6.0 LATIN SQUARE DESIGN

(Updated Spring 2001)

So far:
- Independent t-test (one variable studied, 2 levels for variable)
- Paired t-test (one variable studied, 2 levels, blocking)
- One-Way ANOVA (one variable studied, > 2 levels)
- Two-Way ANOVA (2 variables blocking, > 2 levels for each variable)

3 Variables (4 levels each)

Source	degree of freedom
Mean	1
variable 1	3
variable 2	3
variable 3	3
Resid	54
Total	64 = 4 • 4 • 4

Most of the information going into the estimation of S_{res}^2

Can we get the "same information" with less tests?

YES!

Set up Square-Two-Way ANOVA

Introduce 3rd variable so as not to bias the results for the other two variables.

An example: Want to Study MPG

Variable	1st Level	2nd	3rd	4th
Car	1	2	3	4
Driver	I	II	III	IV
Fuel Additive	A	B	C	D

Car(j)

Driver (i)	1	2	3	4
I	A 21	B 26	D 20	C 25
II	D23	C 26	A 20	B 27
III	B 15	D 13	C 16	A 16
IV	C 17	A 15	B 20	D 20

Note that each Fuel Additive appears exactly once in each row and column.

$\underline{H_0}$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ &

$\mu_I = \mu_{II} = \mu_{III} = \mu_{IV}$ &

$\mu_A = \mu_B = \mu_C = \mu_D$

Model Form:

$$y_{ijk} = \eta + K_i + D_j + A_k + \varepsilon_{ijk}$$

Pred MPG only include significant terms $\hat{y}_{ijk} = \hat{\eta} + \hat{K}_i$

(If only car significant)

Means for each variable level

Cars	Drivers	Additives
1: =19	I:=23	A:=18
2: 20	II:24	B:22
3: 19	III:15	C:21
4: 22	IV:18	D:19

$\bar{\bar{y}} = 20$

$S_{cars} = (19-20)^2\, 4 + (20-20)^2\, 4 + (19-20)^2\, 4 + (22-20)^2\, 4 = 24$

Source	S. of S.	DF	MS	F
Average	6400	1	6400	
Cars	24	3	8	3.00
Drivers	216	3	72	27.00*
Additives	40	3	13.33	5.00*
Residuals	16	6	2.67	
Total	6696	16		

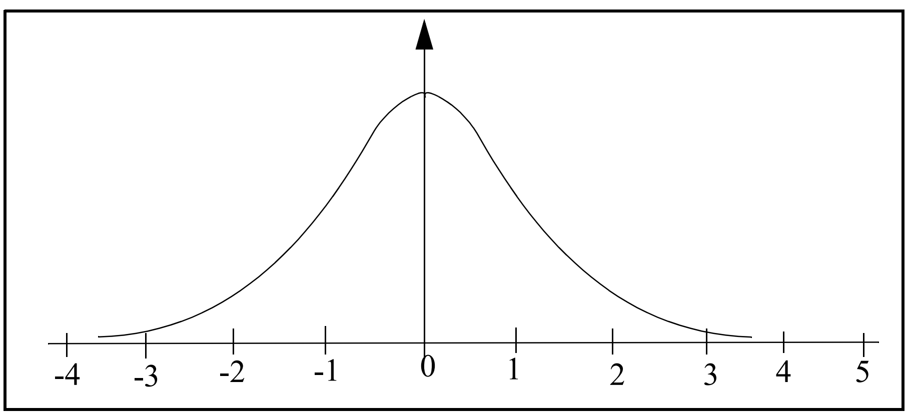
$F_{3,6,.95} = 4.76$ (* Significant F values)
Therefore, Drivers and Additives are significant.
Significant Differences between Driver & Additive Means.
No significant Differences between Cars.

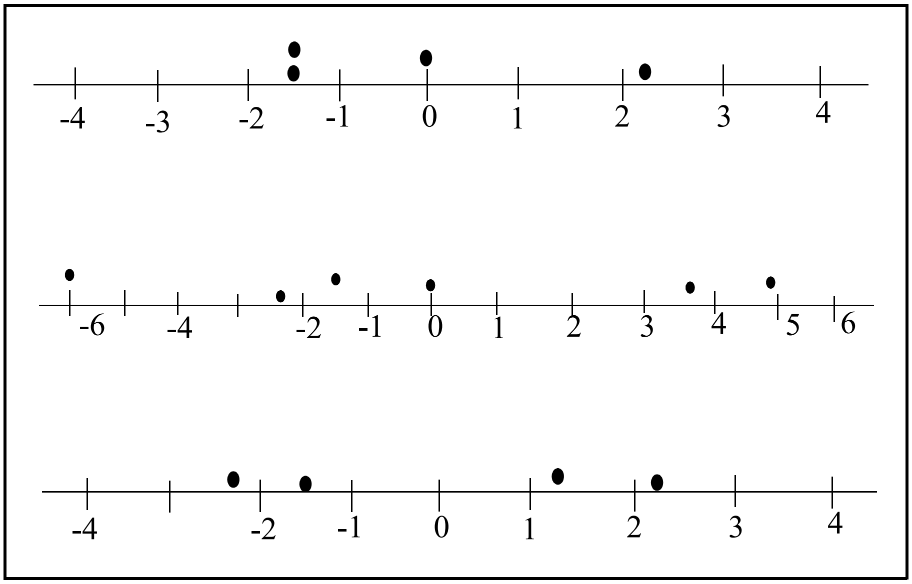
σ_y^2 estimated by $s_{res}^2 = 2.66 = s_y^2$

$s_y = 1.634$ $\quad \bar{\bar{y}}$ estimate of $\eta = 20$

Reference Dist$^{\underline{n}}$ for Car, Driver, & Treatment Additive Means.
Calculate t for each mean & compare to t_6
Each mean based on 4 observations

n	4	4	4	4
$s_{\bar{y}}$	0.817	0.817	0.817	0.817
$\bar{y}_{car}$	19	20	19	22
t_{car}	-1.22	0	-1.22	2.45
$\bar{y}_{Driver}$	23	24	15	18
t_{Driver}	3.67	4.90	-6.12	-2.45
$\bar{y}_{Additive}$	18	22	21	19
$t_{Additive}$	-2.45	2.45	+1.22	-1.22





$$\hat{y}=\begin{bmatrix}20&20&20&20\\20&20&20&20\\20&20&20&20\\20&20&20&20\end{bmatrix}+\begin{bmatrix}3&3&3&3\\4&4&4&4\\-5&-5&-5&-5\\-2&-2&-2&-2\end{bmatrix}+\begin{bmatrix}-2&2&-1&1\\-1&1&-2&2\\2&-1&1&-2\\1&-2&2&-1\end{bmatrix}$$

$$=\begin{bmatrix}21&25&22&24\\23&25&22&26\\17&14&16&13\\19&16&20&17\end{bmatrix}$$

Model Residuals, $e_{ijk} = y_{ijk} - \hat{y}_{ijk}$

$$e=\begin{bmatrix}21&26&20&25\\23&26&20&27\\15&13&16&16\\17&15&20&20\end{bmatrix}-\begin{bmatrix}21&25&22&24\\23&25&22&26\\17&14&16&13\\19&16&20&17\end{bmatrix}=\begin{bmatrix}0&1&-2&1\\0&1&-2&1\\-2&-1&0&3\\-2&-1&0&3\end{bmatrix}$$

<u>Residual Plots</u>

- Dot Diagram for all e's
- Dot diagram for each level of Car
- Dot diagram for each level of Driver
- Dot diagram for each level of Fuel Additive
- e's vs. $\hat{y}$

- e's vs. time
- For replicated data $(y - \overline{Y}_{cell})$ vs. $\overline{Y}_{cell}$
- **Graeco-Latin Sqs. (4 variables)**
- **Hyper Graeco-Latin Sqs. (5 variables)**