8.0 TWO-LEVEL FACTORIAL (2^K) DESIGNS

(UPDATED SPRING, 2005)

Surface finish of a part produced by a turning process is of interest (R_a value in μIN)

How is the surface finish affected, if at all, by the feed rate and the presence/absence of coolant?

Examine:

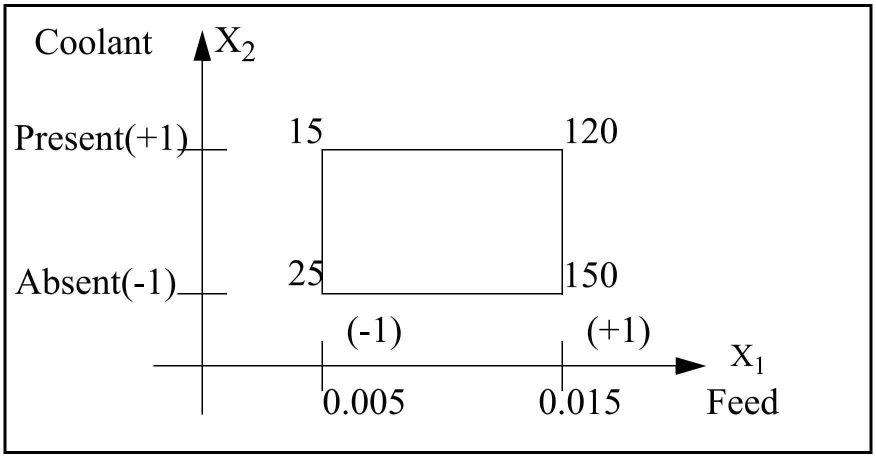
	Low Level (-1)	High Level (+1)	
Feed at 2 levels:	.005 ipr	.015 ipr	Cont. variable
Coolant at 2 levels:	Absent	Present	Disc. variable

Conduct tests for all combinations of the 2 variables: 2 x 2 = 4 tests, k=2, $2^k = 4$

	Actual values			Coded Levels	
Test	Feed	Coolant	SF	X_1	X_2
1	.005	Abs	25	-1	-1
2	.015	Abs	150	+1	-1
3	.005	Prs	15	-1	+1
4	.015	Prs	120	+1	+1

- Is feed important?
- Is coolant important?
- How do the important variables affect the response?

Graphical Representation of the Design



Effect of Feed (X_1)

Compare tests that differ only in the level of feed
At low coolant: 150 - 25 = 125
At High coolant: 120 - 15 = 105
On the average, the effect of increasing the feed from the lo to the hi levels is:

$$E_f = E_1 = \frac{125 + 105}{2} = 115\mu IN$$

Effect of Coolant (X_2)

At low feed: 15 - 25 = -10
At hi feed: 120 - 150 = -30
On the avg., the effect of increasing the coolant from the lo to hi level is:

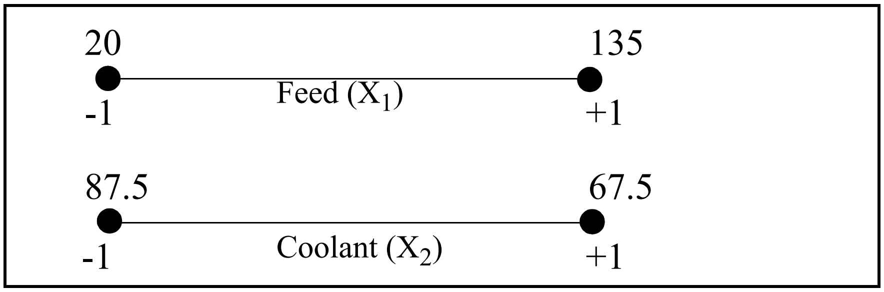
$$E_c = E_2 = \frac{(-10) + (-30)}{2} = -20\mu IN$$

Another Way of looking at it:
• Collapse design in coolant direction:

$$E_f = E_1 = 135 - 20 = 115$$

• Collapse design in feed direction:

$$E_c = E_2 = 67.5 - 87.5 = -20$$



Interaction Effect:

Characterizes the lack of additivity between the feed and coolant effects.

• Effect of feed depends on coolant level
• Effect of coolant depends on feed level

Coolant	Effect of Feed
(+)	105
(-)	125
	Diff = -20

Feed x Coolant

$$\text{Int} = \frac{-20}{2} = -10 = E_{12} = E_{fc}$$

Feed	Effect of Coolant
(+)	-30
(-)	-10
	Diff = -20

Feed x Coolant

$$= \frac{-20}{2} = -10 = E_{21} = E_{cf}$$

Note that $E_{12} = E_{21}$<-- This is always true.

A Quicker Method to Calculate Effects:

Design or Calculation Matrix

Test	Mean or I	X_1	X_2	X_1X_2	Y
1	+	-	-	+	25
2	+	+	-	-	150
3	+	-	+	-	15
4	+	+	+	+	120

• Feed Effect (Product of X_1 and y columns)

$$E_f = \frac{-25 + 150 - 15 + 120}{2} = 115 = E_1$$

of "+" signs in x_1 column

• Coolant Effect (Product of X_2 and y columns)

$$E_c = E_2 = \frac{-25 - 150 + 15 + 120}{2} = -20$$

• Feed x Coolant Interaction Effect

$$E_{fc} = E_{12} = \frac{+25 - 150 - 15 + 120}{2} = -10$$

• Average is

$$Avg = \frac{25 + 150 + 15 + 120}{4} = 77.5$$

So,

Avg. = 77.5 E_2 = -20

E_1 = 115 E_{12} = -10

For a 2 level factorial design, we are characterizing the response as:

$$y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2 + \varepsilon$$

ε is assumed to be NIID (0, σ_y^2) and characterizes the differences between the true functional relationship & the postulated one

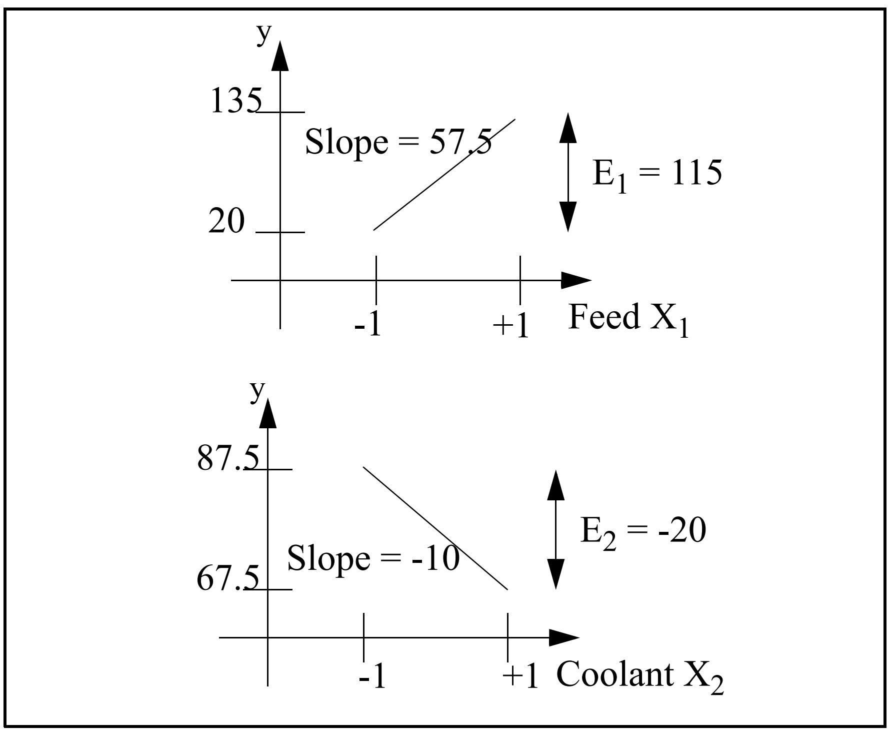
From the data/experiment we get

$$\hat{y} = \hat{b}_0 + \hat{b}_1 x_1 + \hat{b}_2 x_2 + \hat{b}_{12} x_1 x_2$$

$$\text{or } \hat{y} = avg + \frac{E_1}{2} x_1 + \frac{E_2}{2} x_2 + \frac{E_{12}}{2} x_1 x_2$$

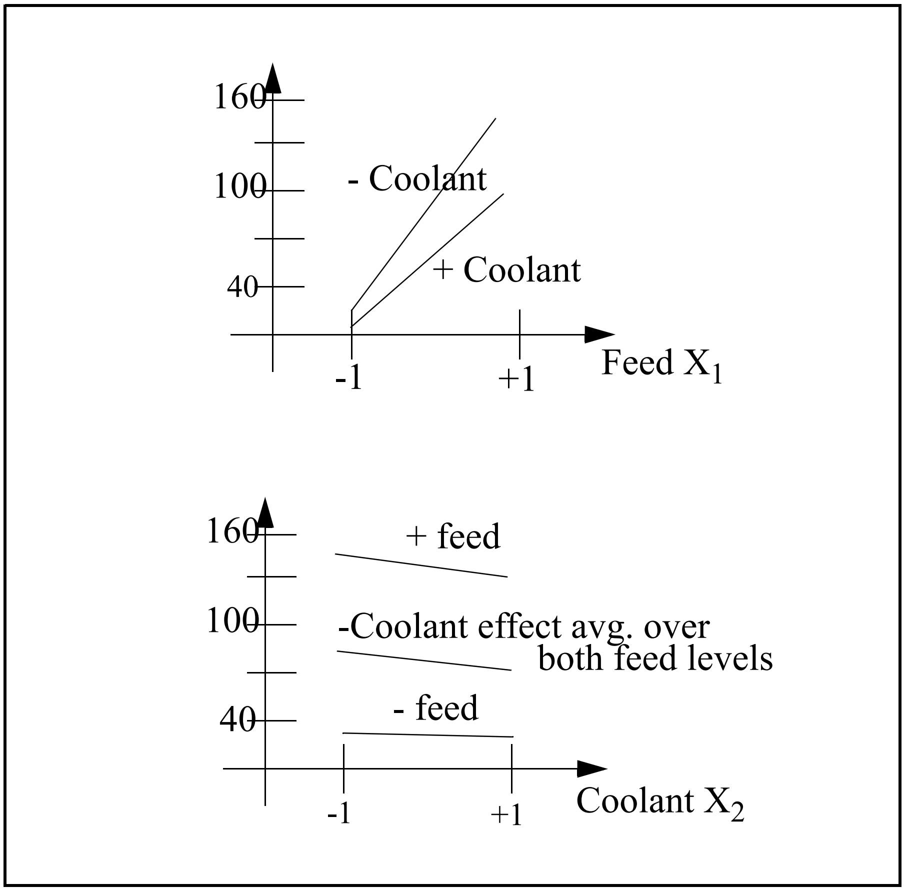
$\hat{b}_i = \frac{E_i}{2}$ is an estimate of b_i

Why divided by 2?



Two-Way Diagram - Helps to interpret 2 - factor interactions

$$b_{12} = \frac{E_{12}}{2} = \frac{-10}{2} = -5; b_0 = avg = 77.5$$



Difference between the individual slopes and the avg. slope characterized by interaction.

Model predictions:

Test	$\hat{y} = 77.5 + 57.5x_1 - 10x_2 - 5x_1x_2$	$\hat{y}$
1	$\hat{y} = 77.5 + 57.5\ (-1) - 10\ (-1) - 5\ (+1)$	25
2	$\hat{y} = 77.5 + 57.5\ (+1) - 10\ (-1) - 5\ (-1)$	150
3	$\hat{y} = 77.5 + 57.5\ (-1) - 10\ (+1) - 5\ (-1)$	15
4	$\hat{y} = 77.5 + 57.5\ (+1) - 10\ (+1) - 5\ (+1)$	120

Note that all the $\hat{y}$'s are = to y's when all the model terms are included.
Return to the Surf. Fin. Case Study Coolant

Calculated effect estimates:

Avg. = 77.5E$_2$ = -20

E$_1$ = 115E$_{12}$= -10

For a 2^2 factorial design, we describe the resp as:

$$y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2 + \varepsilon$$

Based on the Data, we obtain the fitted model response

$$\hat{y} = \hat{b}_0 + \hat{b}_1x_1 + \hat{b}_2x_2 + \hat{b}_{12}x_1x_2$$

Model to predict response $\hat{y} = 77.5 + 57.5x_1 - 10x_2 - 5x_1x_2$

Noted that with all terms in the prediction model, that $\hat{y}_i = y_i$

What is the predicted response when f =.010 and coolant = present?

The General Procedure to Study a Process

- Identify what you believe to be the important variables
- Fix as many factors in the environment as possible - reduce the level of noise - more sensitive comparisons
- Perform a 2-level factorial design - underlying model

$$\begin{aligned} y = {} & b_0 + b_1x_1 + b_2x_2 + b_3x_3 + \dots \\ & + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{14}x_1x_4 + \dots \\ & + b_{123}x_1x_2x_3 + b_{124}x_1x_2x_4 + \dots \\ & + b_{1234}x_1x_2x_3x_4 + \dots \end{aligned}$$

- Based on data develop fitted model.
- Check model adequacy - to be described later
- From experiment or model
  - identify/interpret important variables or interactions - more on this soon
  - Use model to optimize process, drive the response to desired value
- confirmatory tests in actual environment

Once the process has been centered at $\hat{y}$, we will see variation in the response

The effects we calculate attempt to answer the question: "How does the

μ change as a function of x_1, x_2, & x_3".