Lecture # 40

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Dec. 5, 2005

Variable-Sample-Size p Charts

- Speaking of dealing with varying sample sizes, how do we manage this for our p Chart????
- Good news, $\bar{p}$ will not change from sample to sample, but control limits depend on n. How to handle:
  - Separate limits for each subgroup
  - Limits based on n-bar
  - Standardized p-chart

Manufacture of a Package Tray

Wood fiber/polymer composite extruded into thin sheets for use in hatchbacks. Sheets heated then molded into shape of tray. Carpet applied to sheet w/ adhesive. Carpeted tray trimmed to size.

200 - 300 trays per shift.

Defects observed: cloth coverage on hinge, carpet coverage problem, bleed-through, soiled carpet, improper flex on hinge, (chips, scratches, abrasions on back surface), wrinkles

Data for Package Tray Example

Samp.	n	d	p	$\hat{\sigma}_p$	LCL_p	UCL_p
1	238	11	0.046			
2	245	18	0.073			
3	270	17	0.063			
4	207	15	0.072			
: :	: :	: :	: :			

p Chart - Individual Limits

sum of the n's = 7433 **sum of the d's = 582**

$$\bar{p} = \left(\sum_{i=1}^{k} d_i\right) \Big/ \left(\sum_{i=1}^{k} n_i\right) = 582 / 7433 = 0.0783$$

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

For sample 1, 0.0783 ± 0.0527 : 0.026, 0.131

p Chart --- n-bar Limits

- Plot the p's and $\bar{p}$. Put limits on the chart based on $\bar{n}$.

$$\bar{n} = \left(\sum_{i=1}^{k} n_i \right) / k$$ = 7433 / 30 = approx. 248

$$\bar{p} \pm 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad 0.0783 \pm 3 \sqrt{\frac{0.0783(1-0.0783)}{248}}$$

Limits are 0.0271 & 0.1295

Pts near the limits must be interpreted separately.

p Chart --- Standardized Limits

- Calc. standardized p value for each sample

Sample	p	p - pbar	n	$\hat{\sigma}_p$	$Z=(p - pbar)/\hat{\sigma}_p$
1	.046		238		
2	.073		245		
3	.063		270		

Attribute Control Charts

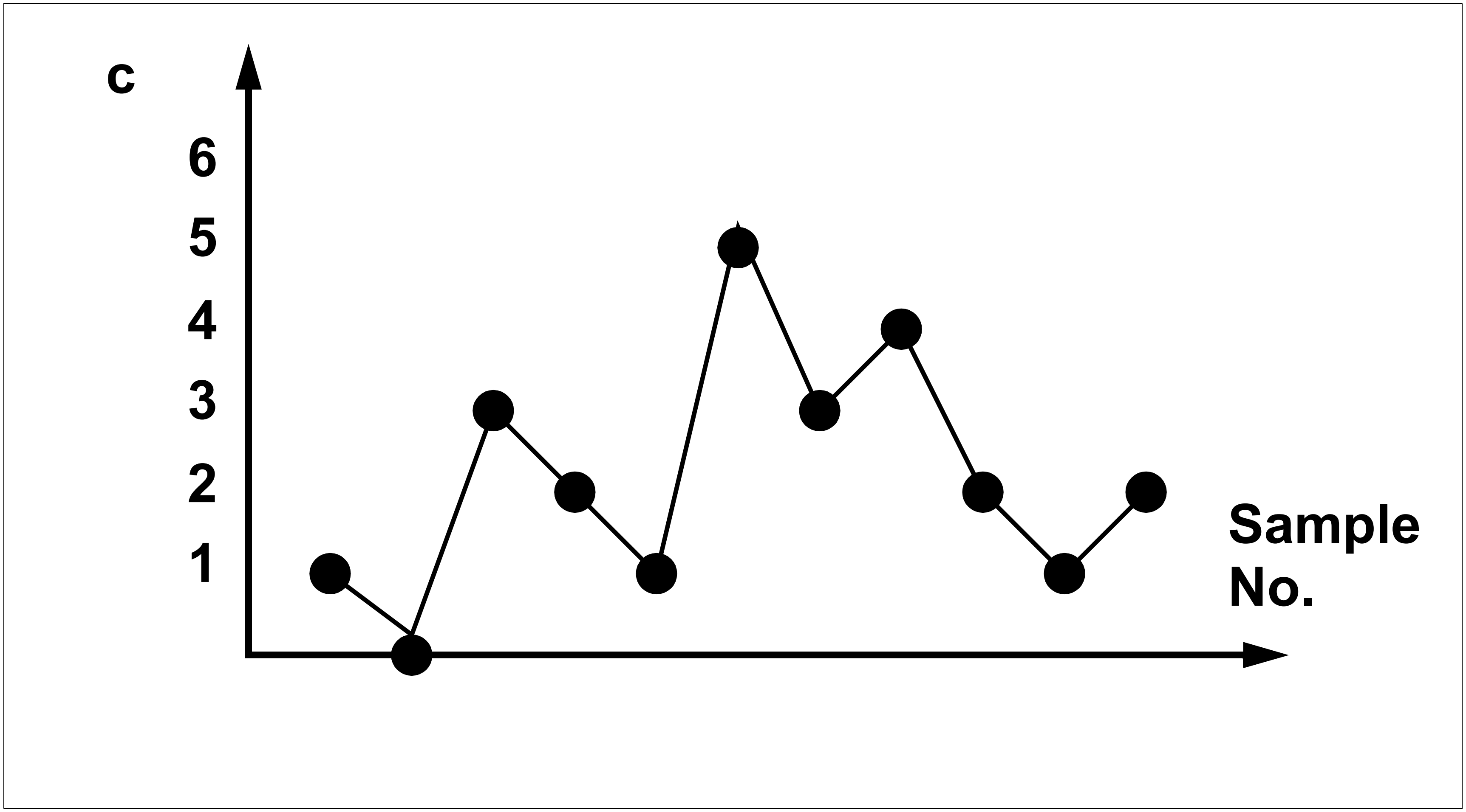
So far,

- p chart

- variable sample size p-charts
  - individual control limits
  - n-bar limits
  - standardized chart

- Now we want to construct a chart for defects, a c-chart

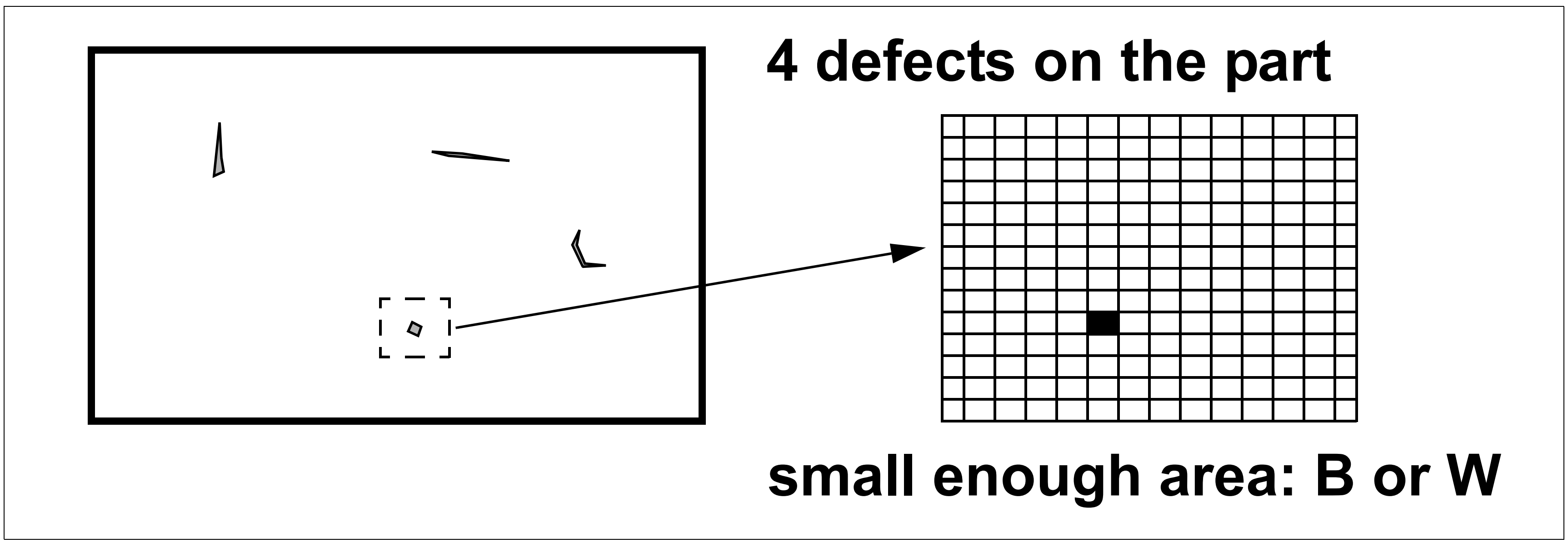
c - Control Chart



c - Control Chart

To find limits for the c Chart we need to more about how defects arise in a probabilistic sense.

Defectives and fraction defective -- binomial distn.



Defects

We decompose the surface into a very large number of sections that are either B or W.

Let's say N=1,000,000 -- on the average 5 black spots, p'=0.000005 (big n, little p', np' from 1 to 20)

P(4) = $\binom{1000000}{4}(0.000005)^{4}(0.999995)^{999996} = 0.175468$

We could use the binomial distn. to work with defects

But, the math can be messy.

Poisson Distribution

$$P(c) = \frac{\lambda^c e^{-\lambda}}{c!}$$

λ is the mean number of defects per sample (c' , μ_c)

 # defects in the sample = c (sometimes x)

So, for c' = 5,

P(c=4) = $\frac{5^c e^{-5}}{c!}$ **= 0.175467 (very close to binomial calc.)**

Sample Size

The mean number of defects, c', on one of our surfaces (say 12" x 12") is 5.

IMPORTANT!!!!

If we were to sample surfaces like this for the purpose of constructing a control chart

OPPORTUNITY SPACE FOR OCCURRENCE OF DEFECTS MUST BE KEPT CONSTANT!!

Surface area must be the same (all 12" x 12")

What is c' if surface is only 1/4 the size??

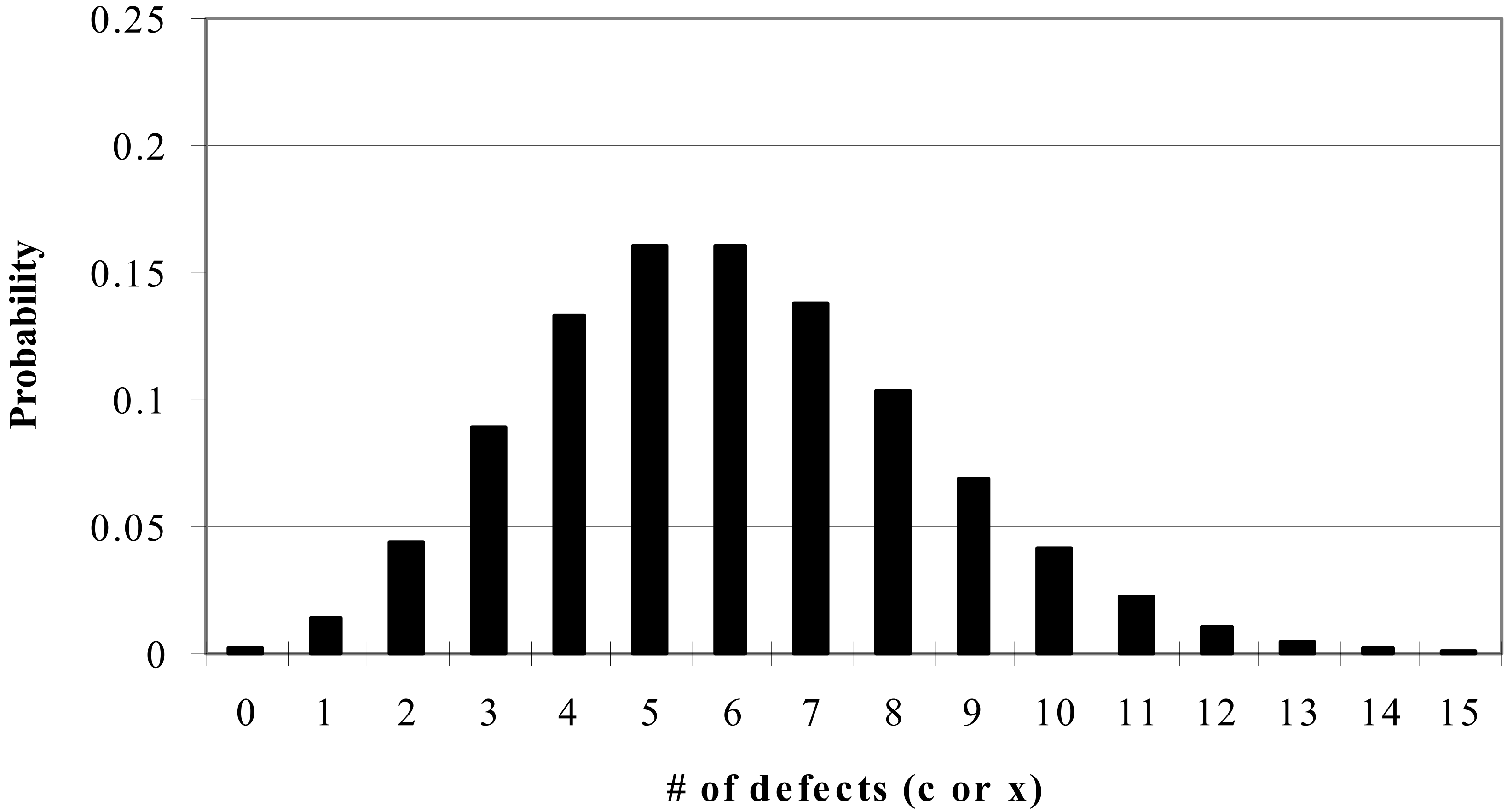
What if surface is twice the size??

c'=2 (# of breaks in a piece of wire that is 20 yds long)

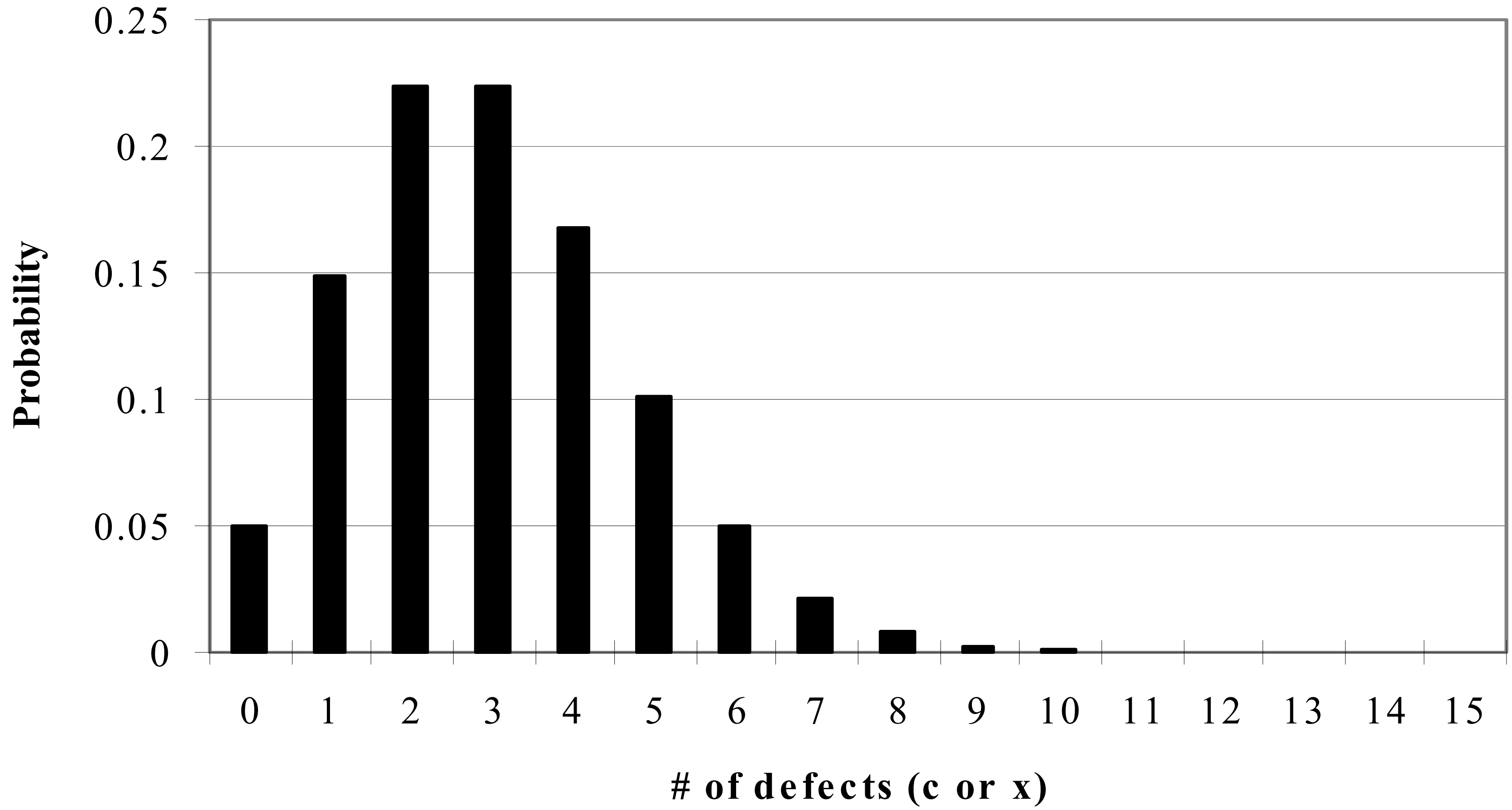
What is probability of 2 breaks in a wire that is 10 yds long?

What is probability of 2 or more breaks in a wire that is 10 yds long?

Poisson (c' = λ = 6)



Poisson ($c' = \lambda = 3$)



Mean and Variance

$$\mu_c = \lambda = c'$$

$$\sigma_c^2 = \lambda = c'$$

In principle, $\mu_c \pm 3\sigma_c$ or $c' \pm 3\sqrt{c'}$

We don't know c' so we must estimate it with $\bar{c}$.

$$\bar{c} = \left(\sum_{i=1}^{k} c_i \right) / k$$

Bumper Installation Case Study

- Sample: # cars built on a given shift (some variation - but small) -- n is about 560. Total of 11 types of defects

1. Mounting plate orientation 180 deg off
2. Front bumper boss gone
3. Rear bumper fit
4. Mounting plate gone
5. Rear bumper damaged
6. Wrong rear bumper
7. Front bumper damaged
8. Bumper no stocks
9. Front bumper loose
10. Rear bumper boss
11. Rear bumper loose

TABLE 13.4 Defect Data for First 25 Samples for the Bumper Assembly Example

Sample	Number of Defects
1	16
2	14
3	28
4	16
5	12
6	20
7	10
8	12
9	30
10	17
11	9
12	17
13	14
14	16
15	15
16	13
17	14
18	16
19	11
20	20
21	11
22	9
23	16
24	31
25	13

Bumper Case Study

From Table 13.4

$$\bar{c} = \frac{\text{total number of defects}}{\text{total number of samples}} = \frac{400}{25} = 16$$

Comment: for all attribute charts the size of the sample should be large enough so that np (or c) is > 1 or 2.

Control Limits

$$\bar{c} \pm 3\sqrt{\bar{c}} \qquad 16 \pm 3\sqrt{16} = 4 \ , \ 28$$

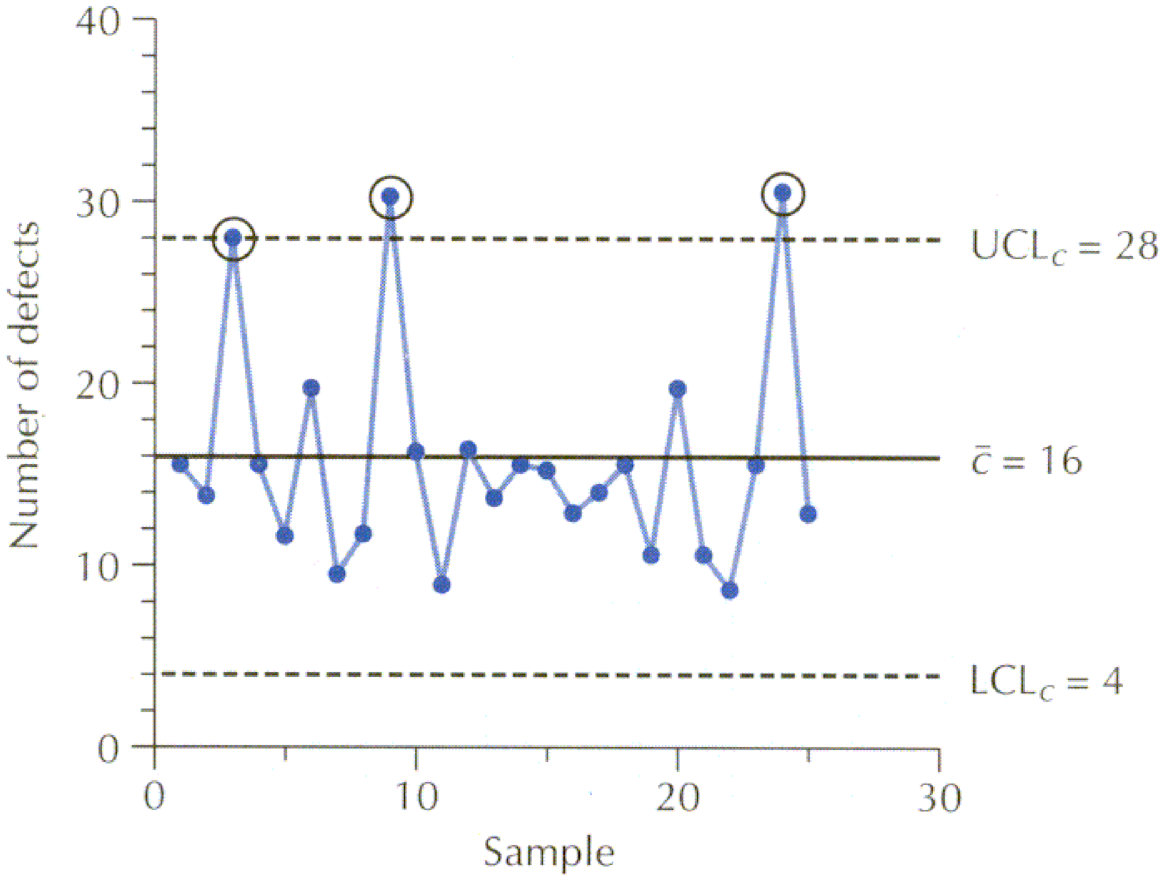
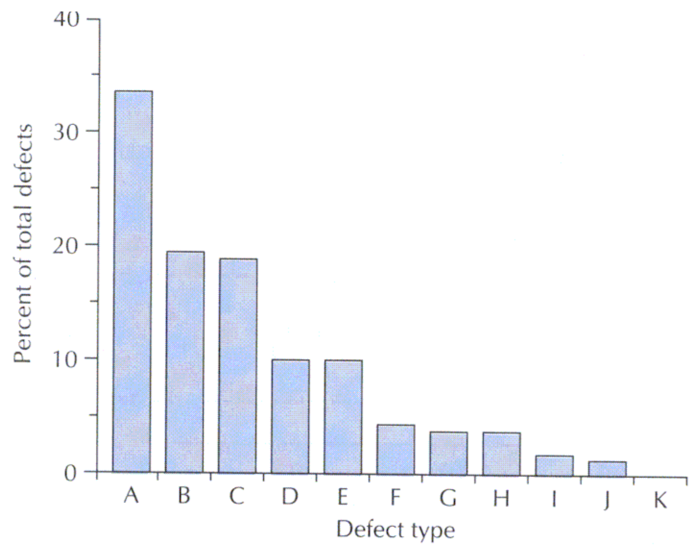


Figure 13.17 *c* Chart for first 25 Samples for the Bumper Assembly Process



Defect type in descending order of importance are

A. Bumper no stocks: This means that the bumper was not on the line at the point that it would typically be installed, and therefore was not placed on the car.

B. Front bumper loose.

C. Rear bumper fit.

D. Rear bumper loose.

E. PGM 180 deg out: One of the mounting plates was placed with the wrong orientation—180 degrees off.

F. Front bumper damaged.

G. Wrong rear bumper

H. Rear bumper boss missing: One of the screws on the bumper was not placed.

I. Front bumper boss missing.

J. Rear bumper damaged.

K. PGM missing: One of the mounting plates is not present.

Figure 13.18 Pareto Diagram for All Defects for the Data from Samples 1 to 25 of the Bumper Assembly Process

Signals on the Chart

Pareto diagram

Bumper Loose defects attributed to problem workers have in aligning bumper with mounting bracket.

Fixture developed to hold bumper in right place. Add'l data collected.

TABLE 13.5 Data for the Bumper Assembly Process After the Loose-Fits Problem Was Addressed

Sample	Number of Defects
26	12
27	9
28	11
29	12
30	13
31	12
32	15
33	8
34	7
35	8
36	11
37	9

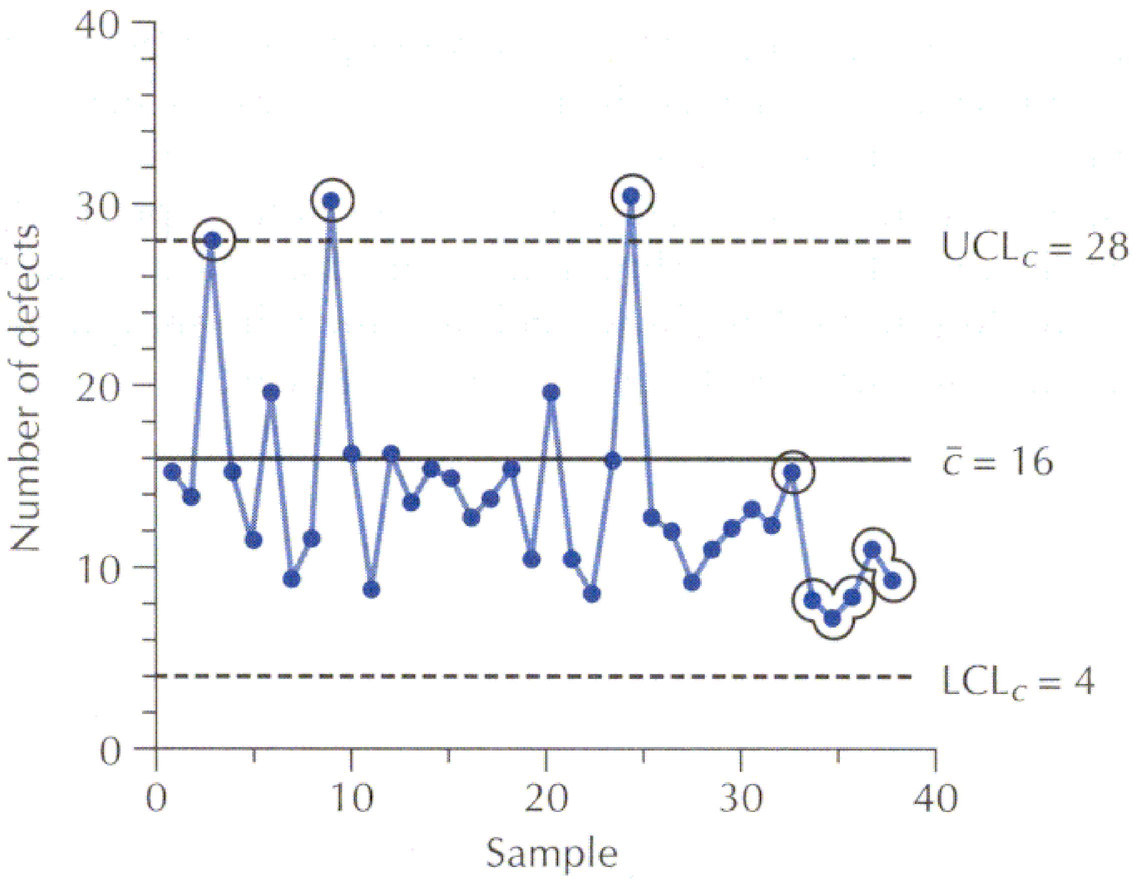


Figure 13.19 c Chart for All 37 Samples of the Bumper Assembly Process