13.0 BICYCLE EXAMPLE

(Updated Spring 2005)

Interested in time it takes to pedal up a hill. 7 variables of interest:

Variable	Low	High
1-Set	Up	Down
2-Dynamo	Off	On
3-Handlebars	Up	Down
4-Gear	Low	Medium
5-Raincoat	On	Off
6-Breakfast	Yes	No
7-Tires	Hard	Soft

Test	I	1	2	3	12	13	23	123
1	+	-	-	-	+	+	+	-
2	+	+	-	-	-	-	+	+
3	+	-	+	-	-	+	-	+
4	+	+	+	-	+	-	-	-
5	+	-	-	+	+	-	-	+
6	+	+	-	+	-	+	-	-
7	+	-	+	+	-	-	+	-
8	+	+	+	+	+	+	+	+

Introduce variables 4,5,6,&7 as follows:

4=12 I=124

5=13 => I=135 Generators

6=23 I=236

7=123 I=1237

Selected generator member of family: I= $\pm$ 124 = $\pm$ 135 = $\pm$ 236 = $\pm$ 1237

Defining relationship:

I=124=135=236=1237

2@ a time =2345=1346=347=1256=257=167

3@ a time =456=1457=2467=3567

4 @ a time =1234567 Resolution = III

Run the experiment.

Test	1	2	3	4	5	6	7	y
1	-	-	-	+	+	+	-	69
2	+	-	-	-	-	+	+	52
3	-	+	-	-	+	-	+	60
4	+	+	-	+	-	-	-	83
5	-	-	+	+	-	-	+	71
6	+	-	+	-	+	-	-	50
7	-	+	+	-	-	+	-	59
8	+	+	+	+	+	+	+	88

Use Base design calc. matrix to calc l_i's

l_1 = (-69 + 52- 60+83-71+50-59+88)/4 = 3.5
l_{123} = (-69+52+60-83+71-50-59+88)/4 = 2.5
l_I = 66.5 est I
l_1 = 3.5 est 1+24+35+67
l_2 = 12.0 est 2+14+36+57
l_3 = 1.0 est 3+15+26+47
l_{12} = 22.5 est 12+4+37+56
l_{13} = 0.5 est 13+5+27+46
l_{23} = 1.0 est 23+6+17+45
l_{123} = 2.5 est 34+25+16+7

- Could plot *l*'s on NPP to find important ones
- In this case, we know from past experiments that $\sigma_y = 3$

$$\sigma_e^2 = \sigma_{effect}^2 = \frac{4\sigma_y^2}{N}$$

therefore

$$\sigma_e = \left[\frac{4 \bullet 3^2}{8}\right]^{\frac{1}{2}} = \sqrt{4.5} \approx 2.1$$

It can be easily found out that l_2 and l_{12} are statistically significant with 95% confidence.

Simple conclusion: μ_2 & μ_4 are important

Could also be: μ_2 & μ_{12} important or μ_2 & μ_{14} important or...

To unconfound, sort out what is/is not important, let's run another 8 tests. Overlaying our knowledge on the experiment results suggests that probably 4 & 14 is important, 2nd experiment will unconfound 4 and its interactions.

Test	1	2	3	4	5	6	7	y
9	-	-	-	-	+	+	-	63
10	+	-	-	+	-	+	+	82
11	-	+	-	+	+	-	+	73
12	+	+	-	-	-	-	-	53
13	-	-	+	-	-	-	+	64
14	+	-	+	+	+	-	-	84
15	-	+	+	+	-	+	-	72
16	+	+	+	-	+	+	+	45

Signs of column 4 in this recipe matrix are opposite/flipped/ "folded" from those of 1st recipe matrix. All the other signs are the same.
Let's examine the recipe matrix to see what the generators are for this 2nd 2^{7-4} design. 4= -12, 5=13,6=23,7=123. So the defining relation for the 2nd design is:

$$I=-124=135=236=1237$$
$$=-2345=-1346=-347=1256=257=167$$
$$=-456=-1457=-2467=3567=-1234567$$

Therefore,

l_I' = 67.0 is the estimate of the effect of I

l_1' = -2.0 is the estimate of the effect of 1-24+35+67

l_2' = -12.5 is the estimate of the effect of 2-14+36+57

l_3' = -1.5 is the estimate of the effect of 3+15+26-47

l_{12}' = -21.5 is the estimate of the effect of 12-4+37+56

l_{13}' = -1.5 is the estimate of the effect of 13+5+27-46

l_{23}' = -3.0 is the estimate of the effect of 23+6+17-45

l_{123}' = -2.0 is the estimate of the effect of -34+25+16+7

$(l_I + l_I')/2$ = 66.75 est I	$(l_I - l_I')$=-0.5 est Higher Order Terms
$(l_1 + l_1')/2$ = 0.75 est 1+35+67	$(l_1 - l_1')/2$ = 2.75 est 24
$(l_2 + l_2')/2$ = -0.25 est 2+36+57	$(l_2 - l_2')$ /2 = 12.25est 14
$(l_3 + l_3')/2$ = -0.25 est 3+15+26	$(l_3 - l_3')/2$ = 1.25 est 47
$(l_{12} + l_{12}')/2$ = 0.5 est 12+37+56	$(l_{12} - l_{12}')/2$ = 22.0 est 4
$(l_{13} + l_{13}')/2$ =-0.5 est 13+5+27	$(l_{13} - l_{13}')/2$ = 1.0 est 46
$(l_{23} + l_{23}')/2$ = -1.0 est 23+6+17	$(l_{23} - l_{23}')/2$ = 2.0 est 45
$(l_{123} + l_{123}')/2$= 0.25 est 25+16+7	$(l_{123} - l_{123}')/2$ = 2.25 est 34

It can be seen that all interactions and main effects of 4 are unconfounded.

If $\sigma_y = 3$, $\sigma_{eff} = \left(\frac{4\sigma^2}{N}\right)^{\frac{1}{2}} = \left(\frac{4 \bullet 3^2}{16}\right)^{\frac{1}{2}} = 1.5$. So, average (I), 14, and 4 are important and estimated to be 66.75, 12.25, and 22.0 respectively.

Model for response is:

$$\hat{y} = 66.75 + \frac{22.0}{2}x_4 + \frac{12.25}{2}x_1x_4$$

Summary:

- First we conducted the "principal fraction" from the family of generators

I= $\pm$ 124 = $\pm$ 135 = $\pm$ 236 = $\pm$ 1237, i.e., we used the generators with all "+" signs:
4=12,5=13,6=23,7=123

- Second we conducted another 2^{7-4} design with generators 4=-12,5=13,6=23,7=123. We noted that the recipe matrix for this 2nd design was identical to that of our first design except that column 4 was folded. We saw that the results of this fold-over design when combined with those of the 1st design unconfounded the effects of 4 and all of its 2-factor interactions.

Another type of follow-up design - to clear up confounding left by a 1st design is the Mirror Image Design which flips all signs in the recipe matrix of the1st design.

Test	1	2	3	4	5	6	7
9	+	+	+	-	-	-	+
10	-	+	+	+	+	-	-
11	+	-	+	+	-	+	-
12	-	-	+	-	+	+	+
13	+	+	-	-	+	+	-
14	-	+	-	+	-	+	+
15	+	-	-	+	+	-	+
16	-	-	-	-	-	-	-

The mirror image design when combined with the 1st design will unconfound all the main effects. Let's examine the recipe matrix to see what the generators happen to be:

4=-12,5=13,6=23,7=123.

So defining relation is:

I=-124=-135=-236=1237
=2345=1346=347=1256=-257=-167
=-456=1457=2467=356
=-1234567

Therefore, assuming 3-factor interactions and higher are negligible,

<u>1st Design</u>	<u>2nd Design: Mirror Image Design</u>
l_I est I	l_I'' est I
l_1 est 1+24+35+67	l_1'' est 1-24-35-67
l_2 est 2+14+36+57	l_2'' est 2-14-36-57
l_3 est 3+15+26+47	l_3'' est 3-15-26-47
l_{12} est 12+4+37+56	l_{12}'' est 12-4+37+56
l_{13} est 13+5+27+46	l_{13}'' est 13-5+27+46
l_{23} est 23+6+17+45	l_{23}'' est 23-6+17+45
l_{123} est 34+25+16+7	l_{123}'' est -34-25-16+7

By combining the two designs, we can obtain

$\frac{1}{2}(l_I + l_I")$ est I	$(l_I - l_I")$ est Higher Order Eff.
$\frac{1}{2}(l_1 + l_1")$ est 1	$\frac{1}{2}(l_1 - l_1")$ est 24+35+67
$\frac{1}{2}(l_2 + l_2")$ est 2	$\frac{1}{2}(l_1 - l_1")$ est +14+36+57
$\frac{1}{2}(l_3 + l_3")$ est 3	$\frac{1}{2}(l_3 - l_3")$ est 15+26+47
$\frac{1}{2}(l_{12} + l_{12}")$ est 12+37+56	$\frac{1}{2}(l_{12} - l_{12}")$ est 4
$\frac{1}{2}(l_{13} + l_{13}")$ est 13+27+46	$\frac{1}{2}(l_{13} - l_{13}")$ est 5
$\frac{1}{2}(l_{23} + l_{23}")$ est 23+17+45	$\frac{1}{2}(l_{23} - l_{23}")$ est 6
$\frac{1}{2}(l_{123} + l_{123}")$ est 7	$\frac{1}{2}(l_{123} - l_{123}")$ est 34+25+16