Lecture # 9

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Expectation

True mean: μ_x **estimated by** $\bar{X}$

True variance: σ_x^2 **estimated by** s_x^2

True standard deviation: σ_x **estimated by** s_x

True mean range: μ_R **estimated by** R

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx = mean = \mu_x$$

$$\text{Variance} = \sigma_x^2 = E\left[(X-\mu_x)^2\right] = Var(X)$$

Expectation (continued)

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

Rules on Expectation

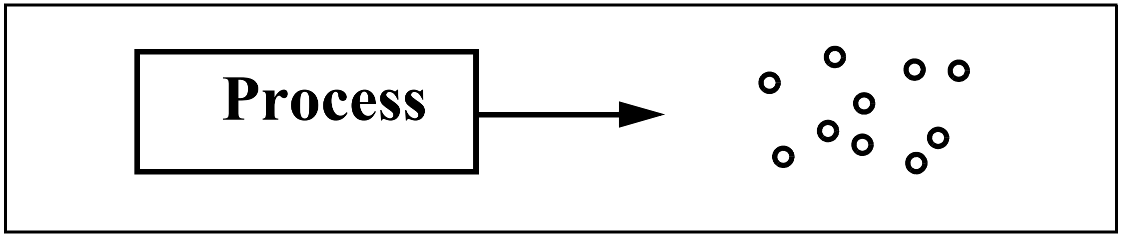
E[cX] =

E[X+Y] =

Var[cX] =

Var[X+Y] =
Var[X-Y] =

Process Study



The following data are collected:
38, 52, 85, 23, 78, 44, 82, 24, 13, 41

n = 10

Average, $\bar{X}$ = 48 , Range, R = 85-13 = 72

Sample variance =s_x^2=670.22, Sample std. dev.= s_x = 25.9

Working with a pdf

- What is the equation for the pdf?
- What is the corresponding cdf?
- What is the mean? Expected value for X, E(X)?

More on Expectation

- **Mean temperature is 50°F with a standard deviation of 9°F. What are the corresponding mean and std. dev. in °C?**

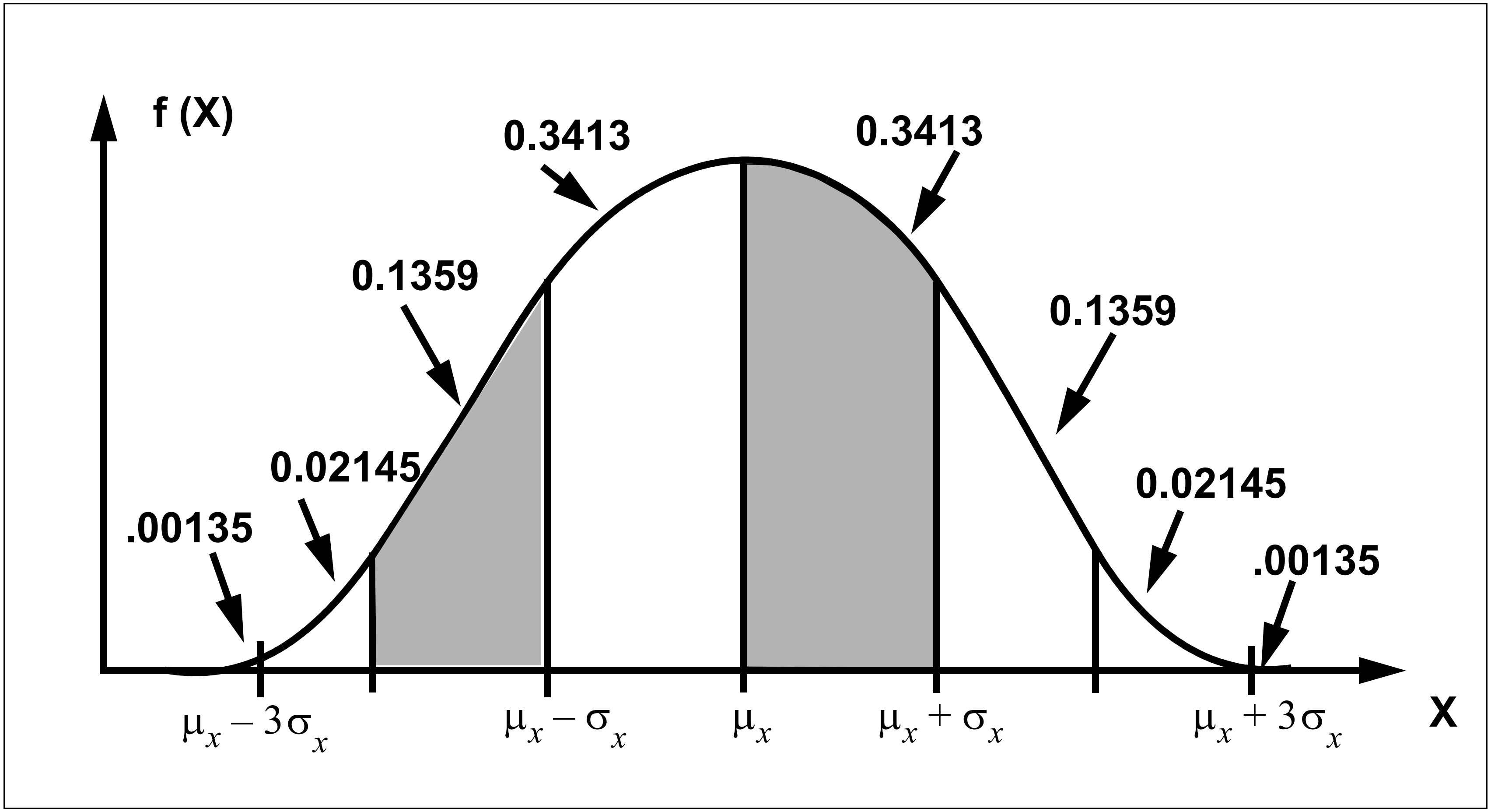
E(F) = μ = 50, Var (F) = σ_F^2 = 9^2

C = (F-32)*5/9

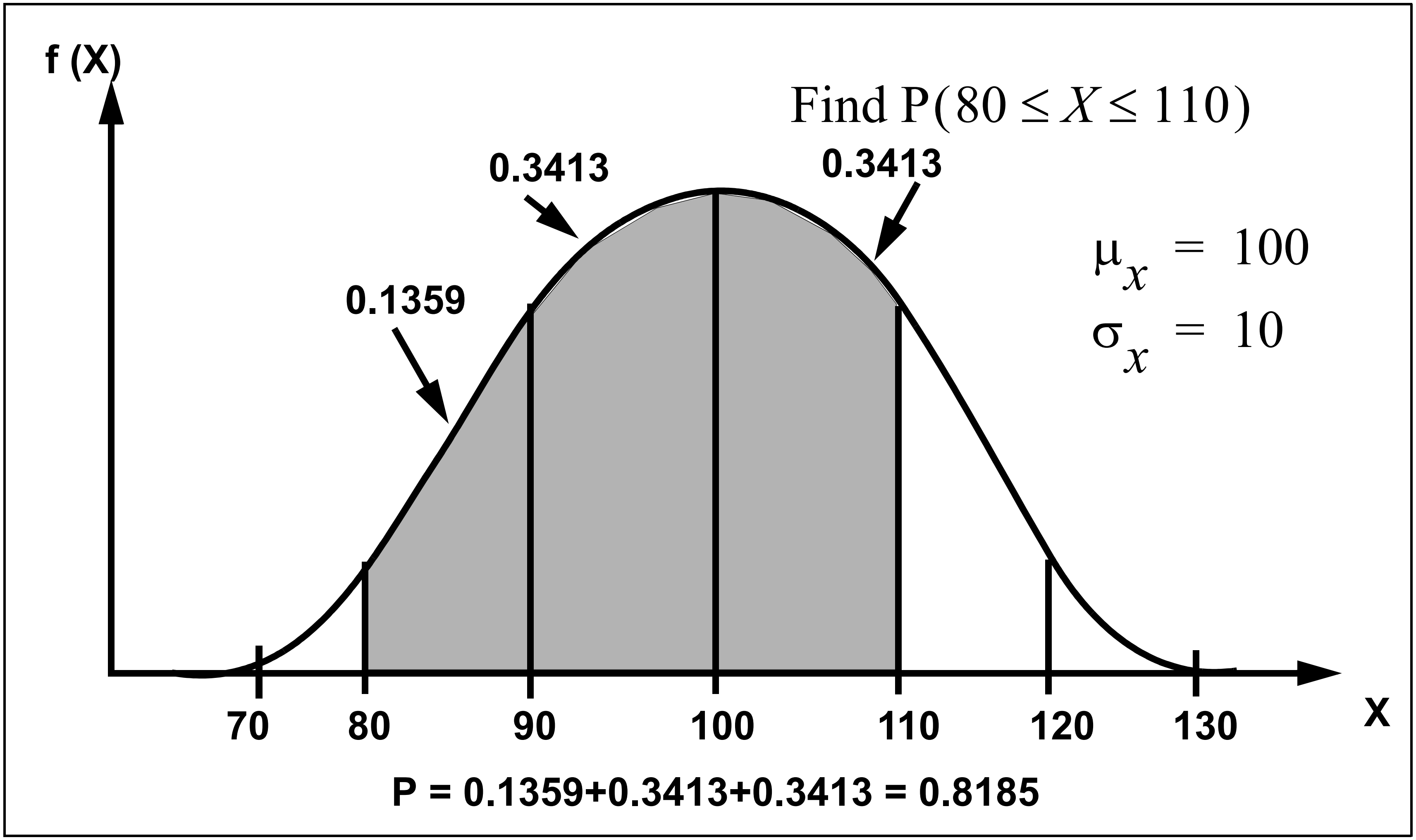
E(C) =

Var(C) =

Normal Distribution pdf

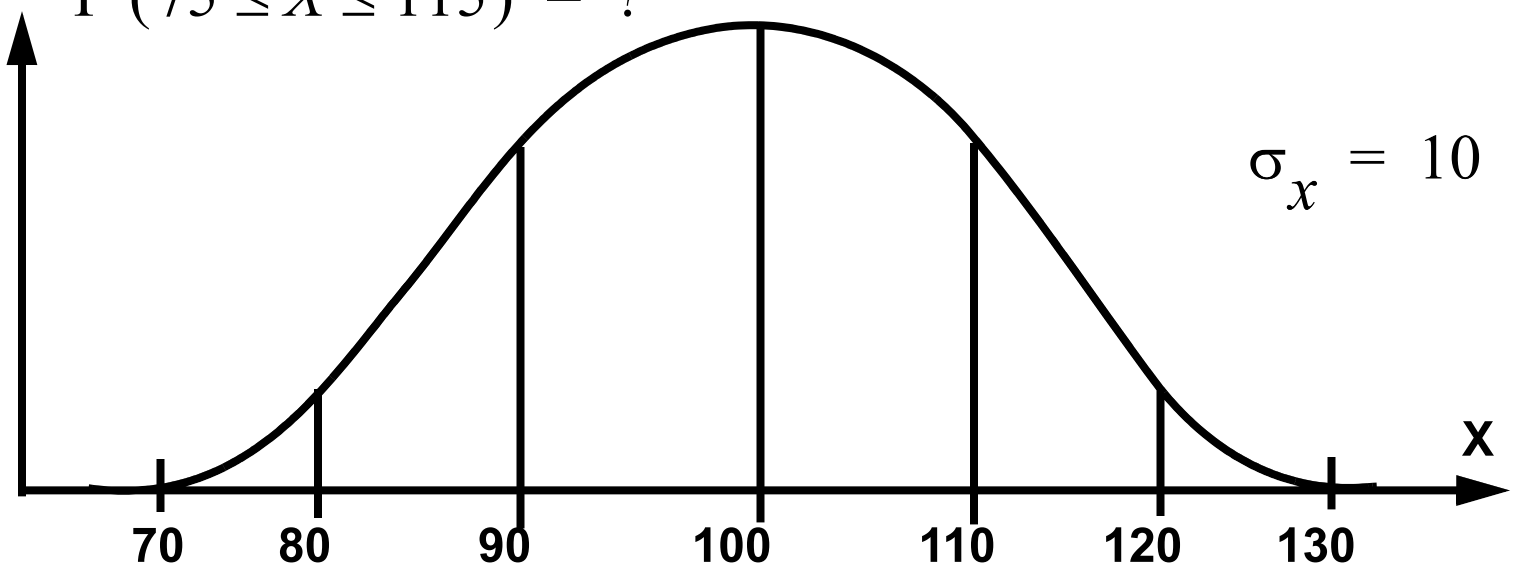


Normal Distribution Example



Second Example

$$P\ (75 \le X \le 115) = ?$$



$$\sigma_x = 10$$

How many std. devs. from the mean ?

75??

115??

Example # 2 Continued

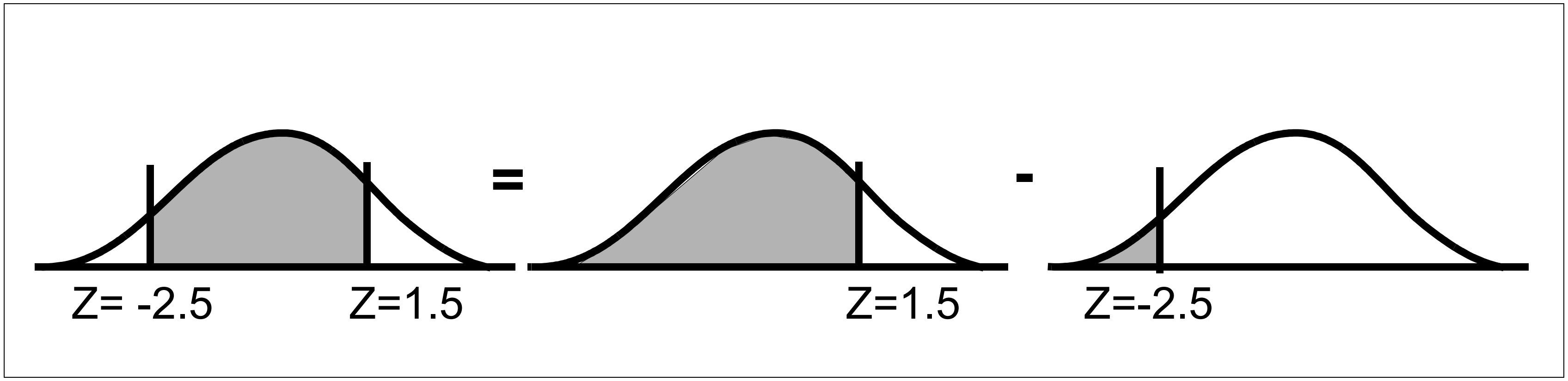
# of Std. Devs., z	Cum. Prob. - area under curve from $-\infty$ to z, F(z)
-3	0.00135
-2	0.0228
-1	0.1587
0	0.50
1	0.8413
2	0.9772
3	0.99865

Table A.1
lists F(z) for various std. dev., z

F(-2.5)=0.0062

F(1.5)=0.9332

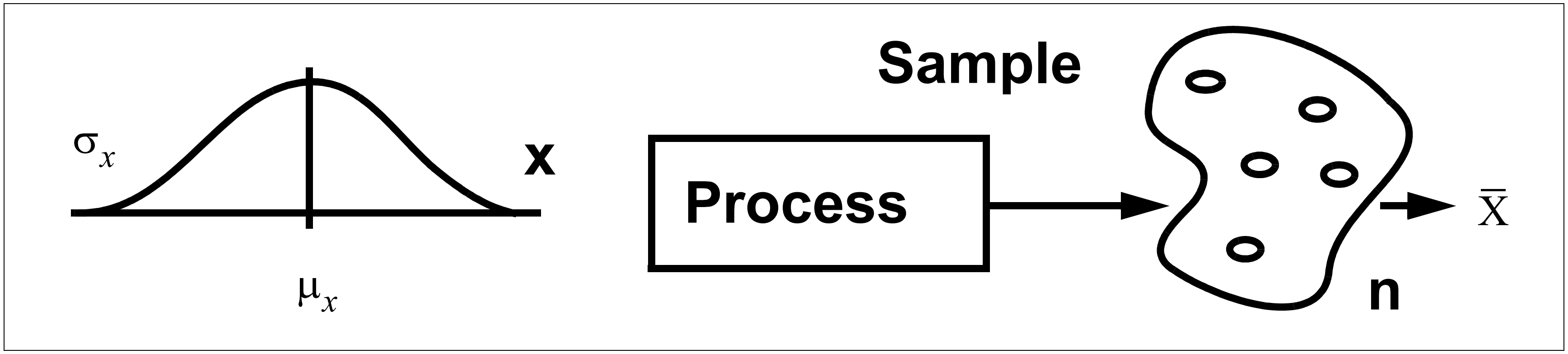
Example # 2 Summary



$$P\,(-2.5 \le Z \le 1.5) \;=\; P\,(Z \le 1.5) - P\,(Z \le -2.5)$$

= 0.9332 - 0.0062 = 0.927

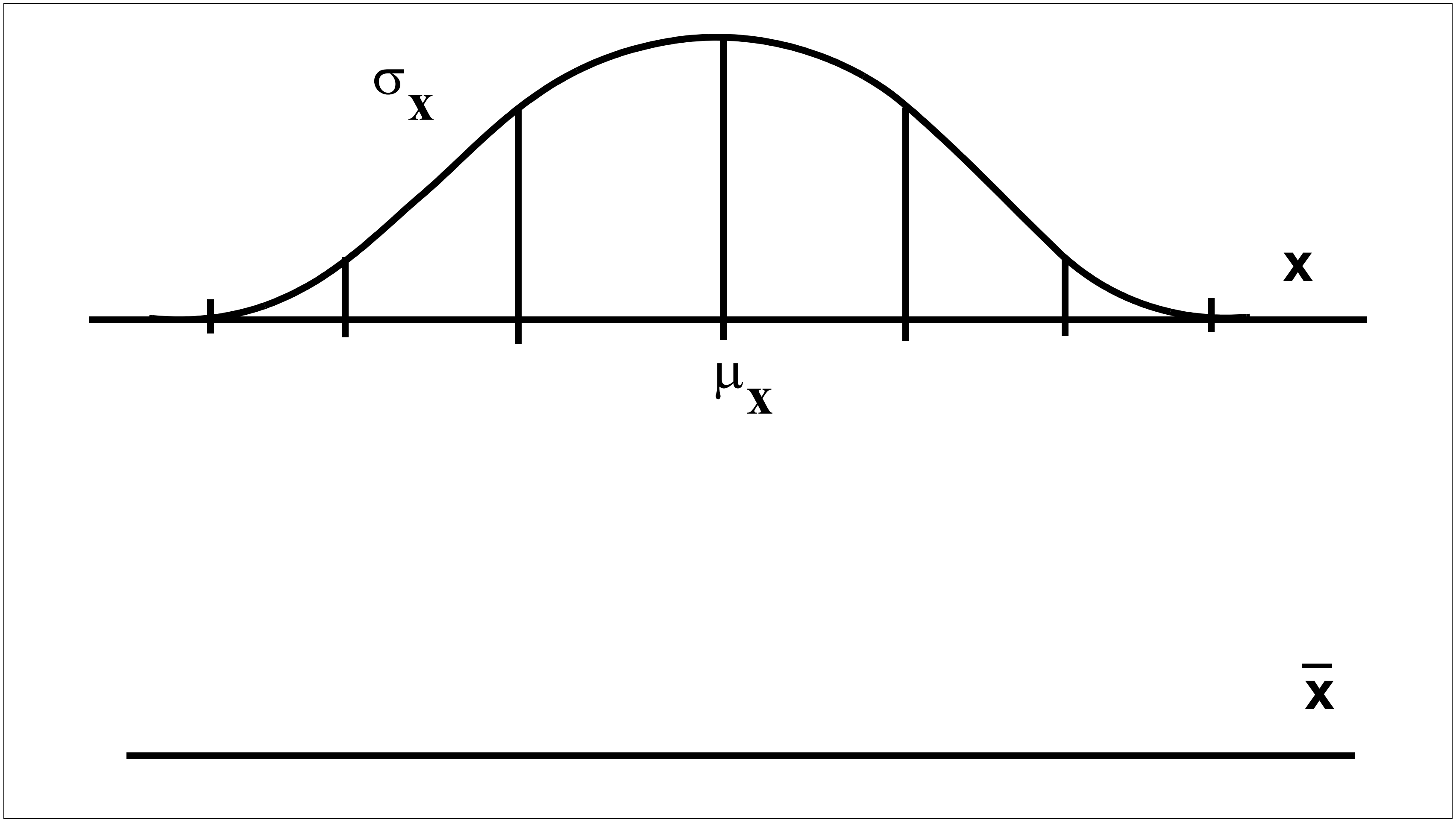
Behavior of Sample Means



How are the $\bar{X}$'s distributed ?

- Central tendency
- Spread
- Shape - distribution of sample means

Distribution of Sample Means



Distribution of Sample Means

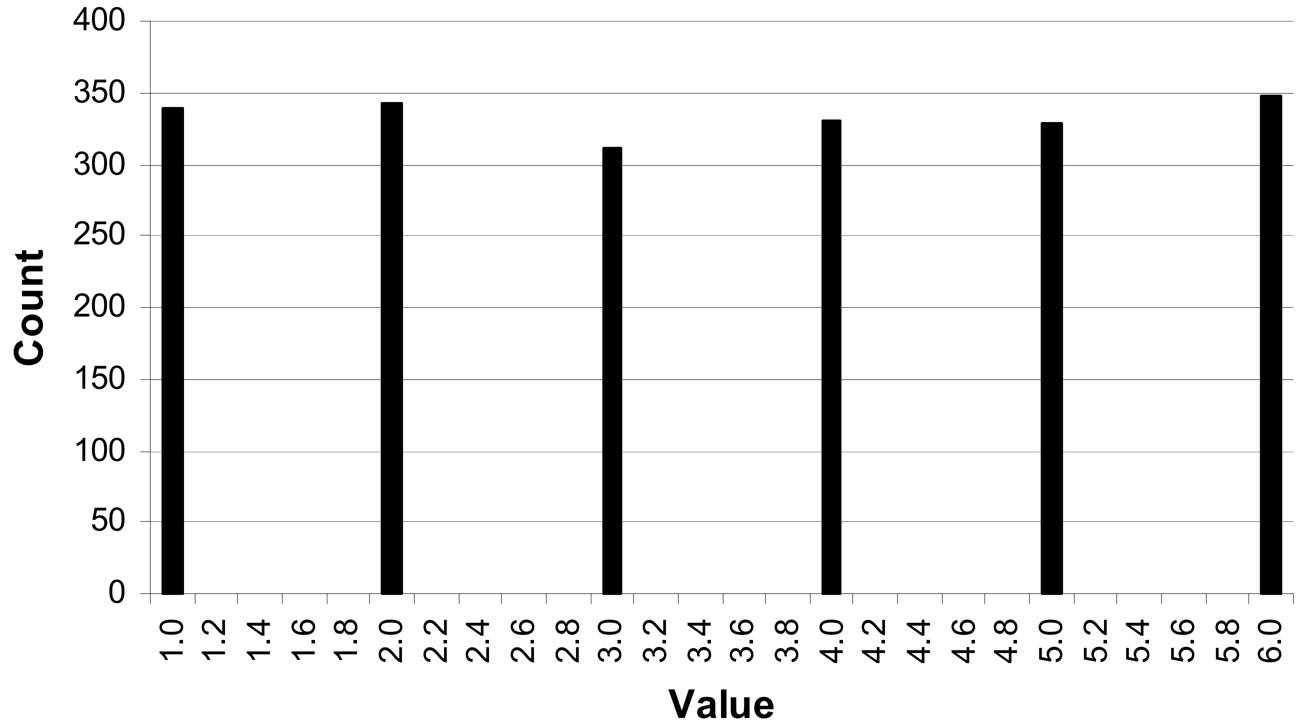
$$\mu_{\bar{x}} = E[\bar{X}]$$

$$\sigma_{\bar{x}}^{2} = Var[\bar{X}]$$

Central Limit Theorem

Averages (in fact, any linear combination of data) tend to be normally distributed regardless of the distribution of X. Tendency towards normality improves as n increases. If X's are normal, averages are also normal.

Example
(2000 throws of a die)



Example
(Throw 2 dice -- find avg)

Example
(Throw 5 dice 2000 times -- find avg)