14.5 ORTHOGONAL ARRAYS

(Updated Spring 2003)

Orthogonal Arrays (often referred to Taguchi Methods) are often employed in industrial experiments to study the effect of several control factors.

Popularized by G. Taguchi. Other Taguchi contributions include:
- Model of the Engineering Design Process
- Robust Design Principle
- Efforts to push quality upstream into the engineering design process

An orthogonal array is a type of experiment where the columns for the independent variables are “orthogonal” to one another.

Benefits:

1. Conclusions valid over the entire region spanned by the control factors and their settings
2. Large saving in the experimental effort
3. Analysis is easy

To define an orthogonal array, one must identify:
1. Number of factors to be studied
2. Levels for each factor
3. The specific 2-factor interactions to be estimated
4. The special difficulties that would be encountered in running the experiment

We know that with two-level full factorial experiments, we can estimate variable interactions. When two-level fractional factorial designs are used, we begin to confound our interactions, and often lose the ability to obtain unconfused estimates of main and interaction effects. We have also seen that if the generators are choosen carefully then knowledge of lower order interactions can be obtained under that assumption that higher order interactions are negligible.

Orthogonal arrays are highly fractionated factorial designs. The information they provide is a function of two things
- the nature of confounding
- assumptions about the physical system.

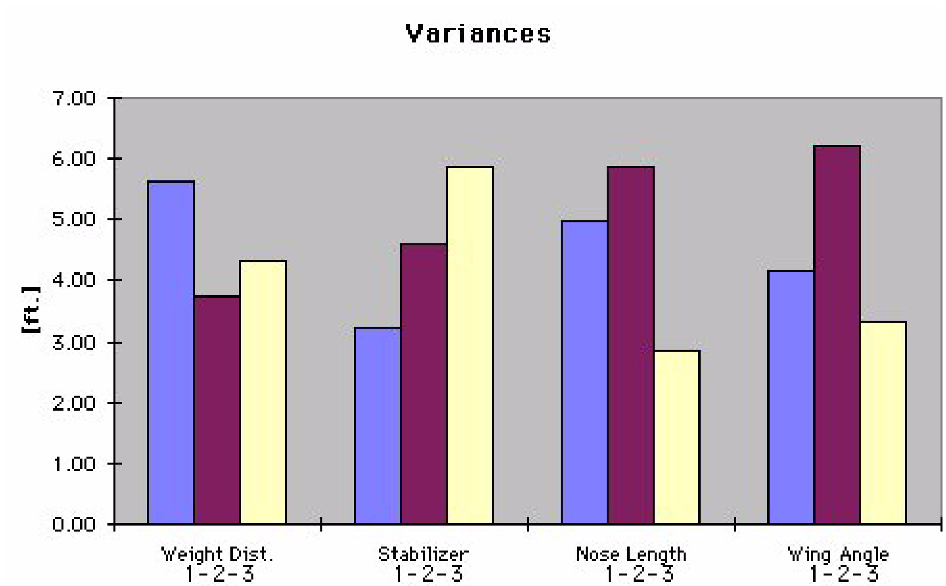
Before proceeding further with a closer look, let’s look at an example where orthogonal arrays have been employed.

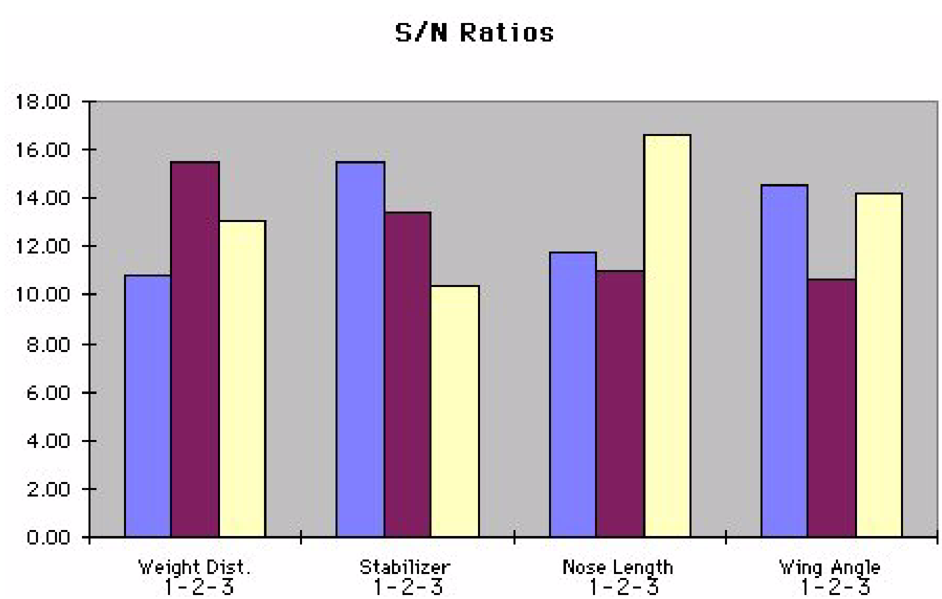
Example taken from students of Alice Agogino at UC-Berkeley

Airplane Taguchi Experiment

This experiment has 4 variables at 3 different settings. A full factorial experiment would require $3^4 = 81$ experiments. We conducted a Taguchi experiment with a $L_9(3^4)$ orthogonal array (9 tests, 4 variables, 3 levels). The experiment design is shown below.

Experi-ment #	Weight A	Stabilizer B	Noise C	Wing D
1	A1	B1	C1	D1
2	A1	B2	C2	D2
3	A1	B3	C3	D3
4	A2	B1	C2	D3
5	A2	B2	C3	D1
6	A2	B3	C1	D2
7	A3	B1	C3	D2
8	A3	B2	C1	D3
9	A3	B3	C2	D1





The students that performed this experiment suggest that the S/N ratio graph should be critically examined to select the desired variable levels.

Variable levels: A2 / B1 / C3 / D1

PROBLEM: Traditionally those that employ orthogonal arrays adopt a "pick the winner" approach. Don't take advantage of the full information provided by the experiment. No model, no residuals, no sequential approach to experimentation.

With Orthogonal Arrays / Taguchi Methods, the approach to design of experiments, the philosophy towards interaction effects is different. Unless an interaction effect is explicity identified a-priori and assigned to column in the orthogonal array, it is assumed to be negligible. Thus, unless assumed otherwise, interactions at all levels are assumed to be negligible.

Taguchi encourages users to select variables that do not interact with one another. A VERY INTERESTING IDEA! However, this idea may be hard to realize in practice.

Orthogonal arrays are frequently listed in handbooks/manuals. Users often extract the listed designs and use them blindly without thinking. NOTE OF CAUTION: choose orthogonal arrays wisely -- select the array with highest possible resolution so as to minimize the effect of any erroneous assumptions we make regarding effects being negligible.

Example

An arc welding experiment performed by national railway cooperation of Japan in 1959. Nine variables were studied as shown in the table:

Factor	Lower level	Higher Level
A - Kind of welding tool	J100	B17
B - Drying method of rods	no drying	one-day
C - Welded material	SS41	SB35
D - Thickness of welded material	8 mm	12 mm
E - Angle of welding Device	70 deg.	60 deg.
F - Opening of Welding Device	1.5 mm	3 mm
G - Current	150 A	130 A
H - Welding methods	weaving	single
I - Preheating	no preheating	150 deg. pre-heating

Nine variables and four two factor interactions were studied using a $L_{16}(2^{15})$ orthogonal array as shown

16 tests (2 levels for each main variable) -- 15 columns (factors) studied

Test	A	G	AG	H	AH	GH	B	D	E	F	I	e	e	C	AC
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
3	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
4	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-
5	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
6	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-
7	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-
8	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+
9	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
10	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-
11	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-
12	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+
13	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-
14	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+
15	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+
16	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-

The columns labeled “e” may be used for error estimation

Let’s take a hard look at this experiment using what we know about fractional factorial designs.

This may be written as 2^{9-5} fractional factorial as follows

Test	D 1	H 2	G 3	A 4	B 5	E 6	F 7	I 8	C 9
1	-	-	-	-	-	-	-	-	-
2	+	-	-	-	-	+	+	+	+
3	-	+	-	-	+	-	-	-	-
4	+	+	-	-	+	+	+	+	-
5	-	-	+	-	+	-	+	+	+
6	+	-	+	-	+	+	-	-	-
7	-	+	+	-	-	-	+	+	-
8	+	+	+	-	-	+	-	-	+
9	-	-	-	+	+	+	-	+	+
10	+	-	-	+	+	-	+	-	-
11	-	+	-	+	-	+	-	+	-
12	+	+	-	+	-	-	+	-	+
13	-	-	+	+	-	+	+	-	-
14	+	-	+	+	-	-	-	+	+
15	-	+	+	+	+	+	+	-	+
16	+	-	+	+	+	+	-	+	-

This design is a 2_{III}^{9-5} fractional factorial design with the following generators.

$I = 2345$, $I = -146$ $I = -137$ $I = 1348$ $I = -12349$

Defining relations

$I = 2345 = -146 = -137 = 1348 = -12349$
$= -12356 = -12457 = 1258 = -159 = 3467 = -368 = 2369 = -478 = 2479$
$= -289 = 2567 = -24568 = 4569 = -23578 = 3579 = -34589 = 1678$
$= -12679 = 124689 = 123789 = 12345678 = -135689 = 125689 = 145789$
$= -2346789 = -56789$

Assuming third and higher order interactions to be negligible the confounding pattern is as follows:

l_I est I(mean)	l_8 est 23 + 45 + 69
l_1 est 1 - 46 - 37 - 59	l_9 est 24 + 35 + 79
l_2 est 2 - 89	l_{10} est 34 + 25 + 18 + 67
l_3 est 3 - 17 - 68	l_{11} est -27 - 49 - 56
l_4 est 4 - 16 - 78	l_{12} est -26 -39 -57 (error)
l_5 est 12 + 58(error)	l_{13} est 8 - 36 - 47 - 29
l_6 est -7 + 13 + 48	l_{14} est 5 - 19
l_7 est -6 + 14 + 38	l_{15} est -9 + 15 +28

What happened to our standard procedure?? We should be using
$l_I\ l_1\ l_2\ l_3\ l_4\ l_{12}\ l_{13}\ l_{14}\ l_{23}\ l_{24}\ l_{34}\ l_{123}\ l_{124}\ l_{134}\ l_{234}\ l_{1234}$

Clearly the above design is a resolution III design.

It is not possible to have a 16-run two-level fractional factorial design with nine variables and resolution greater than III. However, we could have chosen better gereators. We can pick generators in such a way so that the confounding is better than what we obtain "by default".

As an alternative, consider a design with the following generators

I = 1235, I = 2346 I = 1347 I = 1248 I = 12349

The confounding relations are as follows

l_I est I(mean)	l_8 est 23 + 15 + 46 + 78
l_1 est 1 + 69	l_9 est 24 + 36 + 18 + 57
l_2 est 2 + 79	l_{10} est 34 + 26 + 17 + 58
l_3 est 3 + 89	l_{11} est 5 + 49
l_4 est 4 + 59	l_{12} est 8 + 39
l_5 est 12 + 35 + 48 + 67	l_{13} est 7 + 29
l_6 est 13 + 25 + 47 + 68	l_{14} est 6 + 19
l_7 est 14 + 37 + 28 + 56	l_{15} est 9 + 45 + 16 + 27 + 38

The main effects are still confounded with at least one two factor interaction. But 8 out of the 9 main effects are confounded with only one 2 factor interaction.

Another point to note is that all the 2 factor interactions (associated with the main effects) have variable 9 in them

Hence if we know that there is some variable that is least likely to be important -- or that does not interact with the other variables -- then we can assign it to be variable 9.

CONCLUSION:

- Idea of publishing a standard set of designs (orthogonal arrays is a great idea) -- Taguchi brought the field of DOE to the masses
- Select variables that don’t interact -- interesting!! Assuming that interactions do not exist doesn’t mean they aren’t there!
- As a precaution select design generators that give best design design resolution and that provide desirable confounding structure.
- Other??