5.0 TWO-WAY ANOVA WITH NO REPLICATION

(Updated Spring, 2001)

The effect of different medical treatments on the survival times of animals given different poisons is under investigation. 12 animals were randomly assigned to the 3 Poisons and 4 Treatments. The survival time (in hours) is given below:

		Treatment				
Poisons		A	B	C	D	
	I	3.7	8.8	5.8	6.1	$\bar{y}_1 = 6.1$
	II	3.2	8.2	3.8	6.8	$\bar{y}_2 = 5.5$
	III	2.1	3.4	2.4	3.3	$\bar{y}_3 = 2.8$
		$\bar{y}_A = 3.0$	$\bar{y}_B = 6.8$	$\bar{y}_C = 4.0$	$\bar{y}_D = 5.4$	$\bar{\bar{y}} = 4.8$

H_0: $\mu_A = \mu_B = \mu_C = \mu_D$, and $\mu_1 = \mu_2 = \mu_3$

$S_{AVG} = 12(4.8)^2 = 276.48$ $\qquad\nu_{AVG} = 1$

$$S_{TOT} = \sum_{i=1}^{p} \sum_{i=1}^{t} y_{ij}^2 = 332.76 \qquad \nu_{TOT} = 12$$

$$S_T = p \sum_{j=1}^{t} (\bar{y}_j - \bar{\bar{y}})^2 = 24.72 \qquad \nu_T = 3 \qquad \text{(Note p=3)}$$

$$S_P = t \sum_{i=1}^{p} (\bar{y}_i - \bar{\bar{y}})^2 = 24.72 \qquad \nu_P = 2 \qquad \text{(Note t=4)}$$

Can't calculate S_{PE} (Pure Error) since there is no replication. $S_{res} = S_{TOT} - S_{AVG} - S_T - S_P = 6.84$ includes pure error and P & T interaction if present. Alternatively, we can calculate S_{res} from the square of the residuals for each cell.

$\nu_{Res} = \nu_{TOT} - \nu_{AVG} - \nu_T - \nu_P = 6$

$\nu_{Res} = (t-1)(p-1) = (3)(2) = 6$

Residuals: $y_{ij} + \bar{\bar{y}} - \bar{y}_i - \bar{y}_j$

$$S_{res} = \sum_{i=1}^{p} \sum_{j=1}^{t} (y_{ij} + \bar{\bar{y}} - \bar{y}_i - \bar{y}_j)^2$$

ANOVA Table

Source	SS	DF	MS	F_{calc}
Average	276.48	1	276.48	
Poisons	24.72	2	12.36	10.84
Treatment	24.72	3	8.24	7.23
Residual	6.84	6	1.14	
Total	332.76	12		

$F_{2,6,.95} = 5.14$ and $F_{3,6,.95} = 4.76$

Reject H_0:($\mu_1 = \mu_2 = \mu_3$). Poisons are significant.

Reject $H_{0:}$ (:$\mu_A = \mu_B = \mu_C = \mu_D$), Treatments are significant.

Model For Survival Time

$y_{ij} = \eta + P_i + T_j + \varepsilon_{ij}$

A model for this is:

$$\hat{y}_{ij} = \hat{\eta} + \hat{P}_i + \hat{T}_j$$

which includes only significant terms.

$\hat{\eta} = 4.8$,

$\hat{P}_i = (\bar{y}_i - \bar{\bar{y}}) = (1.3, 0.7, -2)$

$\hat{T}_j = (\bar{y}_j - \bar{\bar{y}}) = (-1.8, 2, -0.8, 0.6)$

		Treatment			
Poisons		A	B	C	D
	1	3.7 (4.3) -.6	8.8 (8.1) .7	5.8(5.3) .5	6.1 (6.7) -.6
	2	3.2(3.7) -.5	8.2(7.5) .7	3.8 (4.7) -.9	6.8(6.1) .7
	3	2.1 (1.0) 1.1	3.4 (4.8) -1.4	2.4(2.0) .4	3.3 (3.4) -.1

The e_{ij}'s should be IIDN $(0, \sigma_y^2)$

- Dot Diag. of All Residuals.
- Dot Diags. for each level of poison/treatment
- Residuals vs. $\hat{y}$
- Residuals vs. time
- Residuals centered about zero?
- Abnormalities present?
  - math/means. error
  - Learn from discrepant result
- Variability constant?
  - Across the levels for each factor
  - As a function of $\hat{y}$ (if% error constant, plot will show a funnel-like appearance)

- Curvilinear Tendencies. Suggests that purely linear model be not adequate to explain data. Perhaps interaction or quadratic terms are required.

$\mu_y = 4.8$ $\quad s_y^2 = 1.14$ $\quad s_y = 1.067708$

To confirm ANOVA results,

$\frac{\bar{y} - \mu}{\sigma}$	A	B	C	D
$\bar{y}_j$	3.0	6.8	4.0	5.4
n	3	3	3	3
$s_{\bar{y}}$	0.61644	.61644	.61644	.61644
t	-2.92	3.24	-1.30	.97

Poison Means	1	2	3
$\bar{y}_i$	6.1	5.5	2.8
n	4	4	4
$s_{\bar{y}}$	.5339	.5339	.5339
t	2.44	1.31	-3.75