Lecture # 39

Prof. John W. Sutherland

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Last time:

Let's say we draw 5 items.

P(5D) = 0.0000003

P(4D,1N) = 0.0000297

P(3D,2N) = 0.0011

P(2D,3N) = 0.0214

P(1D,4N) = 0.2036

P(5N) = 0.7735

The General Form

$$P(d) = \binom{n}{d} (p')^{d} (1 - p')^{n - d}$$

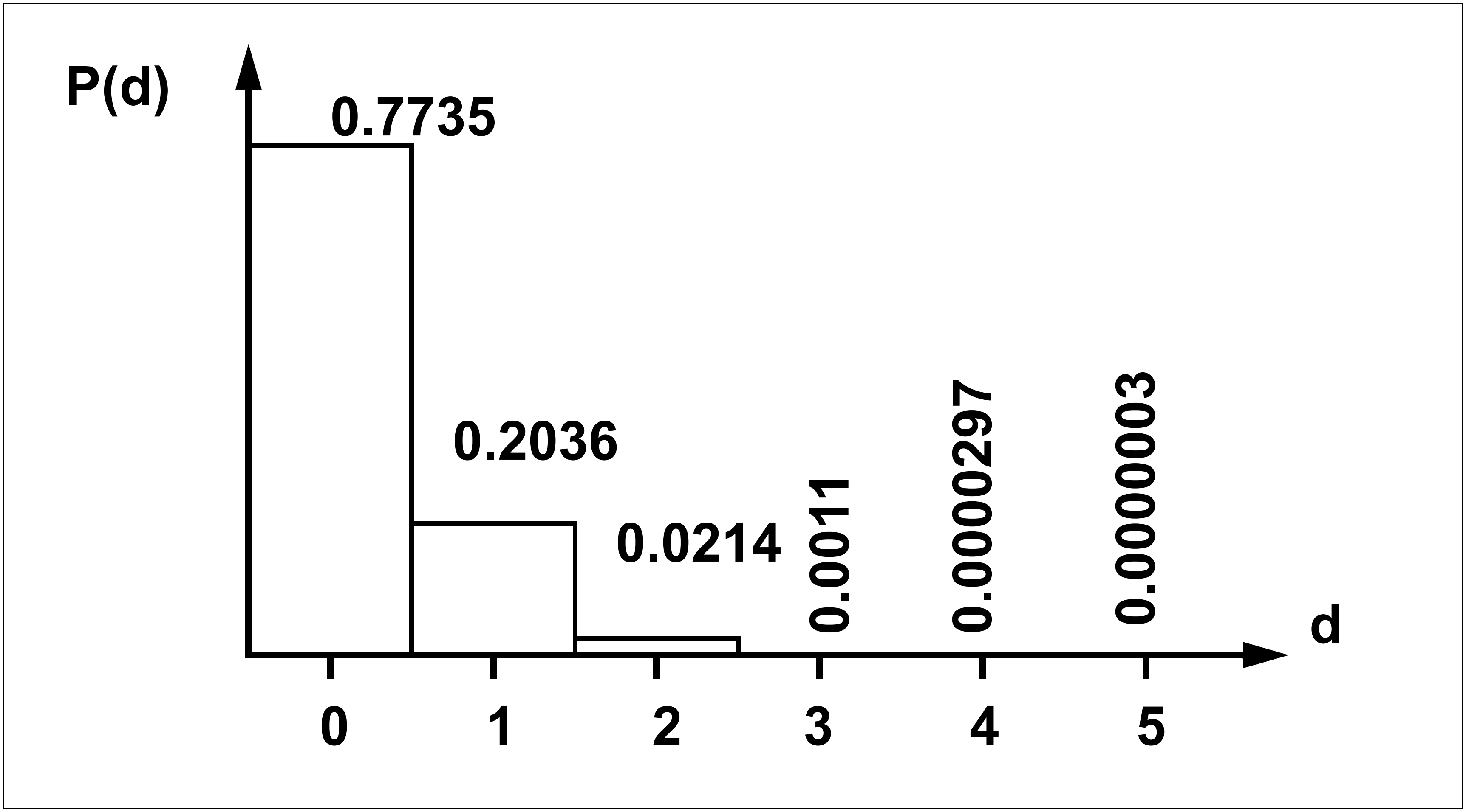
where, $\binom{n}{d} = \dfrac{n!}{d!(n-d)!}$

what if $\binom{1000000}{2} = ?$

So, when n = 5, p'=.10, P(d=1) = ??

When n = 10, p'=.10, P(d=3) = ??

b(d;n=5,p'=0.05)



Mean and Variance

$$\mu_d = np'$$

$$\sigma_d^2 = np'(1 - p')$$

$$\mu_p = E[p] =$$

$$\sigma_p^2 = Var[p]$$

Control Limits - p Chart

In principle, $\mu_p \pm 3\sigma_p$

We don't know μ_p or σ_p so we must estimate them.

Form of the control limits:

$$\hat{\mu}_p \pm 3\hat{\sigma}_p$$

Estimating μ_p for case of constant sample size, n

Estimating μ_p for case of varying sample size, n

Estimating σ_p

p (fraction defective) Chart

- Return to Injection molding example (n=100); instrument panels (flash, splay, voids, & short shots)
- 30 samples collected (100 panels per shift)
- d recorded for each sample; p = d/n calculated

$$\bar{p} = 0.079$$

$$\hat{\sigma}_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.027$$

LCL, UCL

Use 4 rules to interpret chart.

Process stable!!

But average fraction defective is high -- 8%.

Common cause problem. Flash accounts for 50% of the defects.

Mold pressure identified as the problem -- process settings adjusted.

30 addl. samples collected

Data plotted on existing charts -- interpretation

$$\bar{p} = 0.038$$

$$\hat{\sigma}_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.0191$$

LCL, UCL

Control Chart for Number of Defectives

- Sometimes, more convenient to make chart by plotting d rather than p.

- In particular, plotting d appropriate if
  - n is constant
  - p is very small

- Referred to as "np" chart. Basically same as p chart except for scaling factor, n.

- **np chart requires one less calculation since p = d/n need not be calculated for each sample**

- **Recall that**

$$\mu_d = np'$$

$$\sigma_d^2 = np'(1 - p') \quad \text{or} \quad \sigma_d = \sqrt{np'(1 - p')}$$

Form of the control limits (use $\bar{p}$ to estimate p'):

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1 - \bar{p})}$$

- **Revisit the instrument panel assembly example.**

$$\bar{p} = 237/3000 = 0.079$$

$$n\bar{p} = 100(0.079) = 7.9$$

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$7.9 \pm 3\sqrt{7.9(1-0.079)}$$

UCL_{np} = 16.00

LCL_{np} = - 0.20   so use a lower limit of 0.0

Problems with np Chart

- If sample size varies, control limits will vary from sample to sample

- If sample size varies, centerline will vary as well from sample to sample

- This makes chart interpretation difficult

- d value (easier calculations) easy to communicate -- but what does a value of d = 12 mean?? Doesn't mean much unless you also know sample size

Variable-Sample-Size p Charts

- Speaking of dealing with varying sample sizes, how do we manage this for our p Chart????
- Good news, $\bar{p}$ will not change from sample to sample, but control limits depend on n. How to handle:
  - Separate limits for each subgroup
  - Limits based on n-bar
  - Standardized p-chart

Manufacture of a Package Tray

Wood fiber/polymer composite extruded into thin sheets for use in hatchbacks. Sheets heated then molded into shape of tray. Carpet applied to sheet w/ adhesive. Carpeted tray trimmed to size.

200 - 300 trays per shift.

Defects observed: cloth coverage on hinge, carpet coverage problem, bleed-through, soiled carpet, improper flex on hinge, (chips, scratches, abrasions on back surface), wrinkles

Data for Package Tray Example

Samp.	n	d	p	$\hat{\sigma}_p$	LCL_p	UCL_p
1	238	11	0.046			
2	245	18	0.073			
3	270	17	0.063			
4	207	15	0.072			
: :	: :	: :	: :			

p Chart - Individual Limits

sum of the n's = 7433 **sum of the d's = 582**

$$\bar{p} = \left(\sum_{i=1}^{k} d_i\right) \Big/ \left(\sum_{i=1}^{k} n_i\right) = 582 / 7433 = 0.0783$$

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

For sample 1, 0.0783 ± 0.0527 : 0.026, 0.131

p Chart --- n-bar Limits

- Plot the p's and $\bar{p}$. Put limits on the chart based on $\bar{n}$.

$$\bar{n} = \left(\sum_{i=1}^{k} n_i \right) / k$$ **= 7433 / 30 = approx. 248**

$$\bar{p} \pm 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad 0.0783 \pm 3 \sqrt{\frac{0.0783(1-0.0783)}{248}}$$

Limits are 0.0271 & 0.1295

Pts near the limits must be interpreted separately.

p Chart --- Standardized Limits

- Calc. standardized p value for each sample

Sample	p	p - pbar	n	$\hat{\sigma}_p$	$Z=(p - pbar)/\sigma_p$
1	.046		238		
2	.073		245		
3	.063		270		