

Lecture #33

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Facility Location Techniques

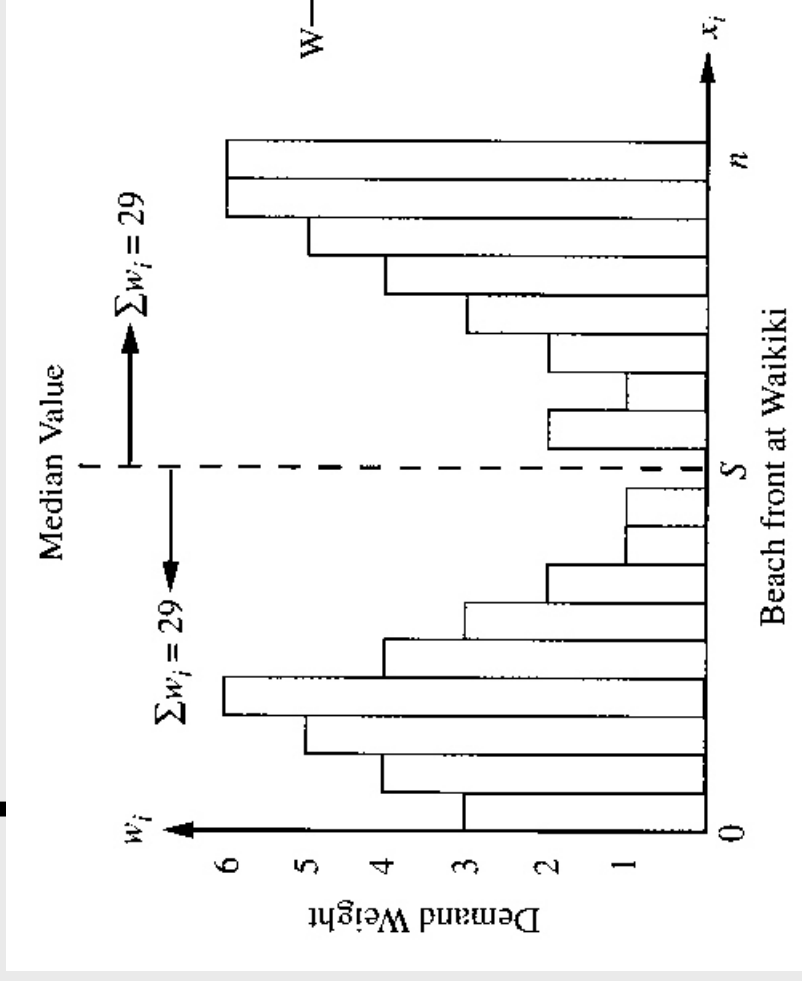
- ❖ **Locating a single facility on a line**
 - ❑ Consider the problem of locating a beach mat concession along Waikiki beach front.
 - ❑ You wish to find a location that would minimize the average walk to your concession from anywhere on the beach.
 - ❑ Further, suppose you have data showing the density of bathers along the beachfront, which is related to the size and location of hotels.

Example

$$Z = \sum_{i=0}^n |x_i - x_{loc}|$$

❖ The objective is

□ Minimize $Z = \sum_{i=0}^s w_i (x_s - x_i) + \sum_{i=s}^n w_i (x_i - x_s)$



Example

❖ where:

- w_i = relative weight of demand attached to the i th location on the beach
- x_i = location of the i th demand point on the beach in feet from the origin, in this case taken to be the west end of the beach
- s = position in the series of data that corresponds to site of the beach mat concession.

Example

- ❖ The objective function Z is differentiated with respect to x_s and set equal to zero.
- ❖ This yields

$$\square \frac{dZ}{ds} = \sum_{i=0}^s w_i - \sum_{i=s}^n w_i = 0 \quad \text{or} \quad \sum_{i=0}^s w_i = \sum_{i=s}^n w_i$$

- ❖ Weight sums are equal if s corresponds to median

Example

- ❖ Result suggests that site should be located at the median with respect to the density distribution of bathers. Locate site so that 50% of the potential demand is to each side (i.e., 29 in figure).
- ❖ Result for locating a site along a line can be generalized for locating a site on a plane if we use the metropolitan metric. Total travel distance will be minimized if the coordinates of the site correspond to the intersection of the x and y medians for their respective density distributions.

Copying Service Example

- ❖ Copying service has decided to open office in a city. Four office buildings that will generate a major portion of its business have been identified.
- ❖ Figure shows location of these demand points on an X-Y coordinate system.
- ❖ Weights are attached to each point and represent potential demand per month in hundreds of orders. The manager would like to determine a central location that will minimize the total distance per month that customers travel to the copying service.

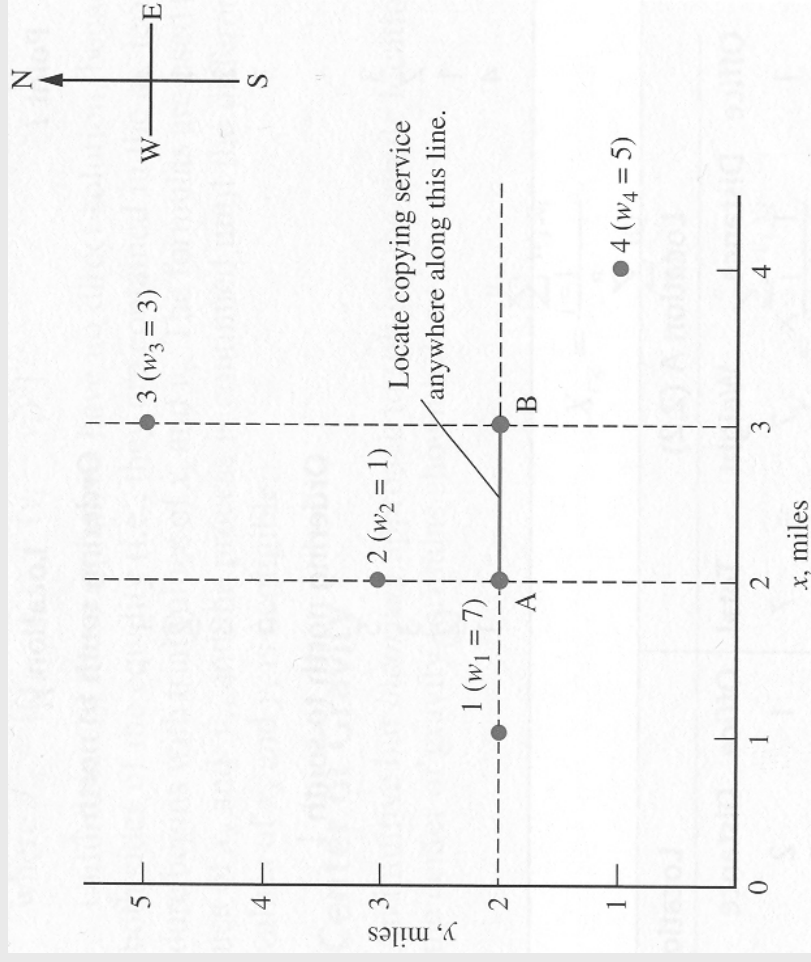
Copying Service Example

- ❖ Because of the urban location, a metropolitan metric is appropriate.
- ❖ Locate site using cross-median approach
- ❖ First, the median is calculated:

$$\square \text{ Median} = \sum_{i=1}^n w_i / 2$$

Copying Service Example

- ❖ Median = $(7+1+3+5)/2=8$
- ❖ To identify the x-coordinate median for x_s , we sum the values of w_i in the x direction both west and east to west.



Copying Service Example

Point i	Location x_i	$\sum w_i$
1	Ordering west to east $\rightarrow$ 1	
2	②	7 = 7
3	3	7 + 1 = 8
4	4	
4	Ordering east to west $\leftarrow$ 4	
3	③	5 = 5
2	2	5 + 3 = 8
1	1	

- ❖ The weights attached to each demand point are summed in descending order until the median value of 8 is reached or exceeded. That happens when the weight of location 2 is added to the weight of location 1: thus, the first x median is established at the value of 2 miles (circled).

Copying Service Example

- ❖ This procedure is repeated with demand points ordered from east to west as shown in table.
- ❖ The second x median is established at the value of 3 miles (circled).
- ❖ The next table shows the same procedure for identifying the y-coordinate median for ys.

Point i	Location y_i	$\sum w_i$
4	1	5
1	②	= 5
2	3	5 + 7 = 12
3	5	
Ordering south to north ↑		
3	5	3
2	3	3 + 1 = 4
1	②	3 + 1 + 7 = 11
4	1	
Ordering north to south ↓		

Copying Service Example

- ❖ Note in the following table that the total weighted travel distance calculated for point A and point B is equal to 35 miles in both instances; thus, any location at either point or along the line between them will be acceptable.

Location A (2,2)				Location B (3,2)			
Office	Distance	Weight	Total	Office	Distance	Weight	Total
1	1	×	7		2	×	14
2	1	×	1	2	2	×	2
3	4	×	3	3	3	×	9
4	3	×	5	4	2	×	10
			<u>35</u>				<u>35</u>

Locating a Retail Outlet

- ❖ When locating a retail outlet such as a supermarket, the objective is to maximize profit.
- ❖ A gravity model is used to estimate consumer demand. Based on the physical analog – gravitational attraction of two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance that separates them.

Locating a Retail Outlet

- ❖ For a service, the attractiveness of a facility may be expressed as

$$A_{ij} = \frac{S_j}{T_{ij}^\lambda}$$

□ where

- A_{ij} = attraction to facility j for customer i
- S_j = size of the facility j
- T_{ij} = travel time for customer i's location to facility j
- λ = parameter estimated empirically to reflect the effect of travel time on various kinds of shopping trips (e.g., where a shopping mall may have a $\lambda = 2$, convenience stores would have a $\lambda = 10$ or larger).

Huff Analysis

- ❖ This gravity model used to predict the benefit that a customer would have for a particular store size and location (D.L. Huff).
- ❖ Knowing that customers also would be attracted to other competing stores, he proposed the ratio P_{ij} . For n stores, this ratio measures the probability of a customer from a given area i traveling to a particular shopping facility j .

$$P_{ij} = \frac{A_{ij}}{\sum_{j=1}^n A_{ij}}$$

Huff Analysis

- ❖ An estimate of E_{jk} , the total annual consumer expenditures for a product class k at a potential shopping facility j, then can be calculated as

$$E_{jk} = \sum_{i=1}^m (P_{ij} C_i B_{ik})$$

- P_{ij} = Probability customers from area i travel to store j
- C_i = Number of customers in area i (e.g. census track)
- B_{ik} = Annual budget for product k for customers in area i
- m = Number of customer areas in the market region

Huff Analysis

- ❖ An estimate of M_{jk} , the market share captured by facility j of product class k sales, can be calculated as

$$M_{jk} = \frac{E_{jk}}{\sum_{i=1}^m (C_i B_{ik})}$$

- ❖ An exhaustive procedure is used to calculate the expected annual profit of each potential site for various possible store sizes at the site.

Copying Service Example

- ❖ Assume $x=2$, $y=2$
- ❖ Also assume that each customer order represents an expenditure of approximately \$10. Because convenience would be an important customer criterion, assume that $\lambda = 2$.
- ❖ If we wish to open a new store at location $(x=3, y=2)$ but with twice the capacity for the existing copying center, how much market share would we expect to capture?

Copying Service Example

❖ Using the travel distances shown on the next table as input to the Huff model, the calculations on the following 3 tables are obtained

TABLE 9.5
Travel Distance in
Miles (T_{ij}) (Using
Metropolitan
Metric)

Site (j)	Customer Location (i)			
	1	2	3	4
Proposed (3, 2)	2	2	3	2
Existing (2, 2)	1	1	4	3

Copying Service Example

TABLE 9.6

Attraction (A_{ij})

Site (j)	Customer Location (i)			
	1	2	3	4
Proposed ($S_1 = 2$)	0.5	0.5	0.2222	0.500
Existing ($S_2 = 1$)	1.0	1.0	0.0625	0.111
Total attraction	1.5	1.5	0.2847	0.611

TABLE 9.7

Probability (P_{ij})

Site (j)	Customer Location (i)			
	1	2	3	4
Proposed	.33	.33	.78	.82
Existing	.67	.67	.22	.18

TABLE 9.8

Monthly Expenditures (E_{jk}) and Market Share (M_{jk})

Site (j)	Customer Expenditures				Monthly Total	Market Share %
	1	2	3	4		
Proposed	\$2,333	\$ 333	\$2,340	\$4,100	\$ 9,106	0.57
Existing	4,667	667	660	900	6,894	0.43
Totals	\$7,000	\$1,000	\$3,000	\$5,000	\$16,000	1.00

Site Selection Considerations

- | | |
|--|---|
| 1. Access:
Convenient to freeway exit and parking entrance ramps
Served by public transportation | 4. Parking:
Adequate off-street |
| 2. Visibility:
Set back from street
Surrounding clutter
Sign placement | 5. Expansion:
Room for expansion |
| 3. Traffic:
Traffic volume on street that may indicate potential impulse buying
Traffic congestion that could be a hindrance (e.g., fire stations) | 6. Environment:
Immediate surroundings should complement the service |
| | 7. Competition:
Location of competitors |
| | 8. Government:
Zoning restrictions
Taxes |

Breaking the Rules

- ❖ **Competitive Clustering (Among Competitors)**
 - (e.g. Auto Dealers, Motels)
- ❖ **Saturation Marketing (Same Firm)**
 - (e.g. Au Bon Pain, Ice Cream Vendors)
- ❖ **Marketing Intermediaries**
 - (e.g. Credit Cards, HMO)
- ❖ **Substitute Electronic Media for Travel**
 - (e.g. telecommuting, e-Commerce)
- ❖ **Impact of the Internet on Service Location**
 - (e.g. Amazon, eBay, FedEx)

Strategic Location Considerations

	Front Office	Back Office
External Customer (consumer)	Is travel out to customer or customer travel to site? Can electronic media substitute for physical travel? Is location a barrier to entry?	Is service performed on person or property? Is co-location necessary? How is communication accomplished?
Internal Customer (employee)	Availability of labor? Are self-service kiosks an alternative?	Are economies of scale possible? Can employees work from home? Is offshoring an option?

Geographic Information System

- ❖ GIS – originally limited to scientists and cartographers.
- ❖ Now some companies use GIS tools (e.g., ArcView) for business applications
- ❖ GIS can be used for tasks such as mapping customer databases, determining site locations, analyzing demand, and improving delivery services.

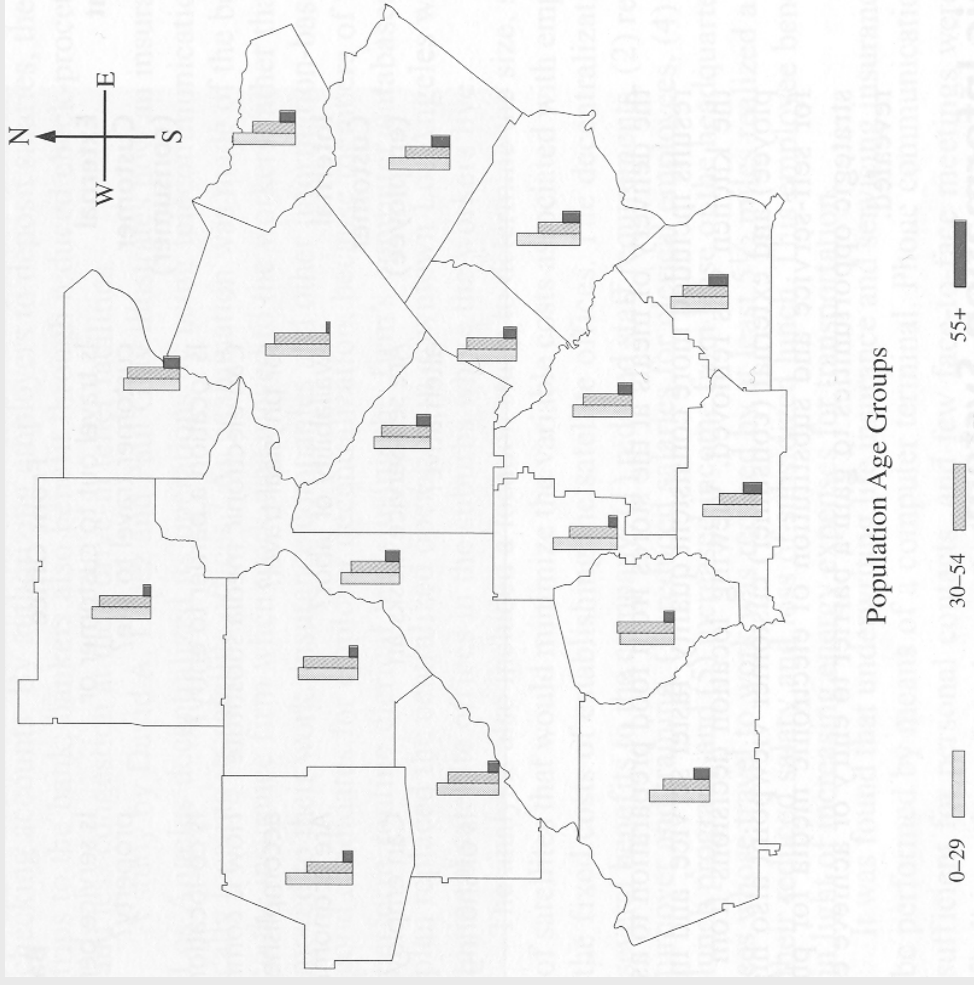
Geographic Information System

- ❖ **ArcView translates data, such as demographic info into a map.**
 - ❑ **e.g., a table that lists street addresses and median home values. It may be possible to discern a pattern in the list, if the data are for a community of 150 people in central Montana**
 - ❑ **If the data are for whole city of Denver, may be difficult to discern a useful pattern for locating a new high-end specialty shop.**
 - ❑ **ArcView transfers the data to a map base and color-codes the median home values according to different zip codes.**

GIS Example

- ❖ Suppose a state agency wants to establish a warehouse to store surplus foods for Meals on Wheels organizations in an 18-county area.
- ❖ Most Meals on Wheels clients are elderly and homebound, so the agency commissioner wants to identify a location that will be most convenient for the MoW units that have the highest demand.
- ❖ One solution is to use ArcView to look at the distribution of the population by age groups

GIS Example



- ❖ Counties on the eastern perimeter of the area tend to have larger populations in the 55+ age group.
- ❖ Based on this info, the warehouse could be located to serve these counties more conveniently.