Lecture # 41

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Poisson Distribution

$$P(c) = \frac{\lambda^{c} e^{-\lambda}}{c!}$$

λ is the mean number of defects per sample (c' , μ_c)

 # defects in the sample = c (sometimes x)

OPPORTUNITY SPACE FOR OCCURRENCE OF DEFECTS MUST BE KEPT CONSTANT!!

Sample Size

The mean number of defects, c', on one of our surfaces (say 12" x 12") is 5.

IMPORTANT!!!!

If we were to sample surfaces like this for the purpose of constructing a control chart

OPPORTUNITY SPACE FOR OCCURRENCE OF DEFECTS MUST BE KEPT CONSTANT!!

Surface area must be the same (all 12" x 12")

What is c' if surface is only 1/4 the size??

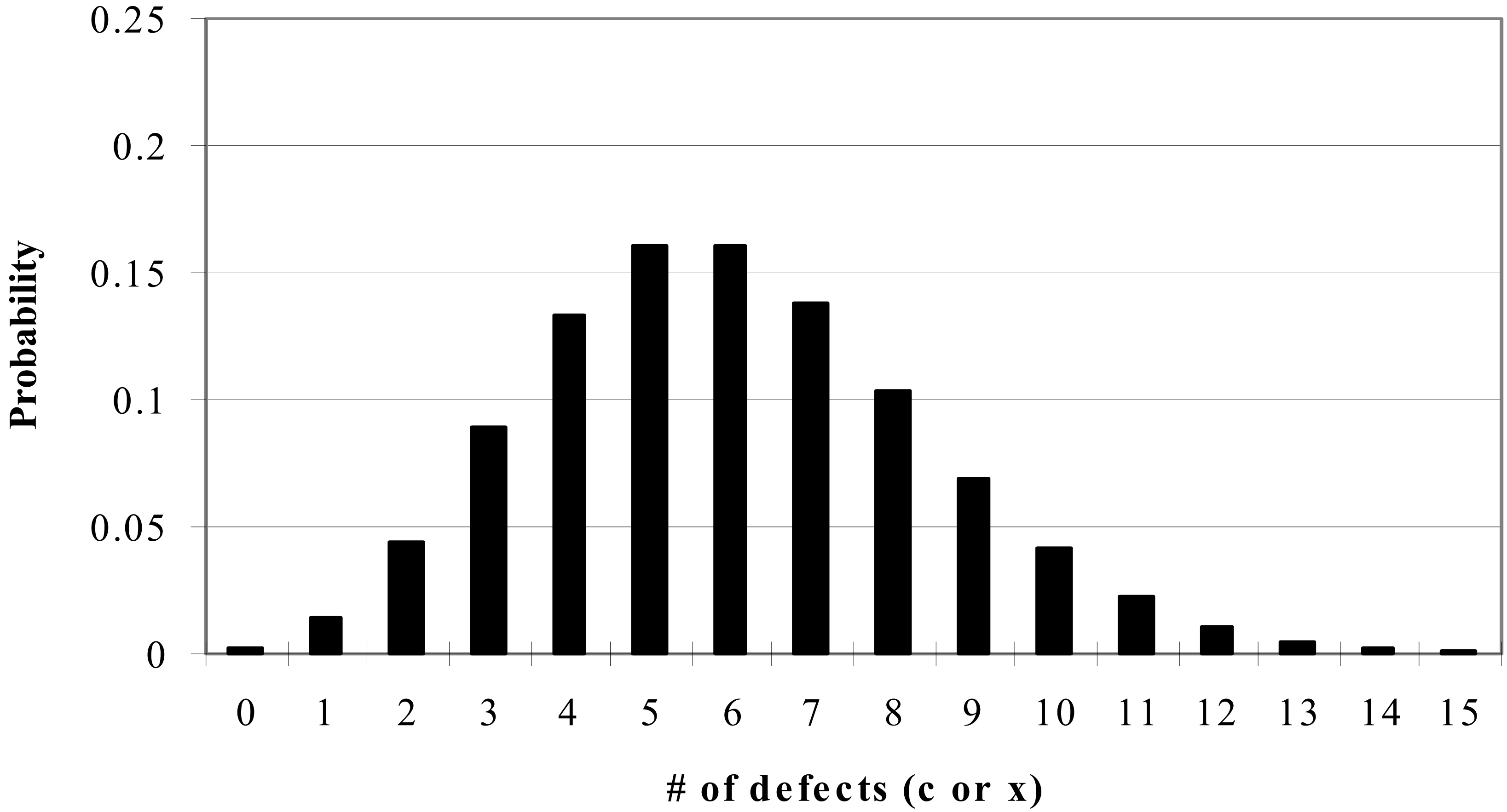
What if surface is twice the size??

c'=2 (# of breaks in a piece of wire that is 20 yds long)

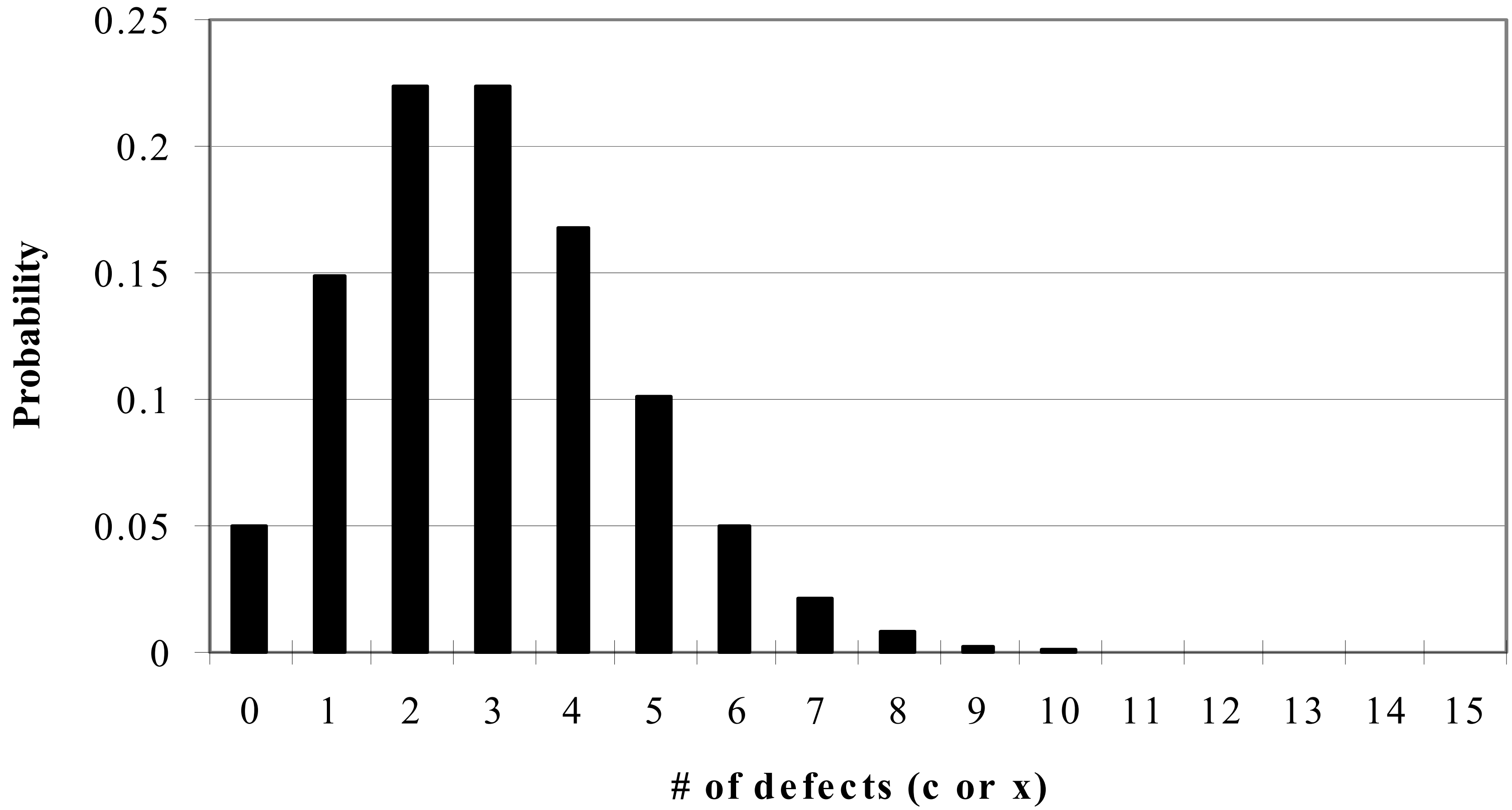
What is probability of 2 breaks in a wire that is 10 yds long?

What is probability of 2 or more breaks in a wire that is 10 yds long?

Poisson ($c' = \lambda = 6$)



Poisson ($c' = \lambda = 3$)



Mean and Variance

$$\mu_c = \lambda = c'$$

$$\sigma_c^2 = \lambda = c'$$

In principle, $\mu_c \pm 3\sigma_c$ **or** $c' \pm 3\sqrt{c'}$

We don't know c' so we must estimate it with $\bar{c}$.

$$\bar{c} = \left(\sum_{i=1}^{k} c_i \right) / k$$

Bumper Installation Case Study

- Sample: # cars built on a given shift (some variation - but small) -- n is about 560. Total of 11 types of defects

1. Mounting plate orientation 180 deg off
2. Front bumper boss gone
3. Rear bumper fit
4. Mounting plate gone
5. Rear bumper damaged
6. Wrong rear bumper
7. Front bumper damaged
8. Bumper no stocks
9. Front bumper loose
10. Rear bumper boss
11. Rear bumper loose

TABLE 13.4 Defect Data for First 25 Samples for the Bumper Assembly Example

Sample	Number of Defects
1	16
2	14
3	28
4	16
5	12
6	20
7	10
8	12
9	30
10	17
11	9
12	17
13	14
14	16
15	15
16	13
17	14
18	16
19	11
20	20
21	11
22	9
23	16
24	31
25	13

Bumper Case Study

From Table 13.4

$$\bar{c} = \frac{\text{total number of defects}}{\text{total number of samples}} = \frac{400}{25} = 16$$

Comment: for all attribute charts the size of the sample should be large enough so that np (or c) is > 1 or 2.

Control Limits

$$\bar{c} \pm 3\sqrt{\bar{c}} \qquad 16 \pm 3\sqrt{16} = 4 \ , \ 28$$

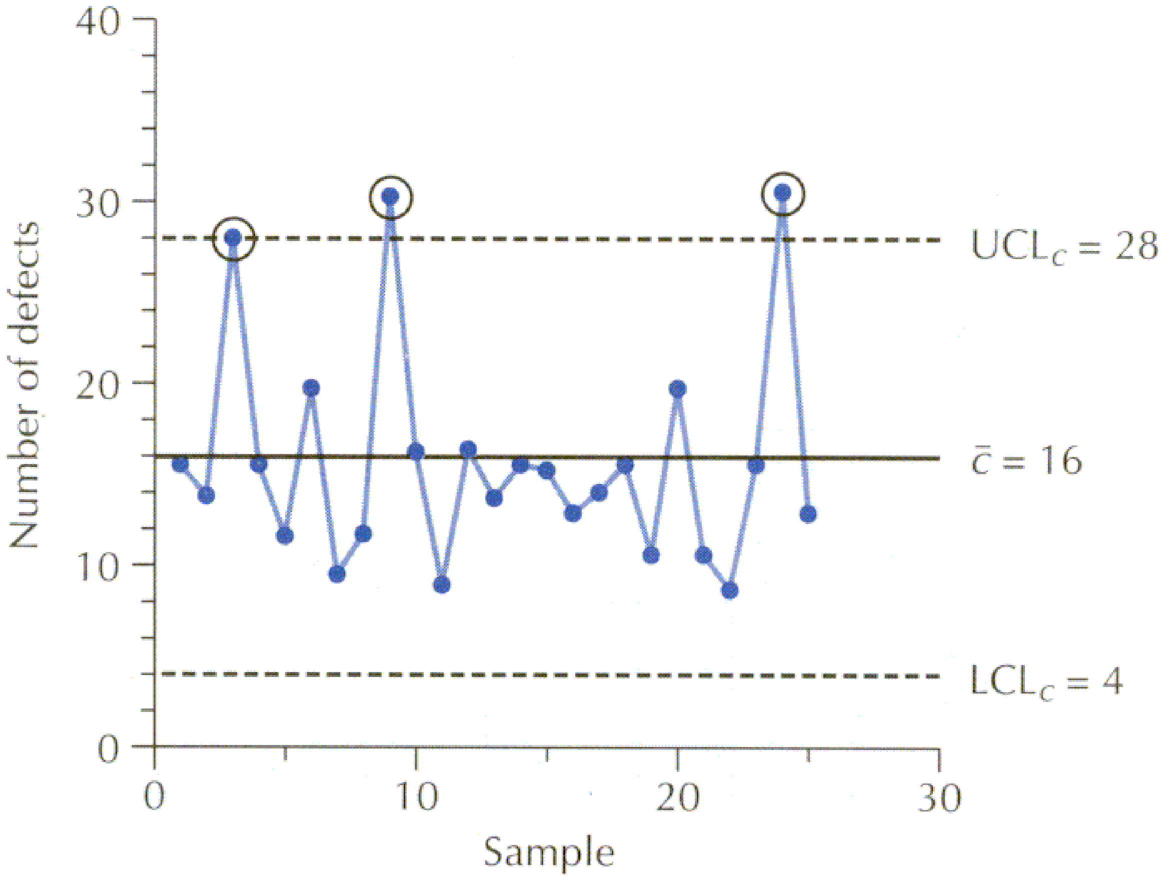
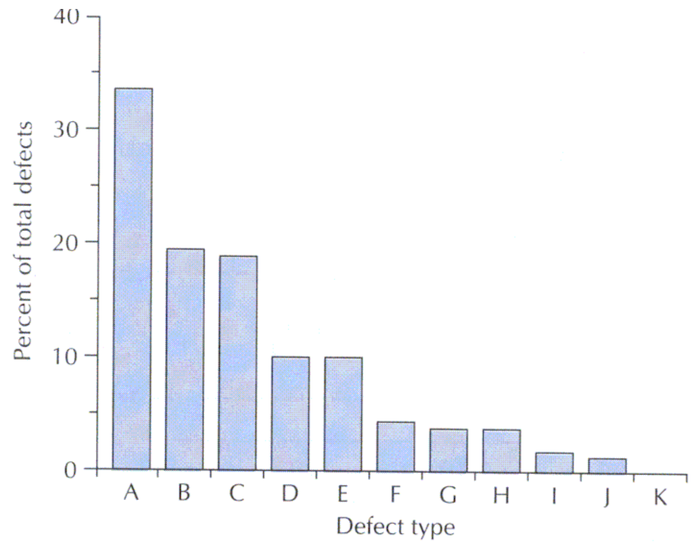


Figure 13.17 *c* Chart for first 25 Samples for the Bumper Assembly Process



Defect type in descending order of importance are

A. Bumper no stocks: This means that the bumper was not on the line at the point that it would typically be installed, and therefore was not placed on the car.

B. Front bumper loose.

C. Rear bumper fit.

D. Rear bumper loose.

E. PGM 180 deg out: One of the mounting plates was placed with the wrong orientation—180 degrees off.

F. Front bumper damaged.

G. Wrong rear bumper

H. Rear bumper boss missing: One of the screws on the bumper was not placed.

I. Front bumper boss missing.

J. Rear bumper damaged.

K. PGM missing: One of the mounting plates is not present.

Figure 13.18 Pareto Diagram for All Defects for the Data from Samples 1 to 25 of the Bumper Assembly Process

Signals on the Chart

Pareto diagram

Bumper Loose defects attributed to problem workers have in aligning bumper with mounting bracket.

Fixture developed to hold bumper in right place. Add’l data collected.

TABLE 13.5 Data for the Bumper Assembly Process After the Loose-Fits Problem Was Addressed

Sample	Number of Defects
26	12
27	9
28	11
29	12
30	13
31	12
32	15
33	8
34	7
35	8
36	11
37	9

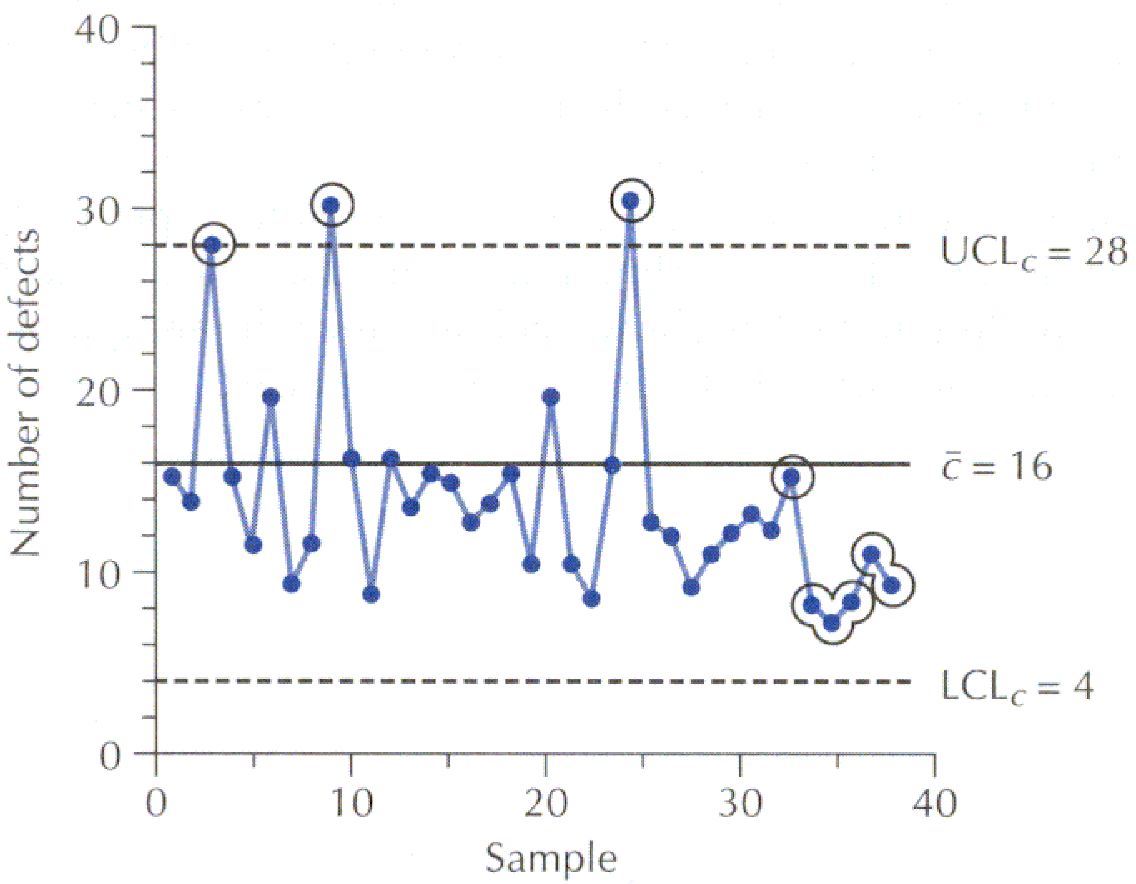


Figure 13.19 c Chart for All 37 Samples of the Bumper Assembly Process

Attribute Control Charts

One chart left to discuss: u chart

Remember, u = c/n = # of defects / unit

Often applied because it is something people can relate to.

Has application to situations where sample size varies

More about u

$$\mu_u \pm 3\sigma_u$$

Since u = c / n

$$\mu_u = \mu_c / n$$

$$\sigma_u = \sigma_c / n = \sqrt{\mu_c} / n = \sqrt{\mu_u / n}$$

Can estimate μ_u with $\bar{u}$, σ_u with $\sqrt{\bar{u} / n}$

u - Control Chart Example Moonroof Installation

$$\bar{u} = \sum_{i=1}^{k} u_i / k$$ **if the sample size n is constant**

$$\bar{u} = \sum_{i=1}^{k} c_i / \sum_{i=1}^{k} n_i$$ **this works all the time**

Defects: Wind Noise, Water Leaks, Binding During Retraction, Squeaks & Rattles

TABLE 13.6 Defect Data for Moonroof Installation Example

Sample	Sample Size, n	Number of Defects per Sample, c	Average Number of Defects per Unit, u	LCL_u	UCL_u
1	16	23	1.44	0.49	2.25
2	20	30	1.50	0.59	2.16
3	26	35	1.35	0.68	2.06
4	8	12	1.50	0.13	2.61
5	22	29	1.32	0.62	2.12
6	29	35	1.21	0.72	2.02
7	31	50	1.61	0.74	2.00
8	13	15	1.15	0.40	2.35
9	28	36	1.29	0.71	2.04
10	23	38	1.65	0.64	2.10
11	19	24	1.26	0.57	2.18
12	23	32	1.39	0.64	2.10
13	14	24	1.71	0.43	2.31
14	29	34	1.17	0.72	0.72
15	27	38	1.41	0.70	2.05
16	15	25	1.67	0.46	2.28
17	22	26	1.18	0.62	2.12
18	22	24	1.09	0.62	2.12
19	14	22	1.57	0.43	2.31
20	16	17	1.06	0.49	2.25
21	22	33	1.50	0.62	2.12
22	16	21	1.31	0.49	2.25
23	14	18	1.29	0.43	2.31
24	5	9	1.80	0.00	2.94
25	13	18	1.38	0.40	2.35
26	19	26	1.37	0.57	2.18
27	10	12	1.20	0.26	2.48

Example Continued

$$\bar{u} = 668/487 = 1.372$$

σ_u will change from sample to sample since n is changing.

This means that we will have separate limits for each sample.

$$\bar{u} \pm 3\sqrt{\bar{u}/n}$$

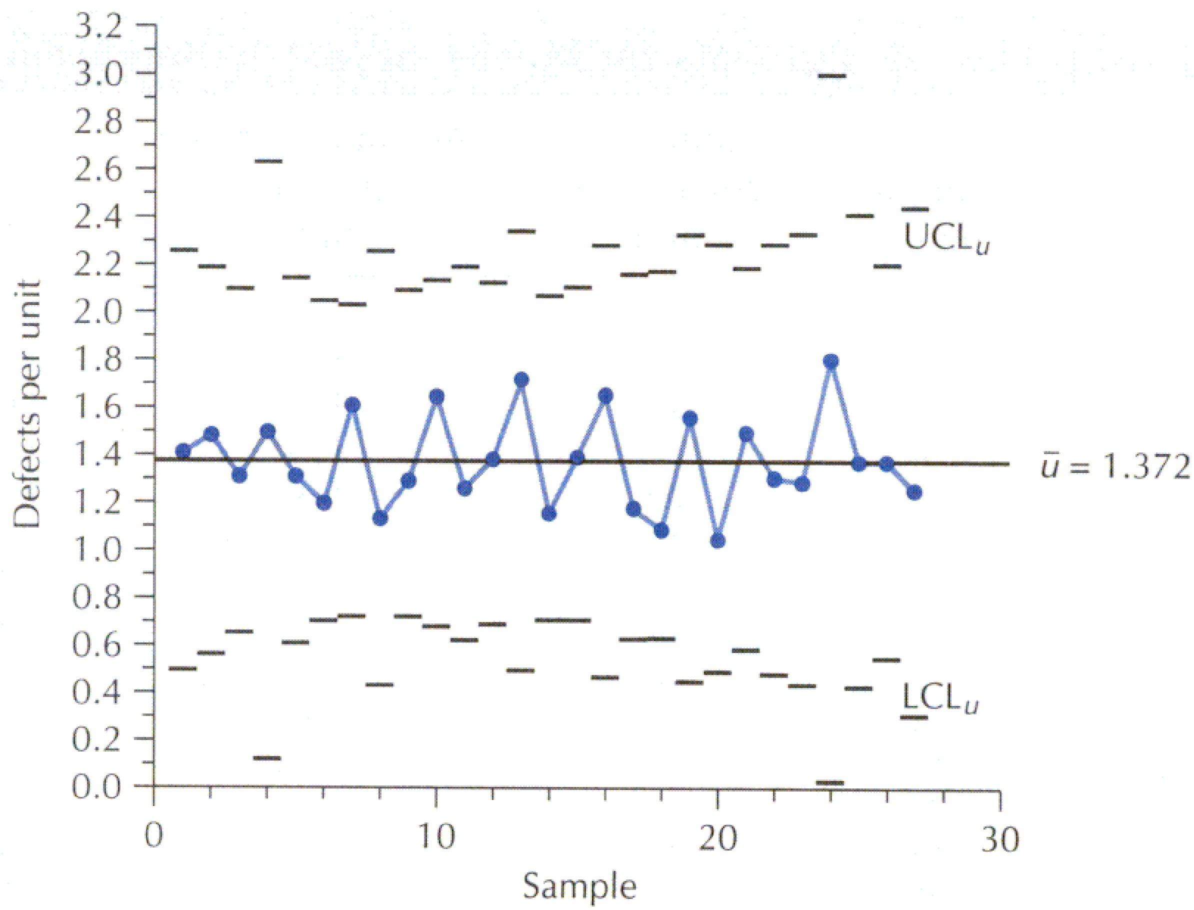


Figure 13.20 *u* Chart for the First 25 Samples for the Moonroof Installation Example

- Chart shows no special causes (statistical signals)
- "Water leaks" most common defect
- New type of seal developed for moonroof
- More data collected from the process (put them on old chart)

TABLE 13.7 Data for the Moonroof Example After the New Seal Was Implemented

Sample	Sample Size, n	Number of Defects per Sample, c	Average Number of Defects per Unit, u	LCL_u	UCL_u
28	10	8	0.80	0.26	2.48
29	14	14	1.00	0.43	2.31
30	11	8	0.73	0.31	2.43
31	29	14	0.48	0.72	2.02
32	19	7	0.37	0.57	2.18
33	19	12	0.63	0.57	2.18
34	45	25	0.56	0.85	1.90

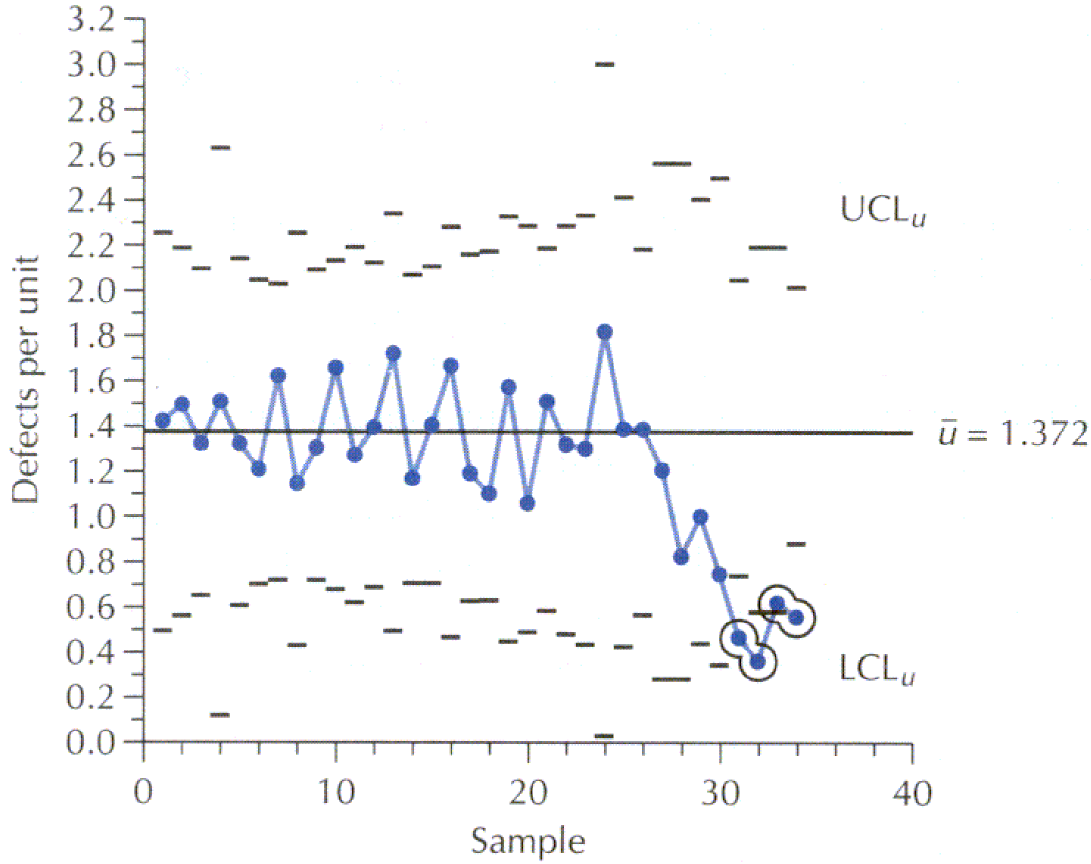


Figure 13.21 Continued *u* Chart for the Moonroof Example After Implementing the New Seal