

Lecture #23

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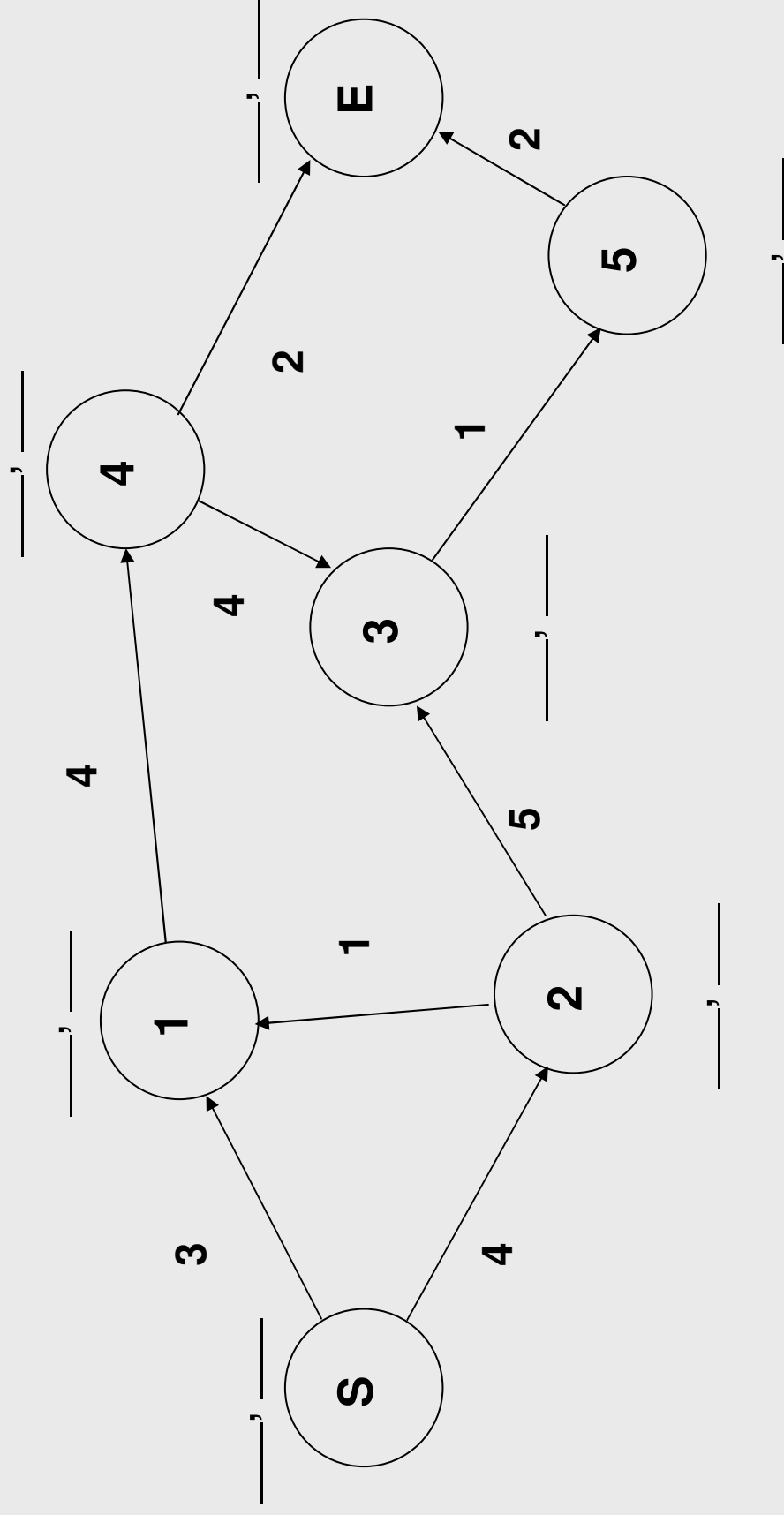
March 15, 2006

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CPM



Critical Path Method

Item	Symbol	Definition
Activity duration	t	The expected duration of an activity
Early start	ES	The earliest time an activity can begin if all previous activities are begun at their earliest times
Early finish	EF	The earliest time an activity can be completed if it is started at its early start time
Late start	LS	The latest time an activity can begin without delaying the completion of the project
Late finish	LF	The latest time an activity can be completed if it is started at its latest start time
Total slack	TS	The amount of time an activity can be delayed without delaying the completion of the project

Critical Path Method

$ES = EF_{\text{predecessor}}$	(max)	(1)
$EF = ES + t$		(2)
$LF = LS_{\text{successor}}$	(min)	(3)
$LS = LF - t$		(4)
$TS = LF - EF$		(5)
or	$TS = LS - ES$	(6)

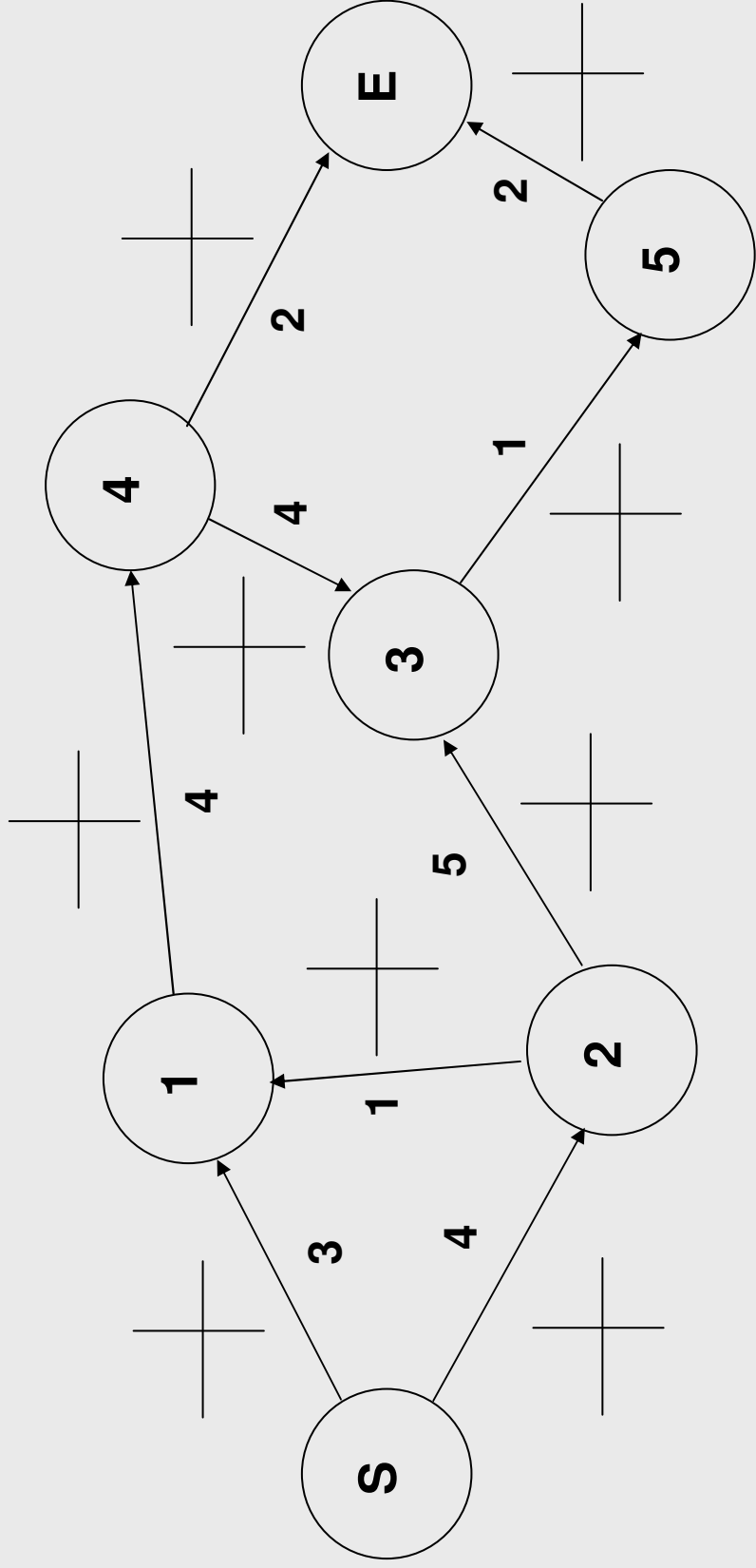
More on CPM

❖ Associated with each activity is a cross to be filled in with the activity schedule times, which are calculated as follows:

$$TS = LS - ES$$

ES	EF
LS	LF

More CPM



More on Activity Networks

- ❖ In many cases t values are assumed to be deterministic (i.e., constant)
- ❖ In other cases, expected duration is mean of known probability distributions
- ❖ PERT Approach – Beta Dist.
 - ❑ 3 time estimates
 - ❑ Most likely time (M), optimistic time (O), and pessimistic time (P)
 - ❑ For each activity: $\text{Mean} = (O + 4M + P) / 6$
 - ❑ For each activity: $\text{Std Dev} = (P - O) / 6$
 - ❑ Add variances along path
 - ❑ End up with Mean and S.D. of Completion Time

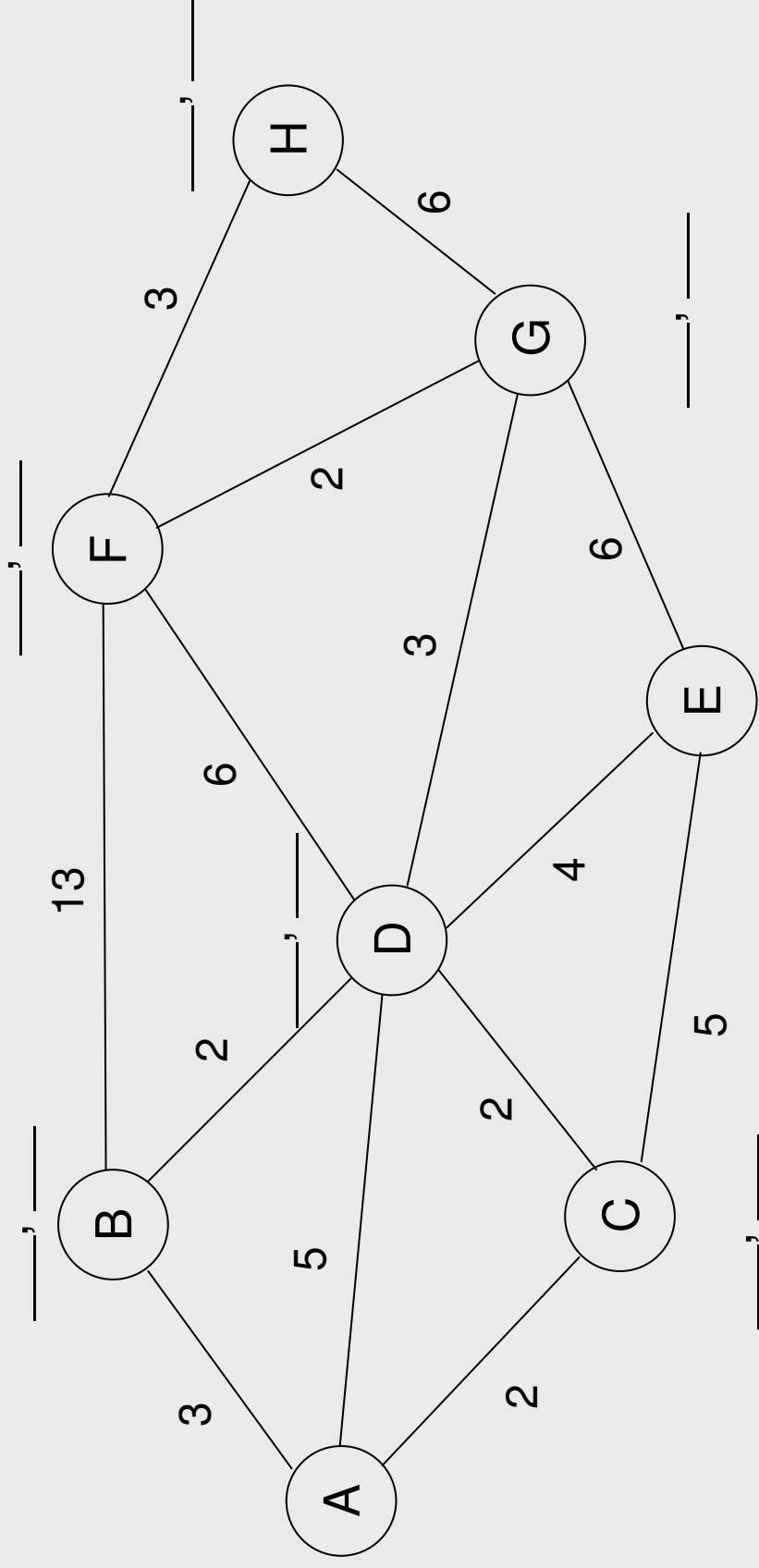
Routing Problems

- ❖ Shortest path problem (SPP)
- ❖ Minimum Spanning Tree (MST)
- ❖ Traveling salesman problem (TSP)
- ❖ Vehicle routing problem (VRP)
- ❖ Chinese postman problem (CPP)

Classifying Routing and Scheduling

- ❖ If the customers being serviced have no time restrictions and no precedence relationships exist, then the problem is a pure routing problem.
- ❖ If there is a specified time for the service to take place, then a scheduling problem exists.
- ❖ Otherwise, we are dealing with a combined routing and scheduling problem.

Shortest Route Problem



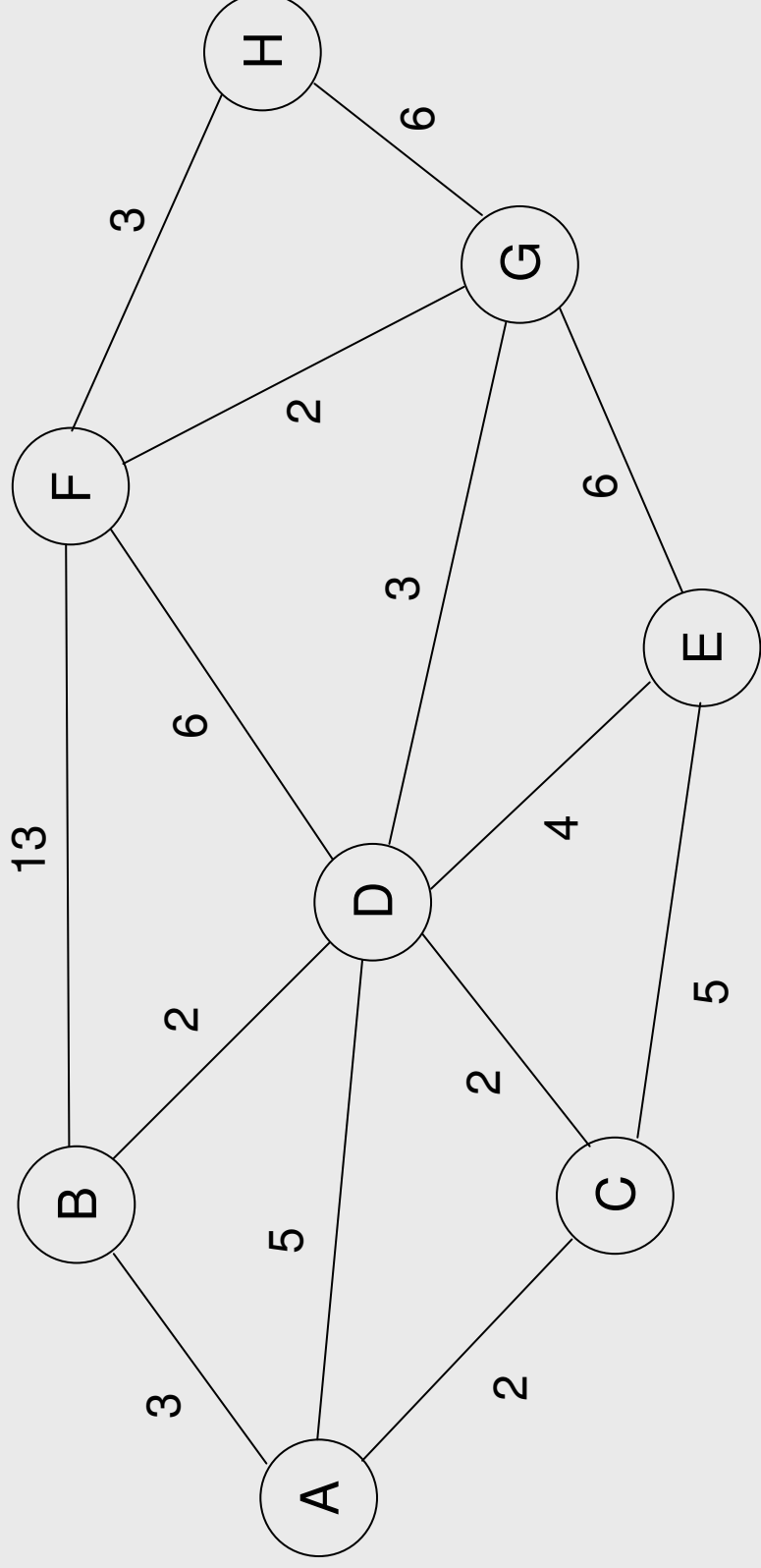
Dijkstra's Algorithm



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Minimum Spanning Tree



Traveling Salesman Problem

- ❖ No simple way to obtain solution
- ❖ Nearest neighbor procedure (NNP)
 - ❑ NNP builds tour based only on cost/ distance of traveling from the last-visited node in network to other nodes
 - ❑ Has disadvantage of being rather shortsighted
 1. Start with node at the beginning of the tour
 2. Find node closest to last node & add to tour
 3. Go to step 2 until all nodes added
 4. Connect first & last nodes to close tour
- ❖ Can use as upper bound for Branch and Bound Soln.

Example of the NNP

- ❖ In this case where the arcs are undirected, the distance from i to j will be the same as the distance from j to i
- ❖ Such a network with undirected arcs is said to be symmetrical

From Node	To Node (distances in miles)					
	1	2	3	4	5	6
1	—	5.4	2.8	10.5	8.2	4.1
2	5.4	—	5.0	9.5	5.0	8.5
3	2.8	5.0	—	7.8	6.0	3.6
4	10.5	9.5	7.8	—	5.0	9.5
5	8.2	5.0	6.0	5.0	—	9.2
6	4.1	8.5	3.6	9.5	9.2	—

Symmetric Distance Matrix

