Lecture # 34

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More on
Control Charts for Individuals

Last time we worked with X and Rm control charts.

Remember -- only makes sense to use such a chart when the formation of a rational sample has no meaning.

The <u>consequences</u> of working with X and Rm charts.....

Very difficult to detect small shifts in the process mean or variability. Also, the charts are not independent of one another.

Exponentially Weighted Moving Average Control Charts

Basic concept:

$$\text{Moving Average}_i = r * X_i + (1 - r) * \text{Moving Average}_{i-1}$$

or

$A_i = r * X_i + (1 - r) * A_{i-1}$ Using our definition....

$A_{i-1} = r * X_{i-1} + (1 - r) * A_{i-2}$ plugging this in

$$A_i = r * X_i + (1 - r) * [r * X_{i-1} + (1 - r) * A_{i-2}]$$

Simplifying:

$$A_i = r X_i + (1 - r) r X_{i-1} + (1 - r)^2 A_{i-2}$$

Using this idea recursively,

$$A_i = r X_i + (1 - r) r X_{i-1} + (1 - r)^2 r X_{i-2} + (1 - r)^3 A_{i-3}$$

$$A_i = r X_i + (1 - r) r X_{i-1} + (1 - r)^2 r X_{i-2} + (1 - r)^3 r X_{i-3} + ...$$

We can express this using summation notation:

$$A_i = \sum_{j=0}^{\#\text{ lags considered}} r(1-r)^j X_{i-j}$$

or

$$A_i = \sum_{j=0}^{\#\text{ lags considered}} W_j \; X_{i-j}$$

where, W_j is the weight associated with jth lag

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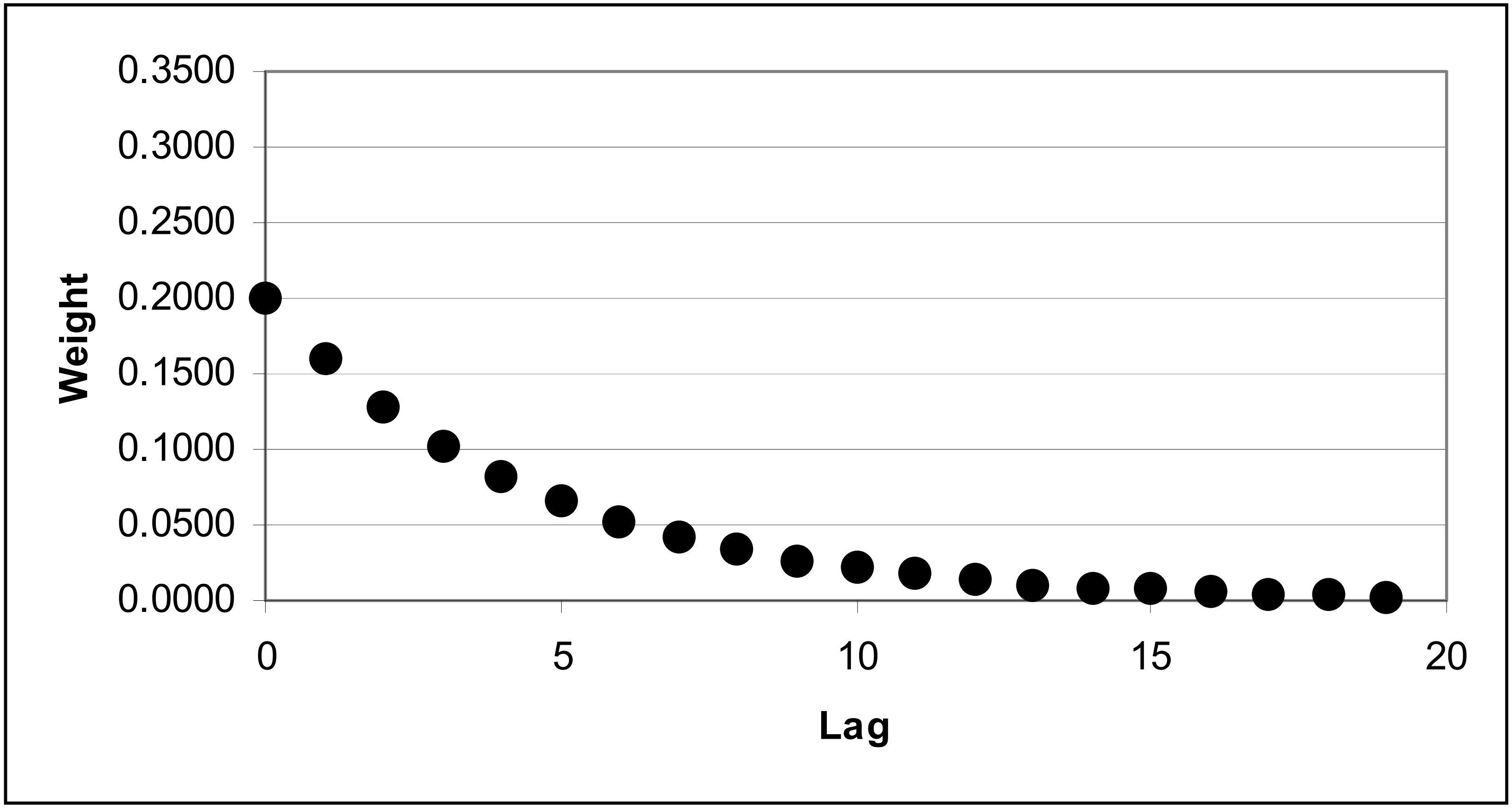
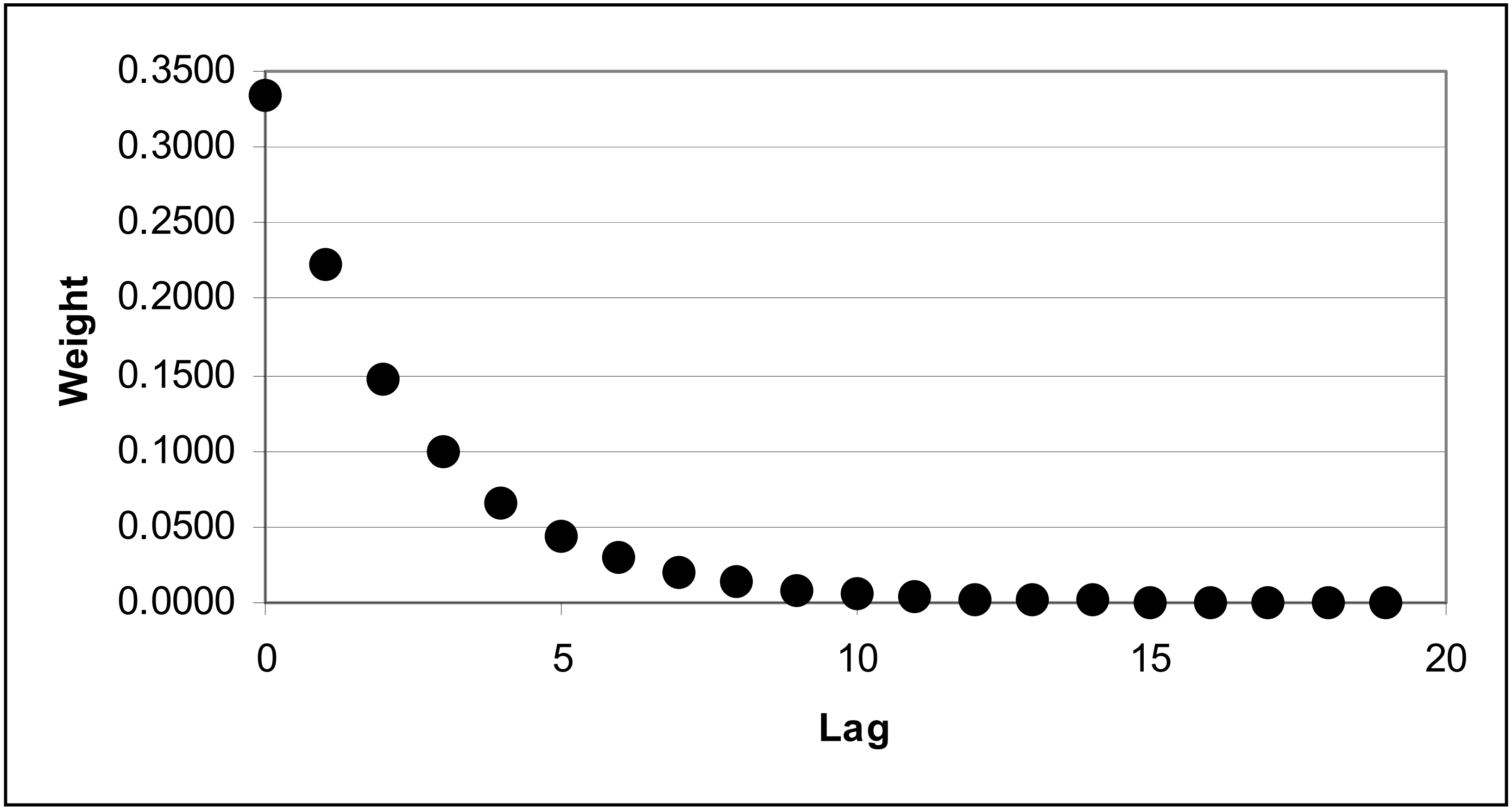

0.3500
0.3000
0.2500
0.2000
0.1500
0.1000
0.0500
0.0000
Weight
0
5
10
15
20
Lag

r =0.2



0.3500
0.3000
0.2500
0.2000
0.1500
0.1000
0.0500
0.0000
Weight
0
5
10
15
20
Lag

r =0.3333

- **Note that when r = 1, all weight is assigned to current observation.**

- **Small values for r, the moving average "forgets" very slowly. Therefore the moving average carries much inertia with it. Insensitive to small, short-lived mean shifts.**

- **Larger values for r (say 0.2 to 0.5), are used when fast response to process shifts is needed.**

$$r = 2/(n+1)$$

Relation between r and Shewhart sample size, n

Constructing EWMA Control Charts

- **Collect k individual measurements, X_i**
- **Calculate estimates of process mean and std. dev.**

$$\bar{X} = \sum_{i=1}^{k} X_i / k$$

$$s_x = \left[\sum_{i=1}^{k} (X_i - \bar{X})^2 / (k-1) \right]^{\frac{1}{2}}$$

- **Compute moving averages, A_i, absolute deviations, D_i, and moving standard deviations, V_i**

$$A_i = rX_i + (1-r)A_{i-1}$$ **use $\bar{X}$ for A_0**

$$D_i = |X_i - A_{i-1}|$$

$$V_i = rD_i + (1-r)V_{i-1}$$ **use s_x for V_0**

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Control Limits

(constants from Table 11.3)

Moving Average (A) Chart: $CL = \bar{X}$

$$LCL_A = \bar{X} - s_x A^*$$

$$UCL_A = \bar{X} + s_x A^*$$

Moving Deviations (V) Chart: $CL = d_2^* s_x$

$$LCL_V = D_1^* s_x$$

$$UCL_V = D_2^* s_x$$

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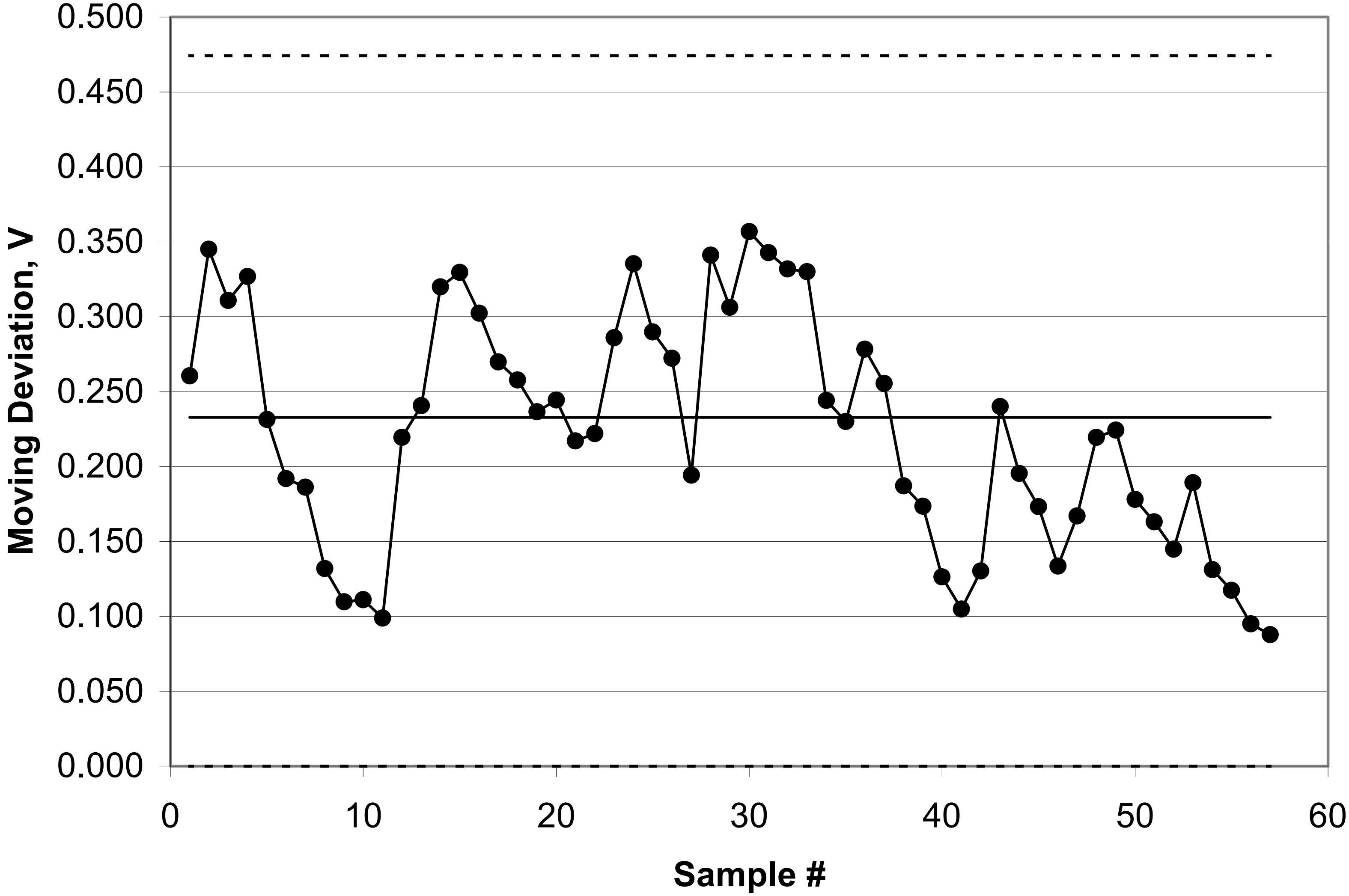
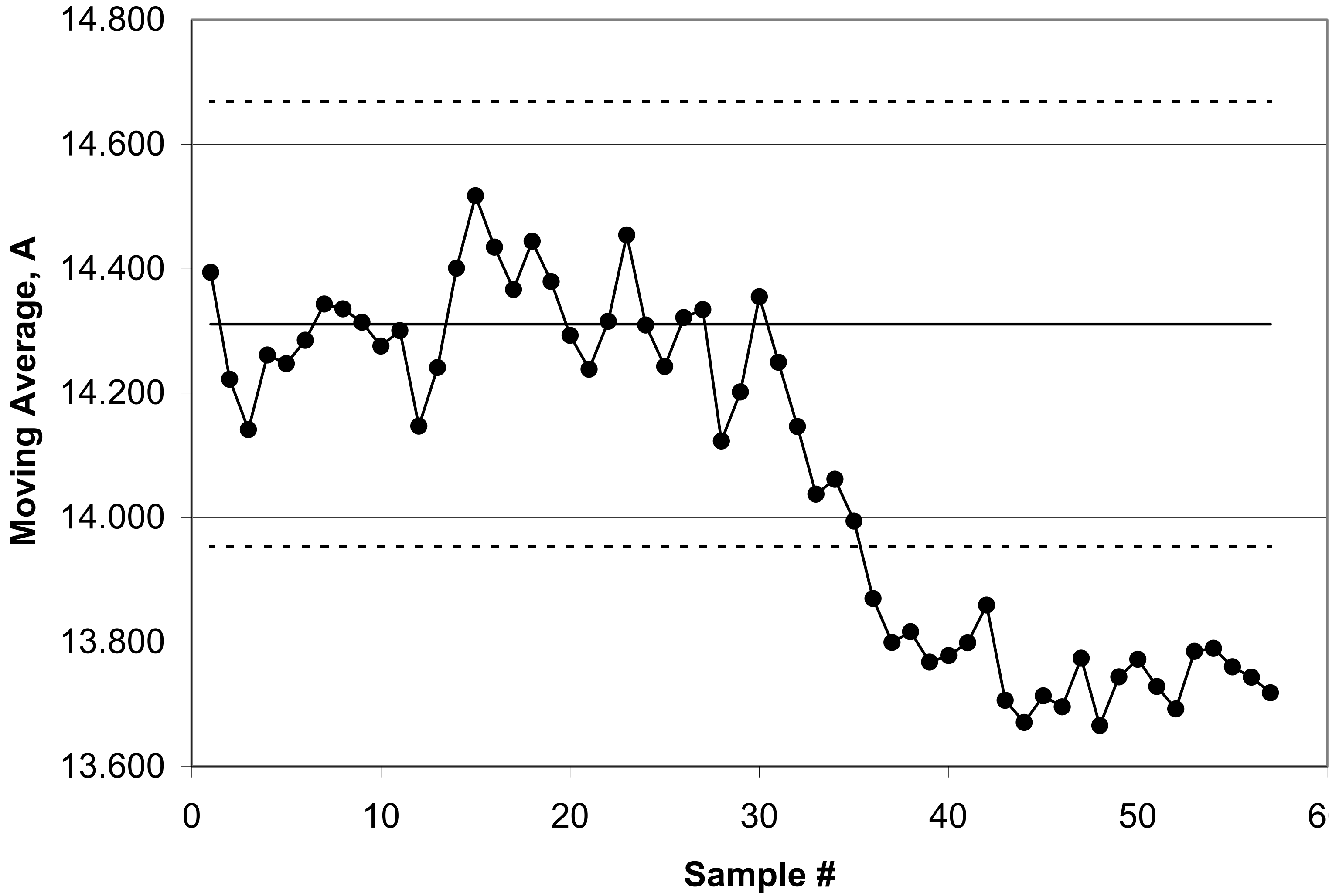
Chart Interpretation

- Shewart Charts -- data are independent. EWMA Charts -- data are correlated -- can’t use our rules!!
- Common to see runs above/below centerline.
- For EWMA charts, generally no points beyond limits. Instead, must look for trends in the data.
- No obvious trends in the V or the A charts.
- Let’s look at the charts after the millbase batch adjustment chart implementation.


0.500
0.450
0.400
0.350
0.300
0.250
0.200
0.150
0.100
0.050
0.000
Moving Deviation, V
0
10
20
30
40
50
60
Sample #



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Conclusion

- From <u>these</u> charts, evidence of change in process mean and process variability evident.
- Notice how moving average and deviation stabilizes at new levels after the change in the process.
- Redisplay A and V charts for samples 31 - 57
- Process looks stable

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CUSUM Charts

- We've looked at control charts for individuals
  - X and Rm charts
  - EWMA charts

  Charted Statistic is dependent on past values

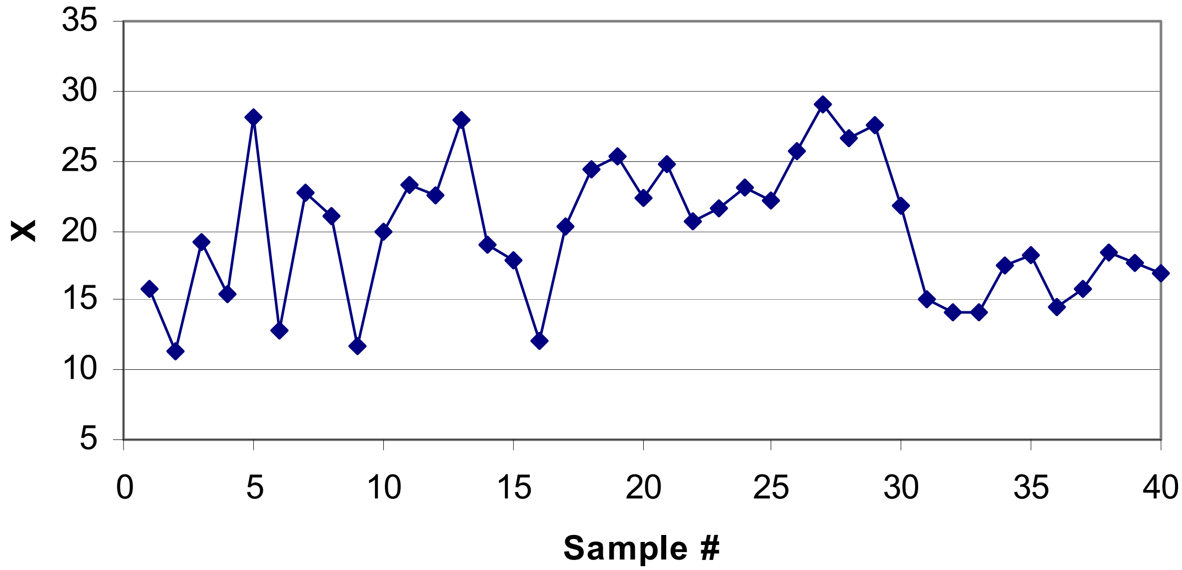
- Time Series

  An analysis technique to study autocorrelated data

  Extract underlying dynamics of the system

- Cumulative Sum -- Cusum -- Statistic is sum of past individual measurements, sample means, sample ranges, etc.

Behavior of Cusums



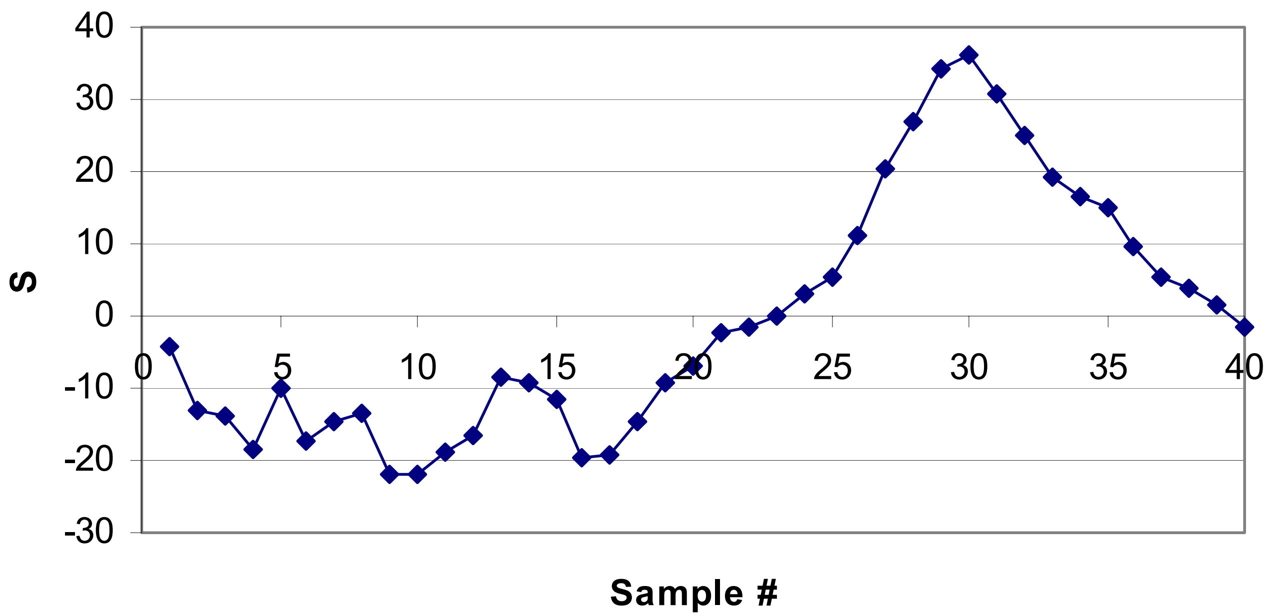
Sample data set #1

Base process: N(20,4^2) for samples 1-20
Shift in mean to 24 at sample 21
Shift in mean to 16 at sample 31

Cusum

Define cumulative statistic as

$$S_t = \sum_{i=1}^{t} (X_i - \mu_x) = S_{t-1} + (X_t - \mu_x)$$



Cusum Chart

- **Same caution as we have discussed recently -- data are correlated -- trends/run rules don't apply**

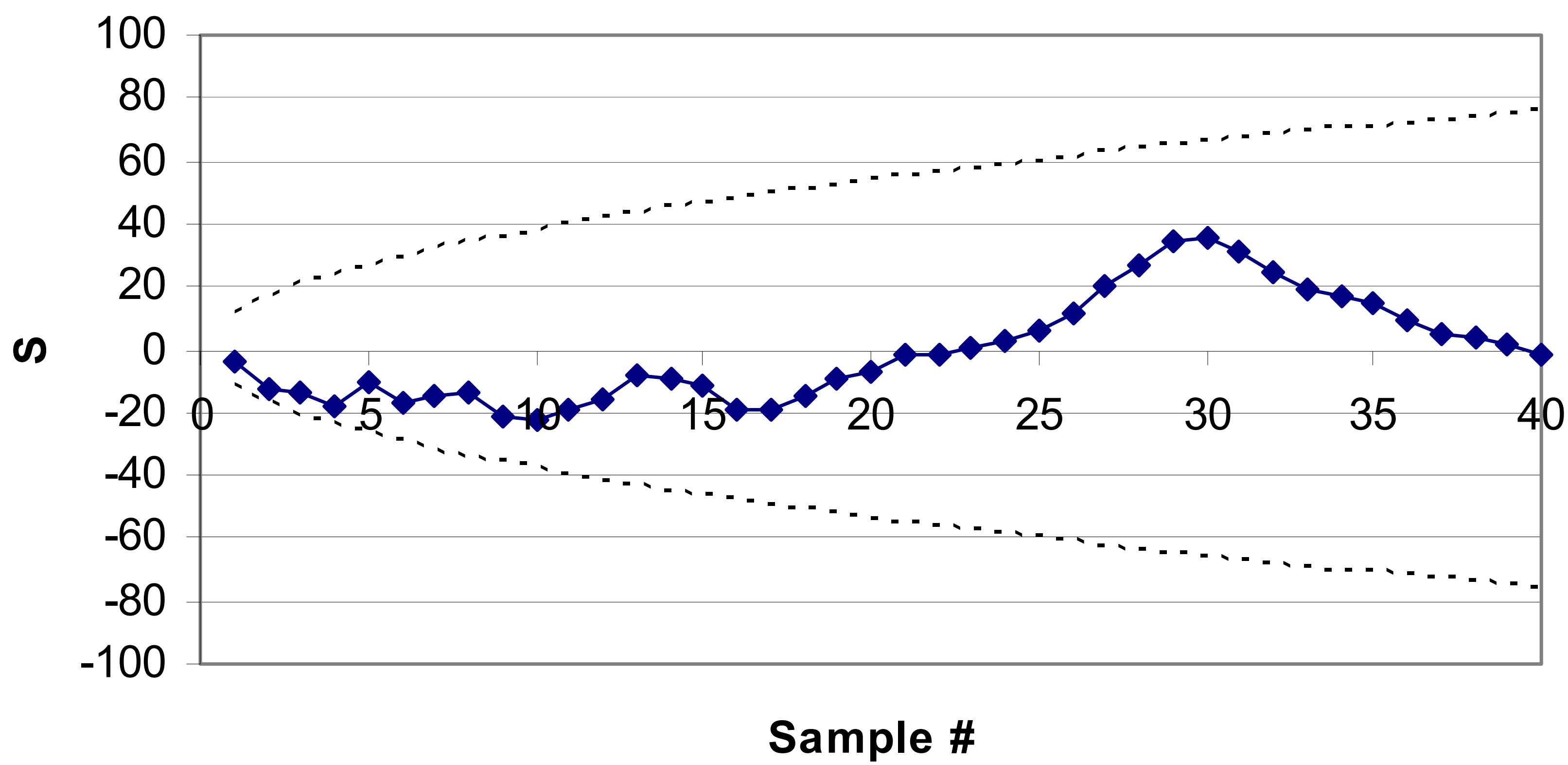
- **Can we construct limits for the chart?**

- **Variability at the time = t**

$$\sigma_t^2 = t \ \sigma_x^2 \qquad\qquad \sigma_t = \sqrt{t} \ \sigma_x$$

- **So, if we set the limits at (± 3 sigma) we obtain the limits** $\pm 3 \ \sqrt{t} \ \sigma_x$

Basic Cusum with Control Limits



Interpreting the Chart

- Chart shows no obvious "signals"
- However, trends up & down (due to mean shifts) are evident -- if they would've lasted longer -- exceed limits
- One problem -- the limits are correct, but not the same. Different limits for different samples can be confusing.
- Would be nice if we had a chart that had constant control limits (i.e., same values for all samples)

Standardized Cusum

Start with the X_i values (mean = μ_X and std dev = σ_X)

$$Z_i = \frac{X_i - \mu_X}{\sigma_X}$$

$$S_t^* = \frac{\sum_{i=1}^{t} Z_i}{\sqrt{t}}$$

Construction of the Cusum Chart: A step by step procedure

1. Collect at least k=25 individual measurements in time order, $X_1, X_2, \ldots, X_k$.

2. Compute X-bar and s_x from the data.

$$\bar{X} = \frac{\sum_{i=1}^{k} X_i}{k}, \qquad s_X = \left[\sum_{i=1}^{k} \frac{(X_i - \bar{X})^2}{k-1} \right]^{\frac{1}{2}}$$

3. Standardize all the X's into Z's for i=1,2,3,...,k

$$Z_i = \frac{X_i - \bar{X}}{s_X}$$

Construction of the Cusum Chart: A step by step procedure

4. Sum the Z's cumulatively for each t, t=1,2,3,...,k

$$sum_t = \sum_{i=1}^{t} Z_i$$

5. Obtain the standardized cusum for each t, t=1,2,3,...,k;

$$S_t^* = \frac{sum_t}{\sqrt{t}}$$

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6. Plot the S_t^* on the standardized cusum chart, where centerline=0; UCL=3; LCL=-3

7. Interpret the Chart, looking expecially for poss. trends

An Example

TABLE 12.2 Standardized Cumulative Sums of the Millbase Batch Weights (Batches 31 to 57)

Sample	X_i	Z_i	sum_t	S_t^*
1	14.04	1.567	1.567	1.567
2	13.94	0.980	2.547	1.801
3	13.82	0.276	2.823	1.630
4	14.11	1.978	4.801	2.401
5	13.86	0.511	5.312	2.376
6	13.62	−0.898	4.414	1.802
7	13.66	−0.663	3.751	1.418
8	13.85	0.452	4.203	1.486
9	13.67	−0.605	3.598	1.199
10	13.80	0.158	3.756	1.188
11	13.84	0.393	4.149	1.251
12	13.98	1.215	5.364	1.548
13	13.40	−2.189	3.175	0.881
14	13.60	−1.015	2.160	0.577
15	13.80	0.158	2.318	0.599
16	13.66	−0.663	1.655	0.414
17	13.93	0.921	2.565	0.622
18	13.45	−1.896	0.669	0.158
19	13.90	0.745	1.414	0.324
20	13.83	0.335	1.749	0.391
21	13.64	−0.781	0.968	0.211
22	13.62	−0.898	0.070	0.015
23	13.97	1.156	1.226	0.256
24	13.80	0.158	1.384	0.283
25	13.70	0.428	0.956	0.191
26	13.71	−0.370	0.586	0.115
27	13.67	−0.605	−0.019	−0.004

Standardized Cusum Control Chart

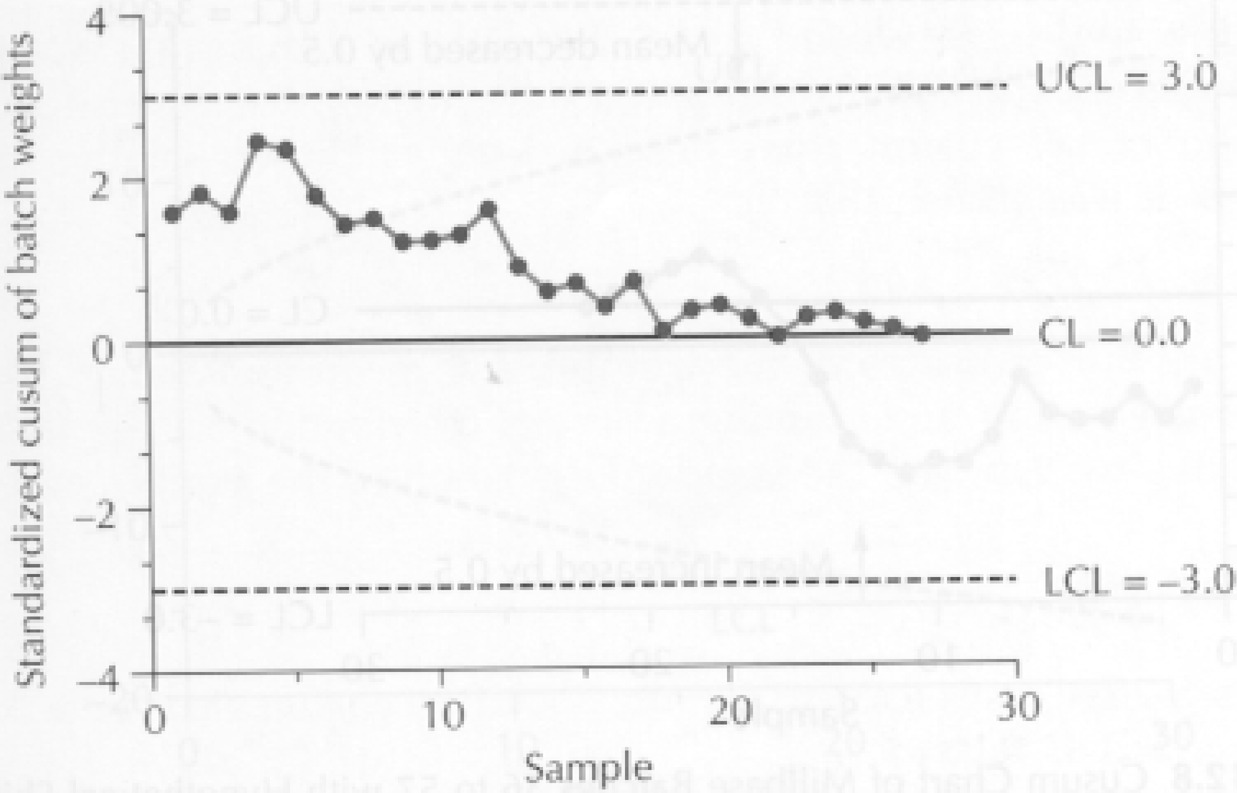


Figure 12.6 Standardized Cusum Control Chart for the Millbase Batches 31 to 57

Revised Cusum Chart - Example

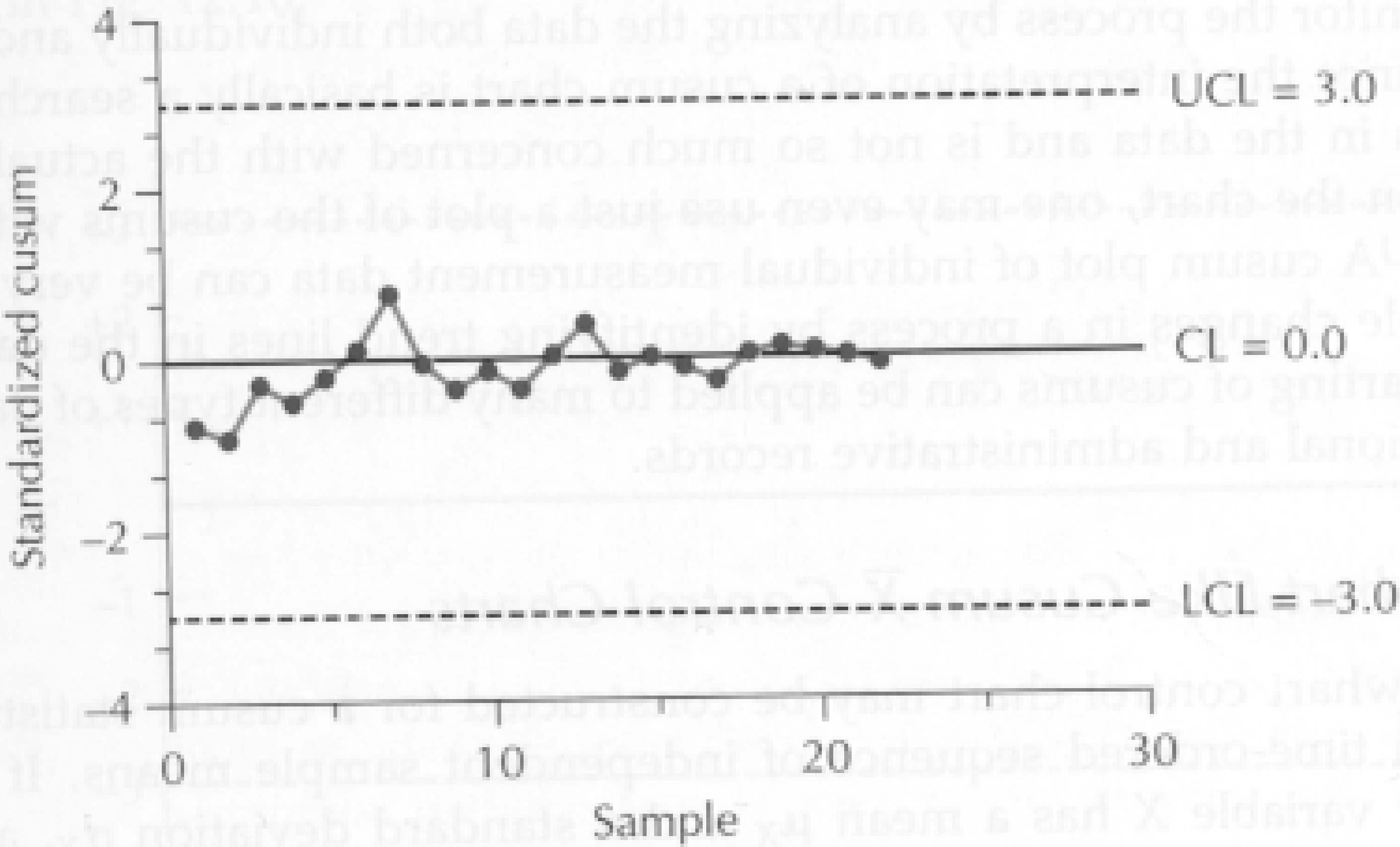


Figure 12.7 Revised Cusum Chart for the Millbase Batches 36 to 57

Cusum Chart -- Hypothetical Shifts

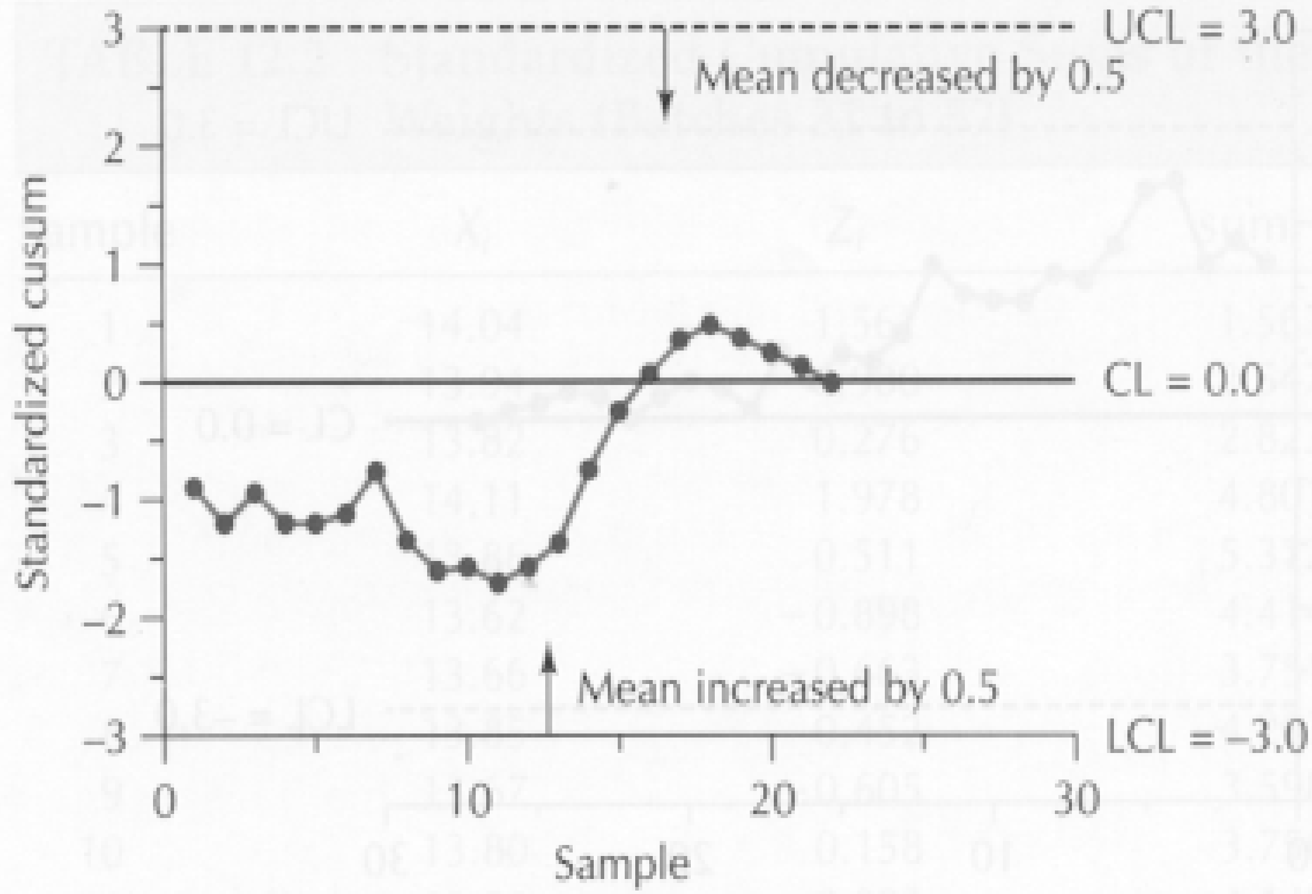
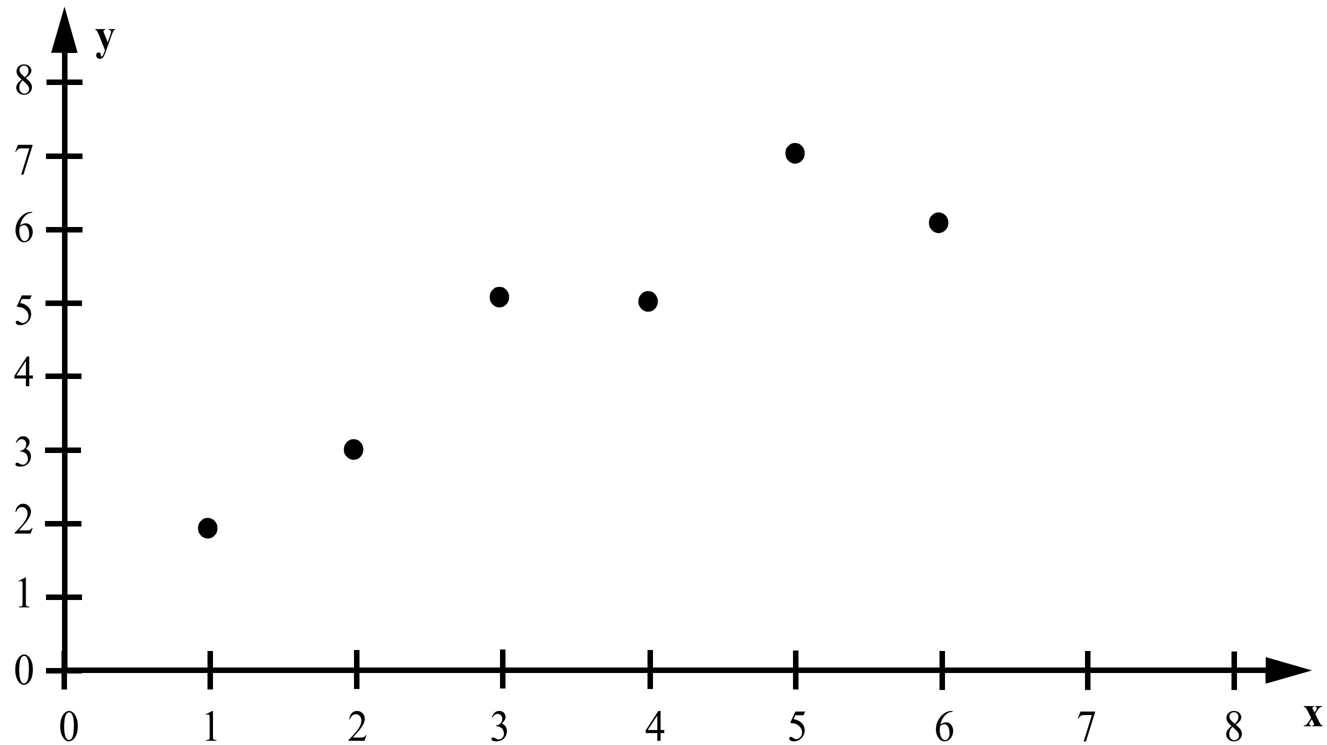


Figure 12.8 Cusum Chart of Millbase Batches 36 to 57 with Hypothetical Shifts

Linear Regression



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