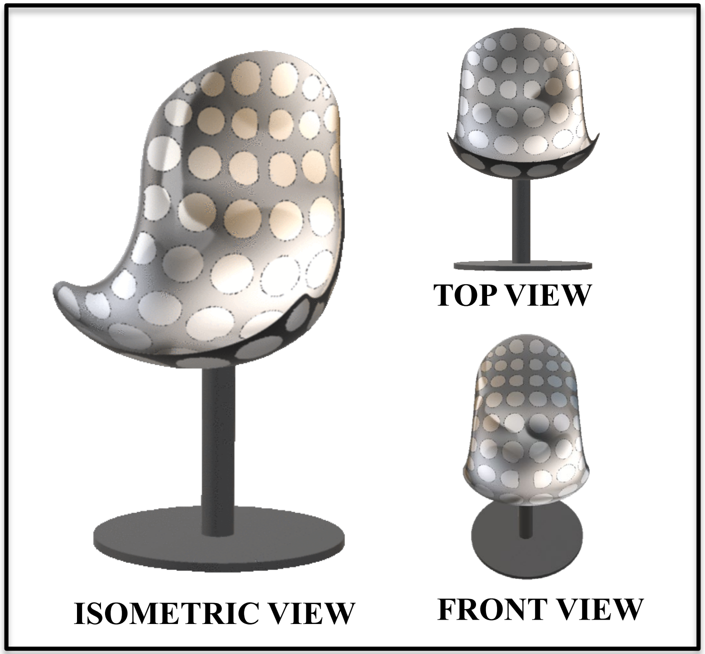
Structure topology optimization of chair surface



Boundary Conditions



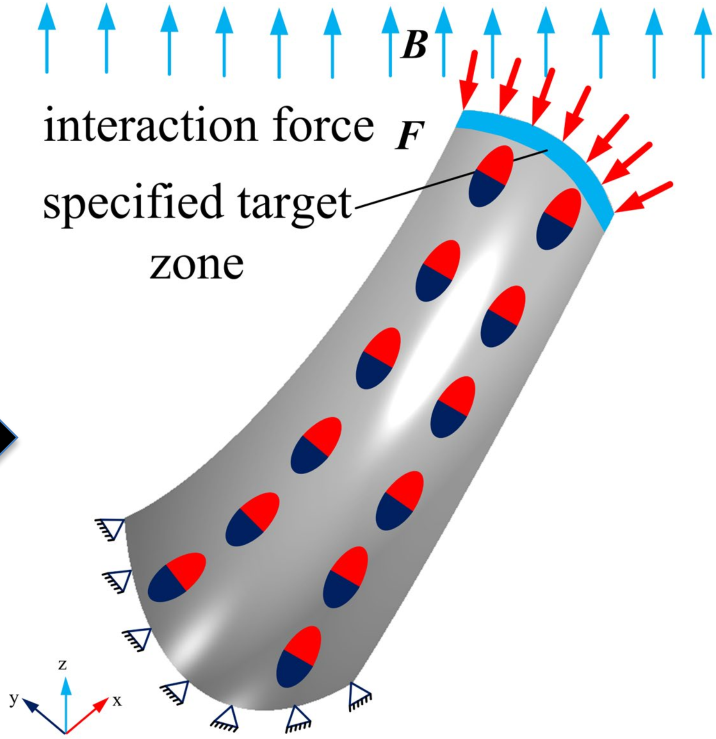
Design evolution on 2D



Design evolution on 3D

Example 3: Ferromagnetic Soft Robot (FerroSoRo)

Flytrap Plant



Boundary Conditions
of the petal like structure

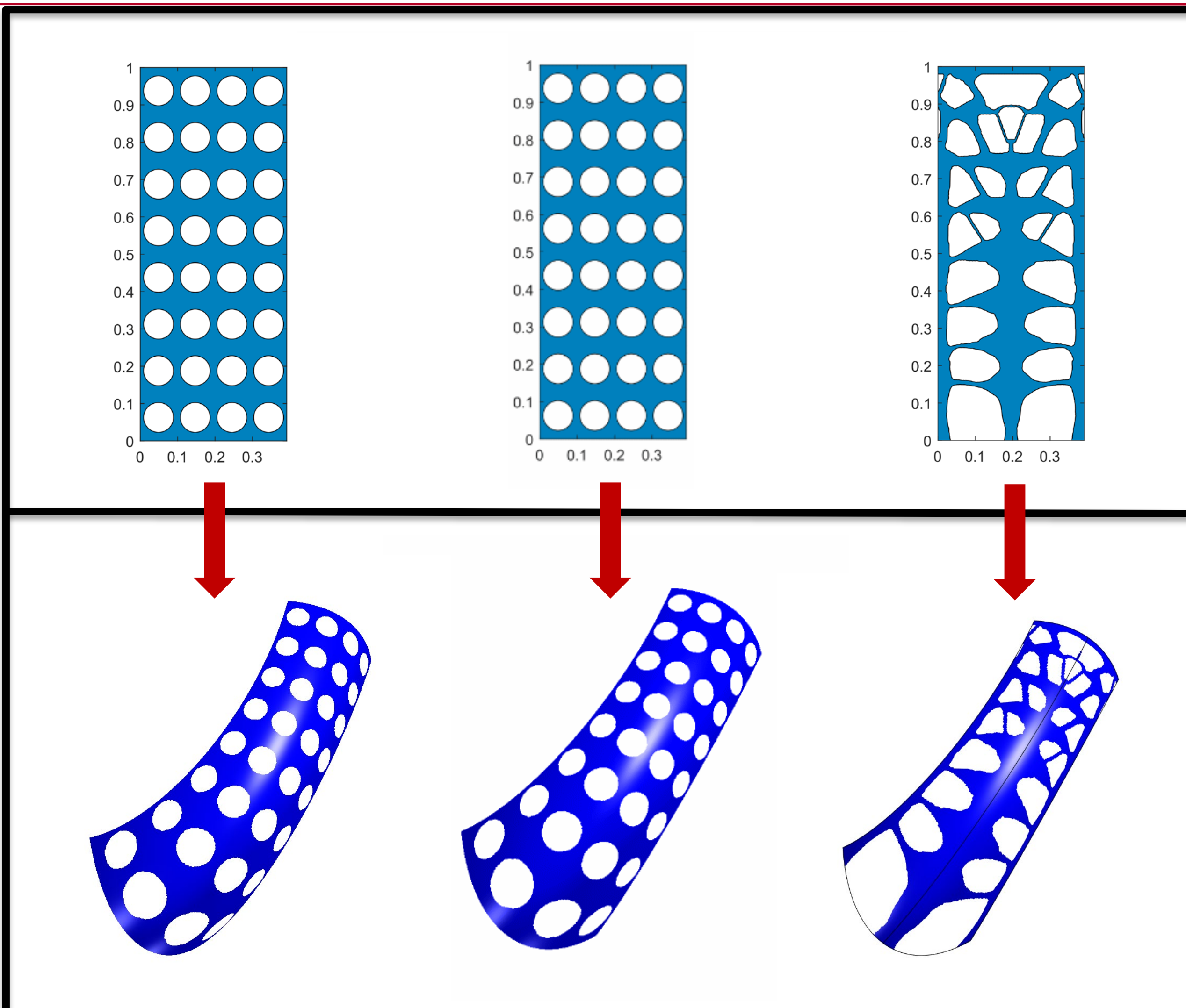
Design Evolution

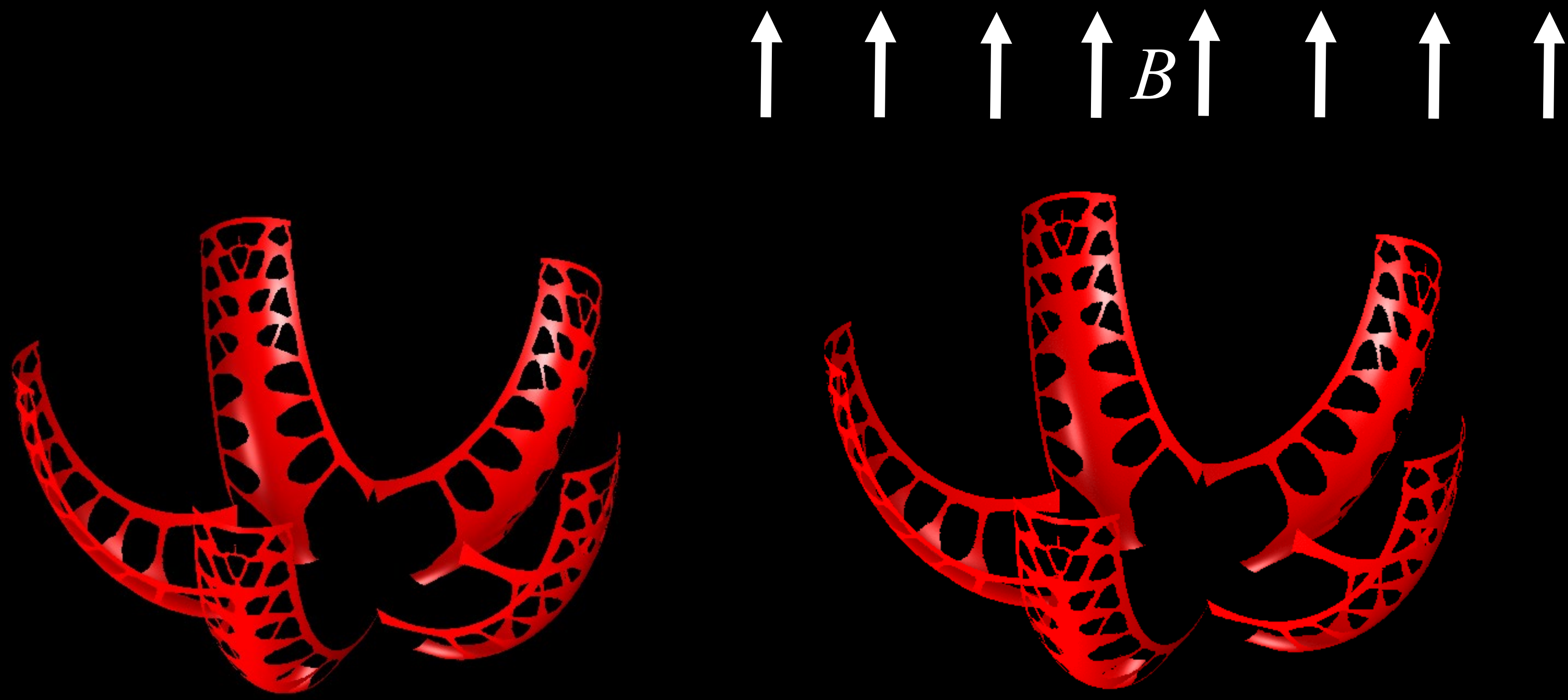
- Design evolution in 2D

  ❖ Level set function
  197 × 501 grids

- Design evolution in petal surface

  ❖ 63422 triangular elements
  ❖ Thickness 0.05 m





No Magnetic Field

Response to Magnetic Field

Flytrap FerroSoRo

Conclusions

Future Work

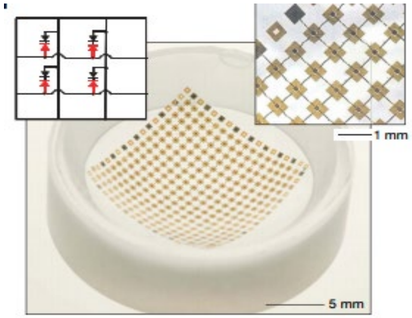
General

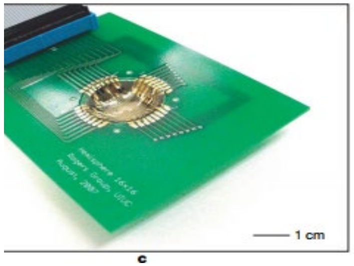
- arbitrary topologies.
- easily transformed to solve other TO problems

A

Optimize the deformation demanded surfaces

e.g.: conformal electronics, stretchable circuits

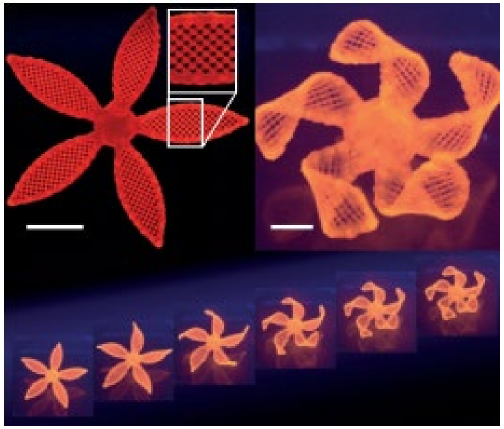




Ko, Heung Cho et al., Nature, 2008.

Design for 4D printing

e.g.: determine the locations of active material and passive material



Ryan and Jennifer et al., Nature, 2016

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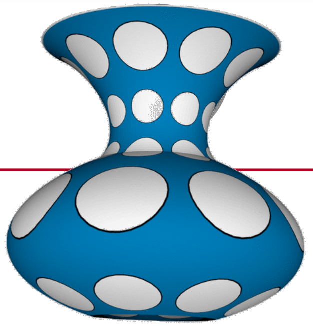

THANKS !

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