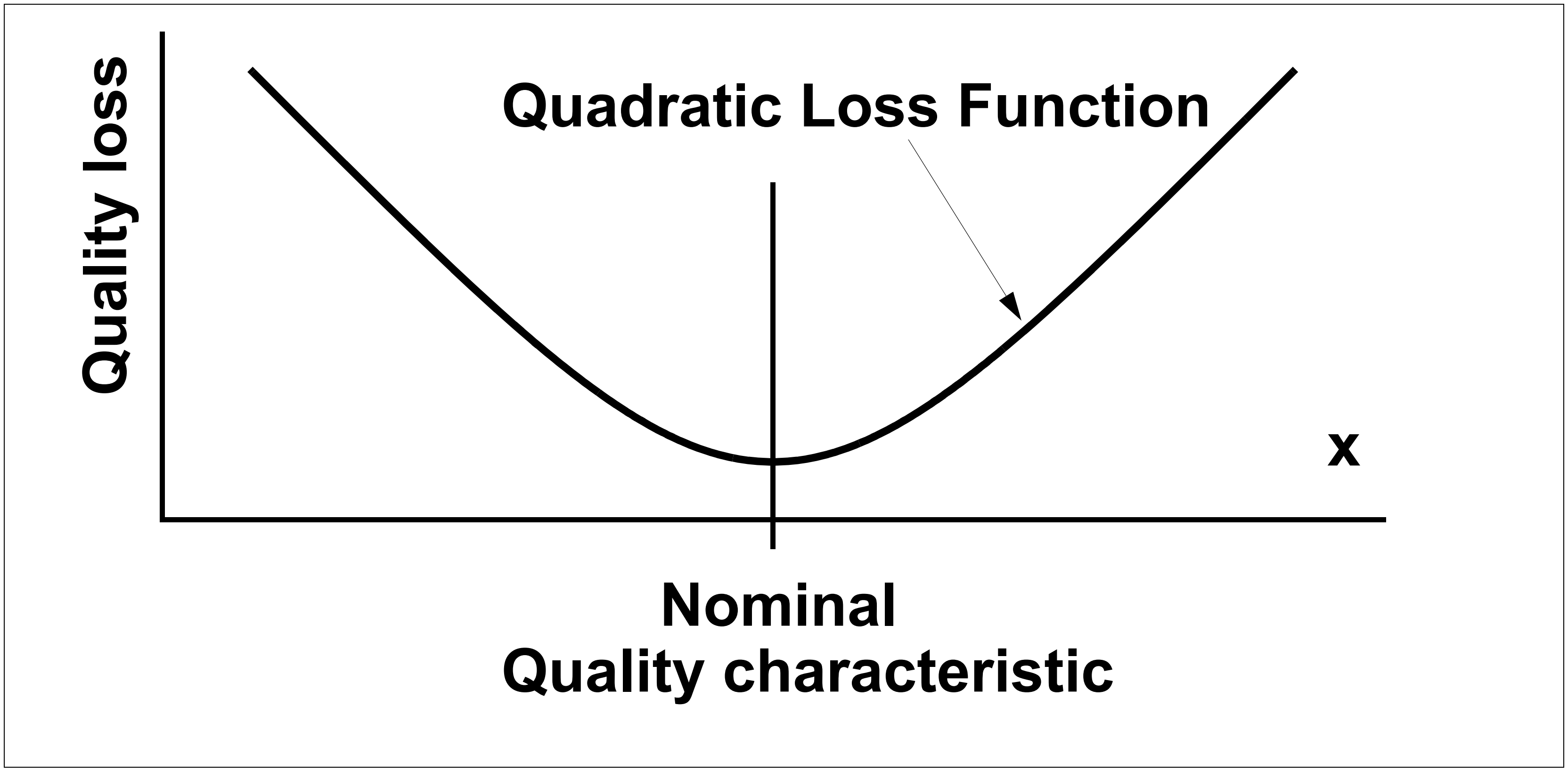
Lecture # 29

Prof. John W. Sutherland

Nov. 2, 2005

Capability Assessment via a Loss Function



Defining the Loss Function

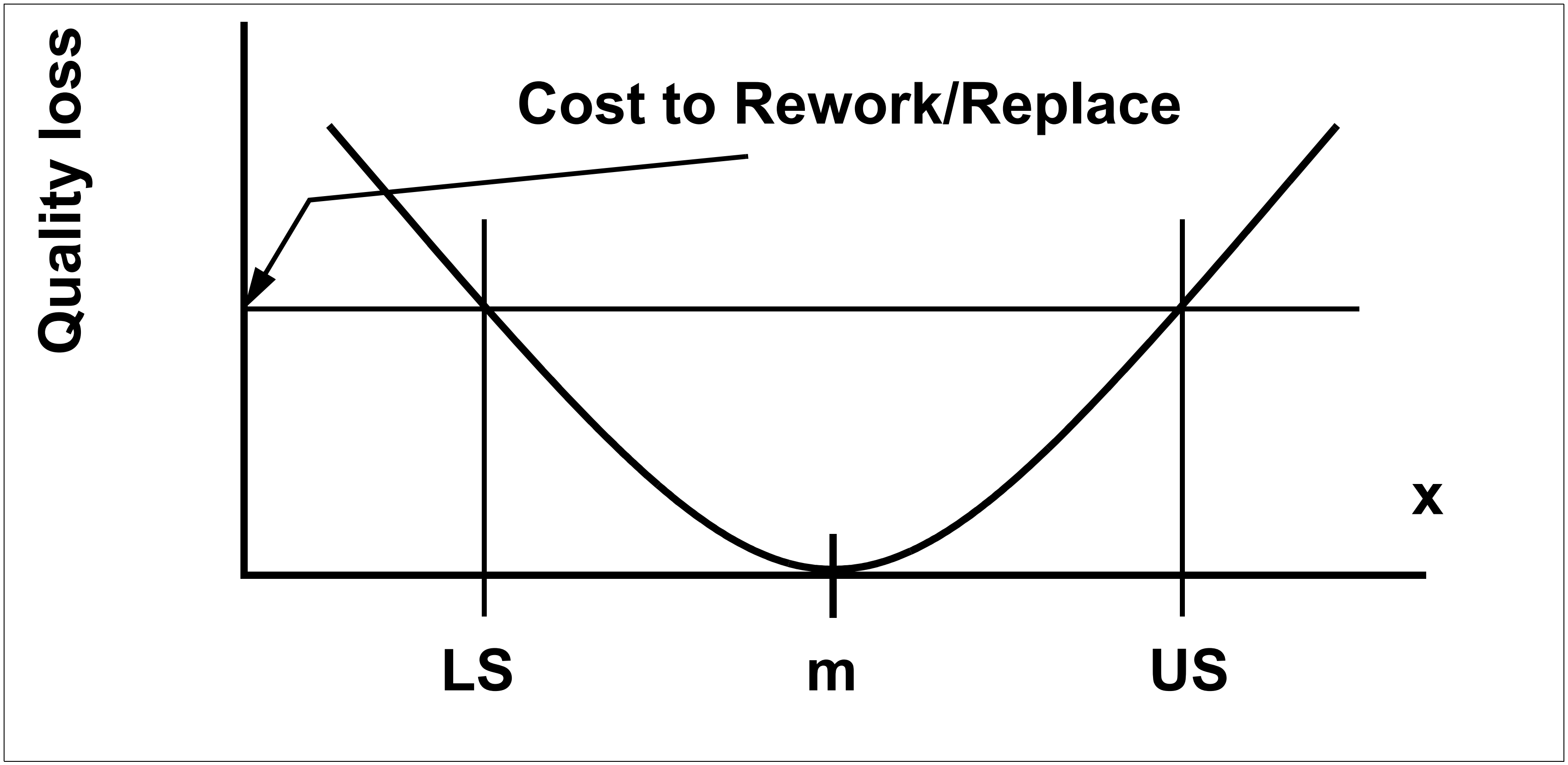
$$L = L_0 + k(x - m)^2$$

- for an x value of 0, the loss, L, is \$0.
  This means that m = 0 and $L_0 = 0$

- when x= 1.5, the loss, L, is \$2.25
  $L/(x\text{-}m)^2 = k$ $k = \$2.25/(1.5 - 0)^2 = 1$

  The \$2.25 loss is obtained from the fact that when x=1.5, the probability of a \$45 complaint is 5%, and \$45 * (0.05) = \$2.25

Loss Function Interpretation of Engineering Specifications



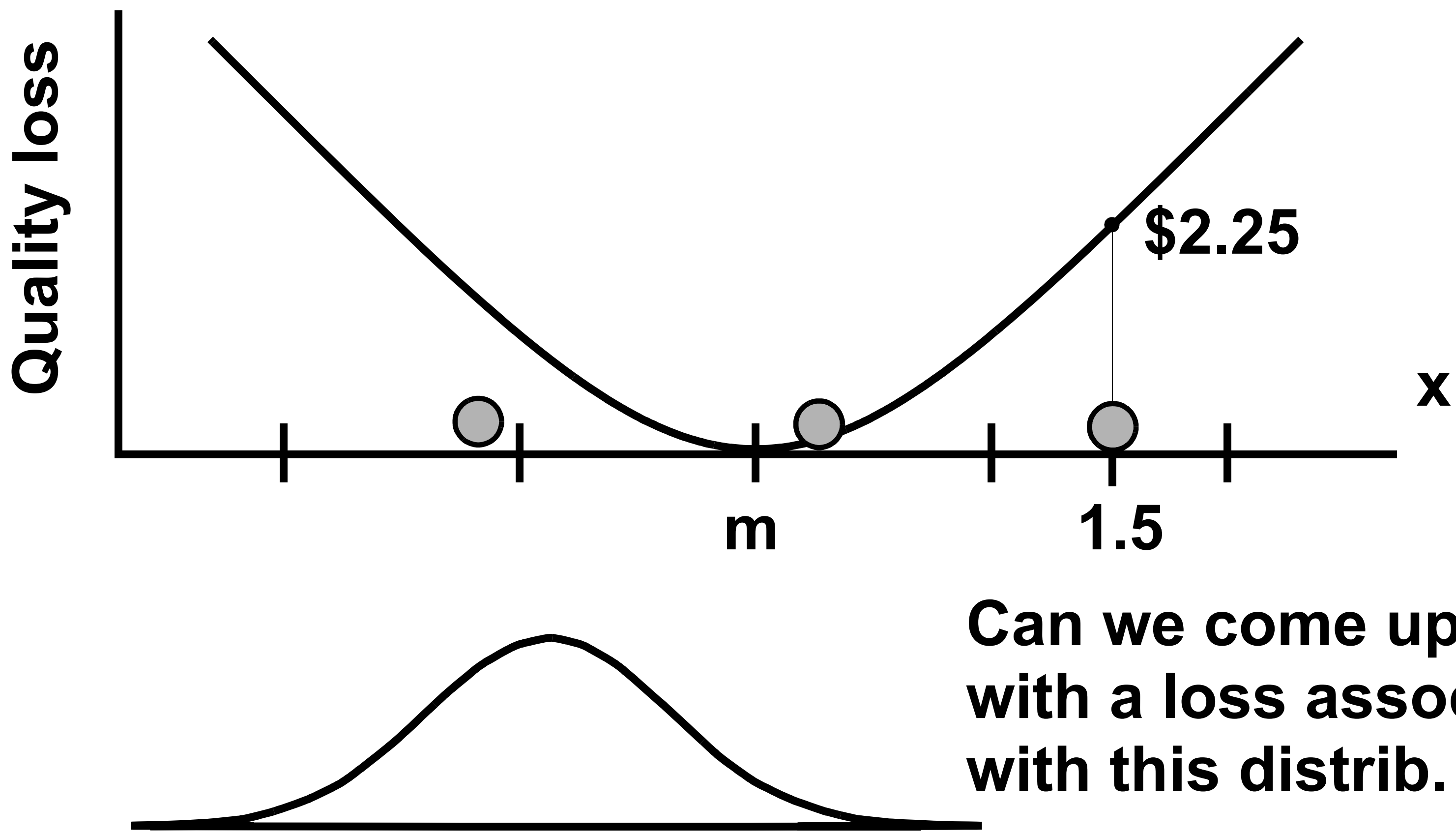
More on Loss / Specifications

So, we want to set our specifications at the point of indifference: where the loss = cost of replacement

If cost of replacement = $2 for our previous example,

$L = 1\,(x - 0)^2 = 2$ ----- $x = \pm 1.414$

More on the Loss Function



Can we come up with a loss assoc. with this distrib.

Expected Process Loss

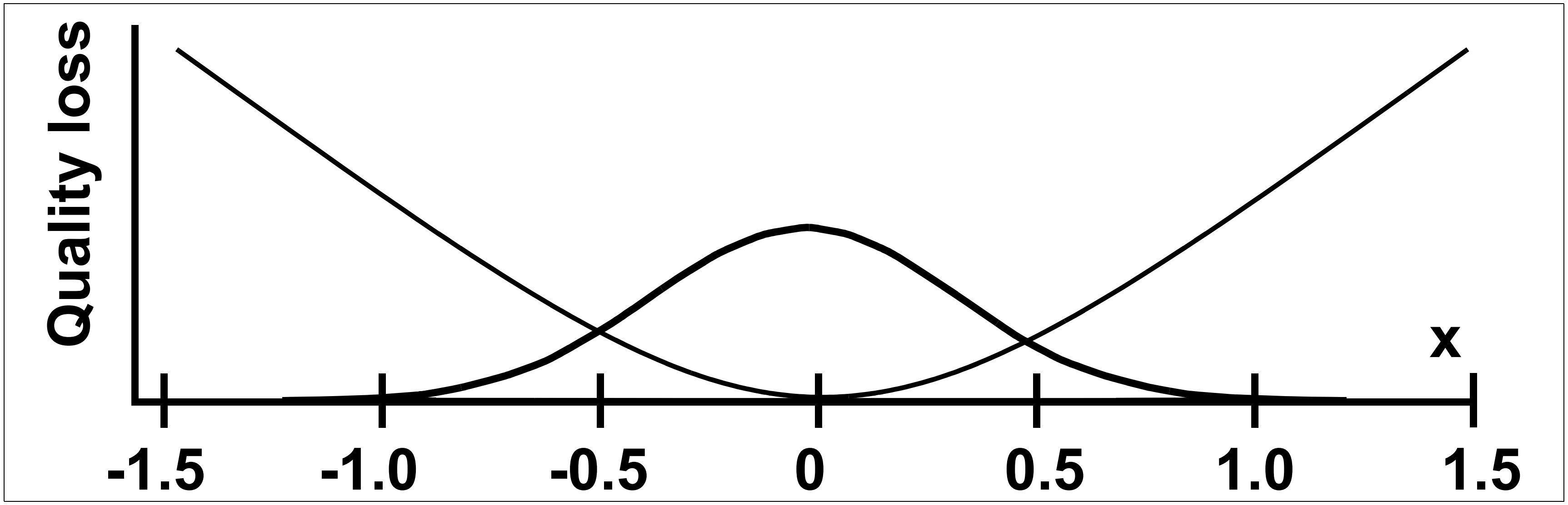
$$E[L(X)] = \int_{\text{all } X} L(t)f(t)\,dt$$ where,

$$L(x) = k(x - m)^2$$ and f(t) is the pdf

after simplification,

$$E[L(X)] = k\left\{(\mu_X - m)^2 + \sigma_X^2\right\}$$

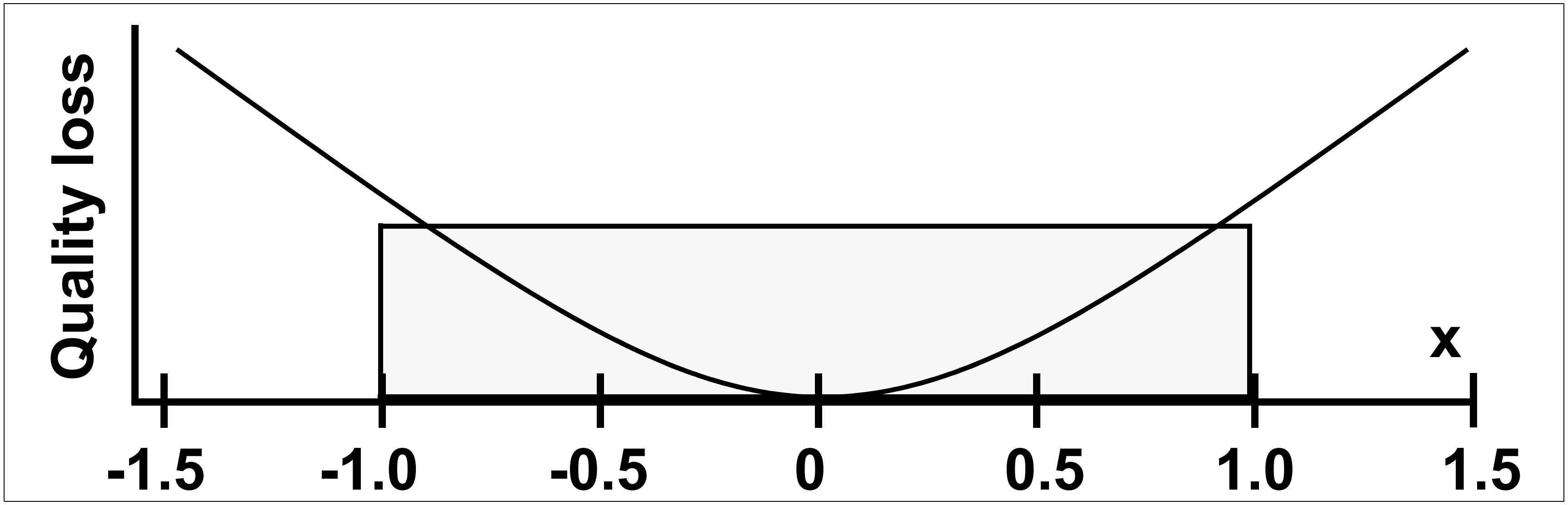
Loss Function: Example #1



m=0, k=$1.0, LSL=-1, USL=1,
X normal w/ mean 0 and σ_x = 0.33

$E[L] = 1\ [0.33^2 + (0-0)^2] = \$0.11$ How to interpret??

Loss Function: Example #2



$\sigma_x = 0.58$

$E[L] = 1\ [0.58^2 + (0\text{-}0)^2] = \$0.33$

Filling in a Past Blank

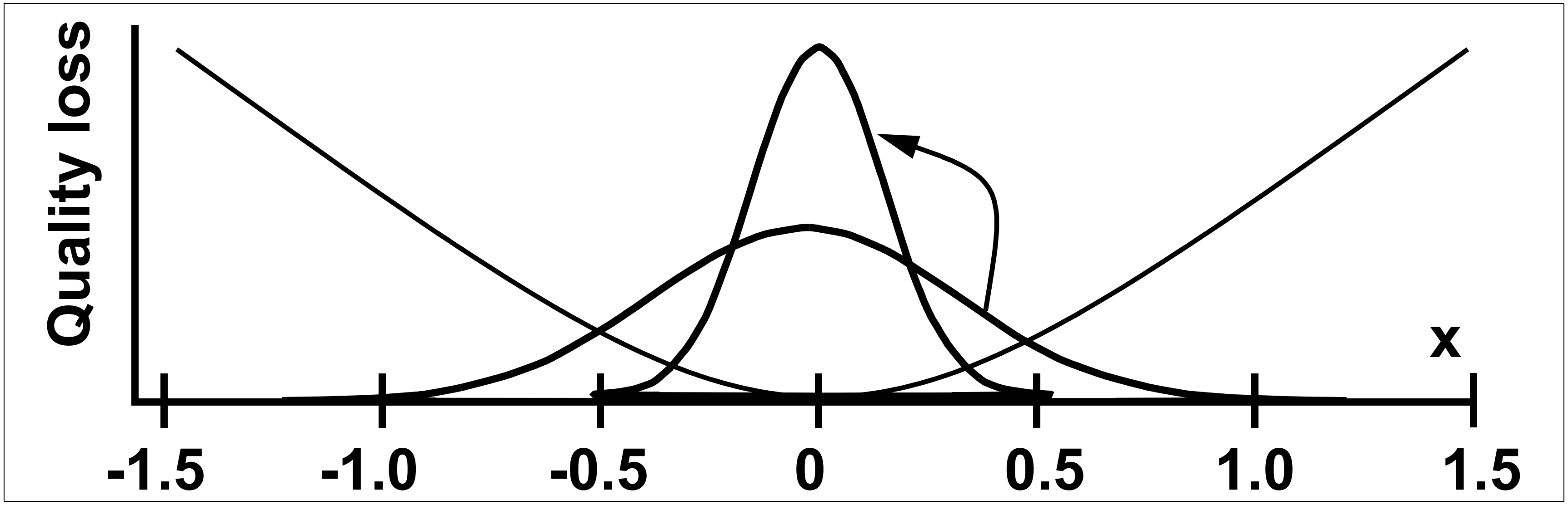
Should know from prior Statistics Class
For a Uniform Distribution between a & b

$$f(x) = 1/(b-a)$$

$$\mu_x = \int_a^b x f(x) dx = \int_a^b \frac{x}{(b-a)} dx = \left. \frac{x^2}{2(b-a)} \right|_a^b = \frac{(b+a)}{2}$$

$$\sigma_x^2 = \int_a^b (x-\mu_x)^2 f(x) dx = \frac{(b-a)^2}{12}$$

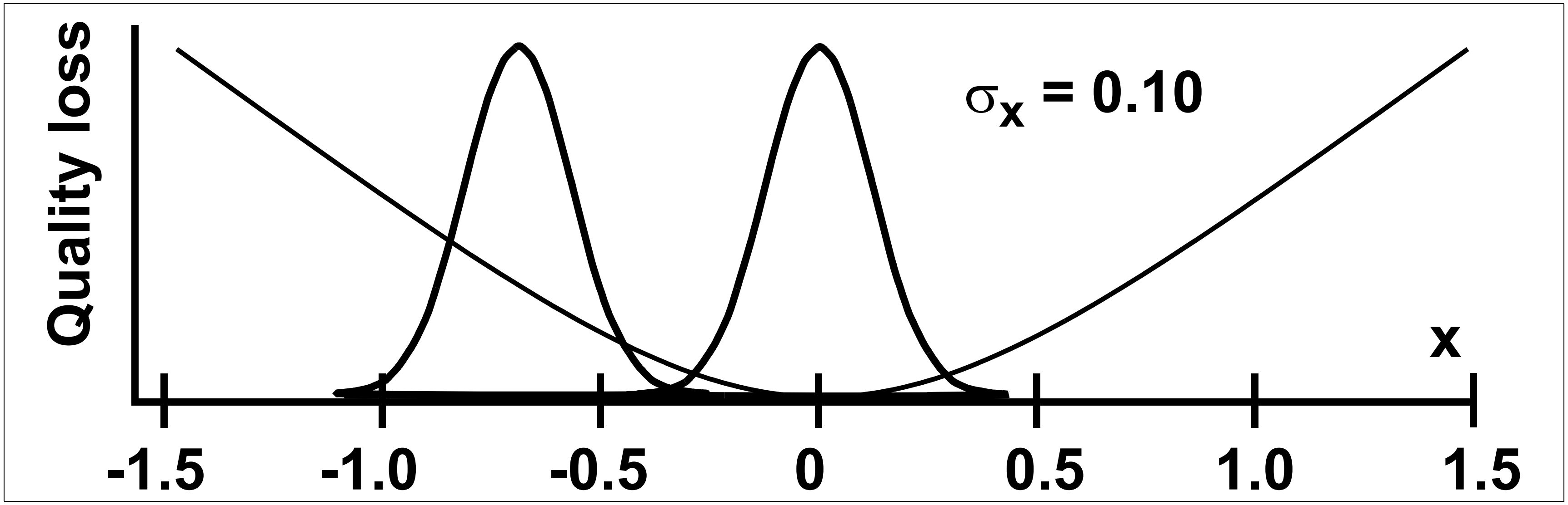
Loss Function: Example #3



Say we reduce $\sigma_x = 0.33$ to $\sigma_x = 0.10$

$E[L] = 1\ [0.10^2 + (0-0)^2] = \$0.01$

Loss Function: Example #4



Can save \$0.10 by reducing process mean from 0 to -0.7

$E[L] = 1\ [0.10^2 + (-0.7 - 0)^2] = \$0.50$ WOW!!

Taguchi's comment:

Chapter 10

Bring text to class!!

Roll Mill Process Case Study

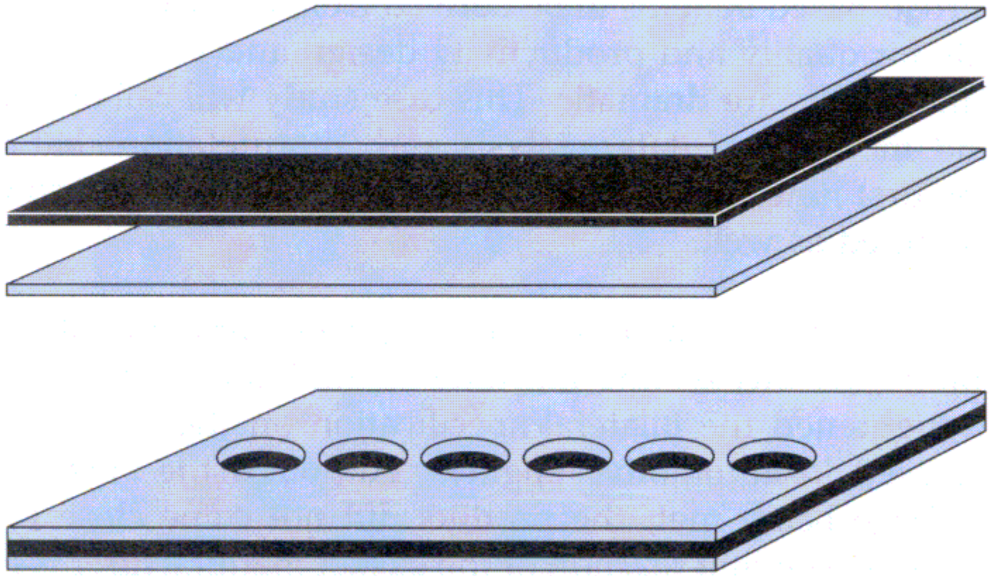


Figure 10.1 The Product: A Hard Gasket of Three-Piece Construction for Automotive Applications

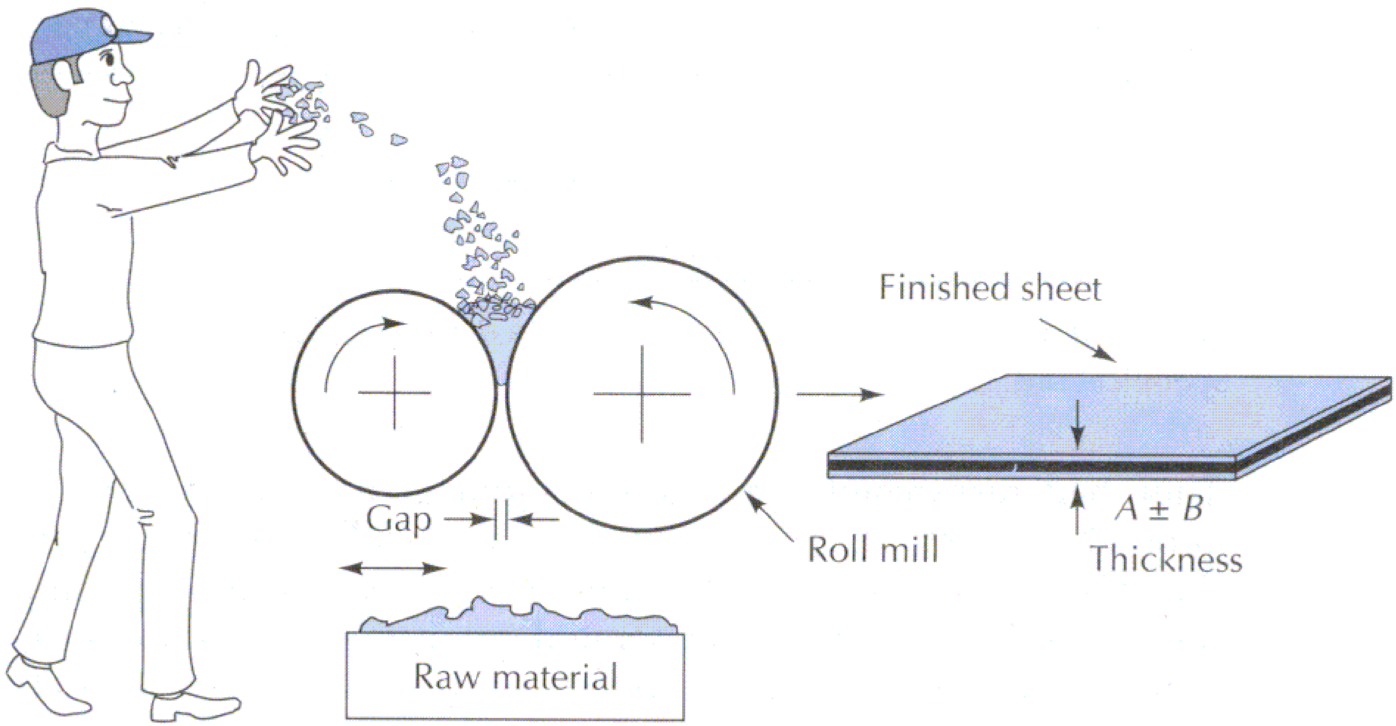


Figure 10.2 The Process: Roll Mill for the Manufacture of Soft Gasket Material

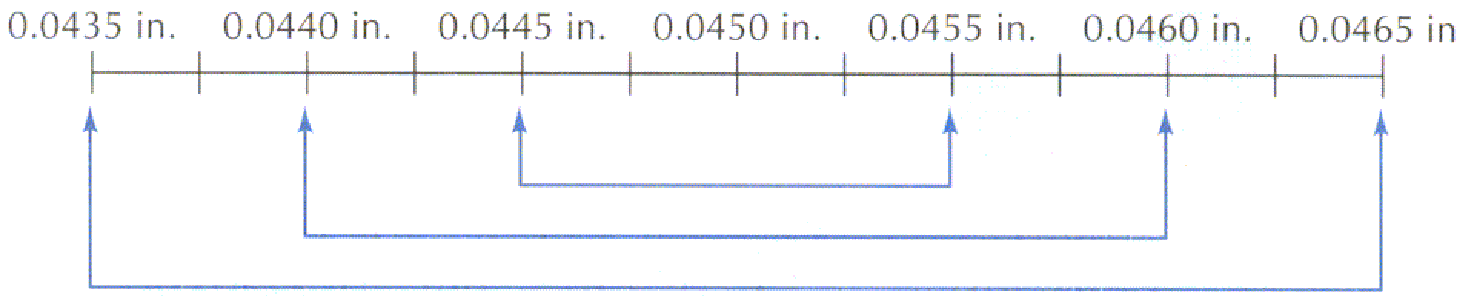


Figure 10.3 Selective Assembly Process Used to Assemble Hard Gaskets That Meet the Specifications

TABLE 10.1 Measurements of Sheet Thickness

Sheet	Thickness (in.)	Sheet	Thickness (in.)	Sheet	Thickness (in.)	Sheet	Thickness (in.)
1	0.0440	19	0.0472	37	0.0459	55	0.0425
2	0.0446	20	0.0477	38	0.0468	56	0.0442
3	0.0437	21	0.0452	39	0.0452	57	0.0432
4	0.0438	22	0.0457	40	0.0456	58	0.0429
5	0.0425	23	0.0459	41	0.0471	59	0.0447
6	0.0443	24	0.0472	42	0.0450	60	0.0450
7	0.0453	25	0.0464	43	0.0472	61	0.0443
8	0.0428	26	0.0457	44	0.0465	62	0.0441
9	0.0433	27	0.0447	45	0.0461	63	0.0450
10	0.0451	28	0.0451	46	0.0462	64	0.0443
11	0.0441	29	0.0447	47	0.0463	65	0.0423
12	0.0434	30	0.0457	48	0.0471	66	0.0447
13	0.0459	31	0.0456	49	0.0427	67	0.0429
14	0.0466	32	0.0455	50	0.0437	68	0.0427
15	0.0476	33	0.0445	51	0.0445	69	0.0464
16	0.0449	34	0.0448	52	0.0431	70	0.0448
17	0.0471	35	0.0423	53	0.0448	71	0.0451
18	0.0451	36	0.0442	54	0.0429	72	0.0428

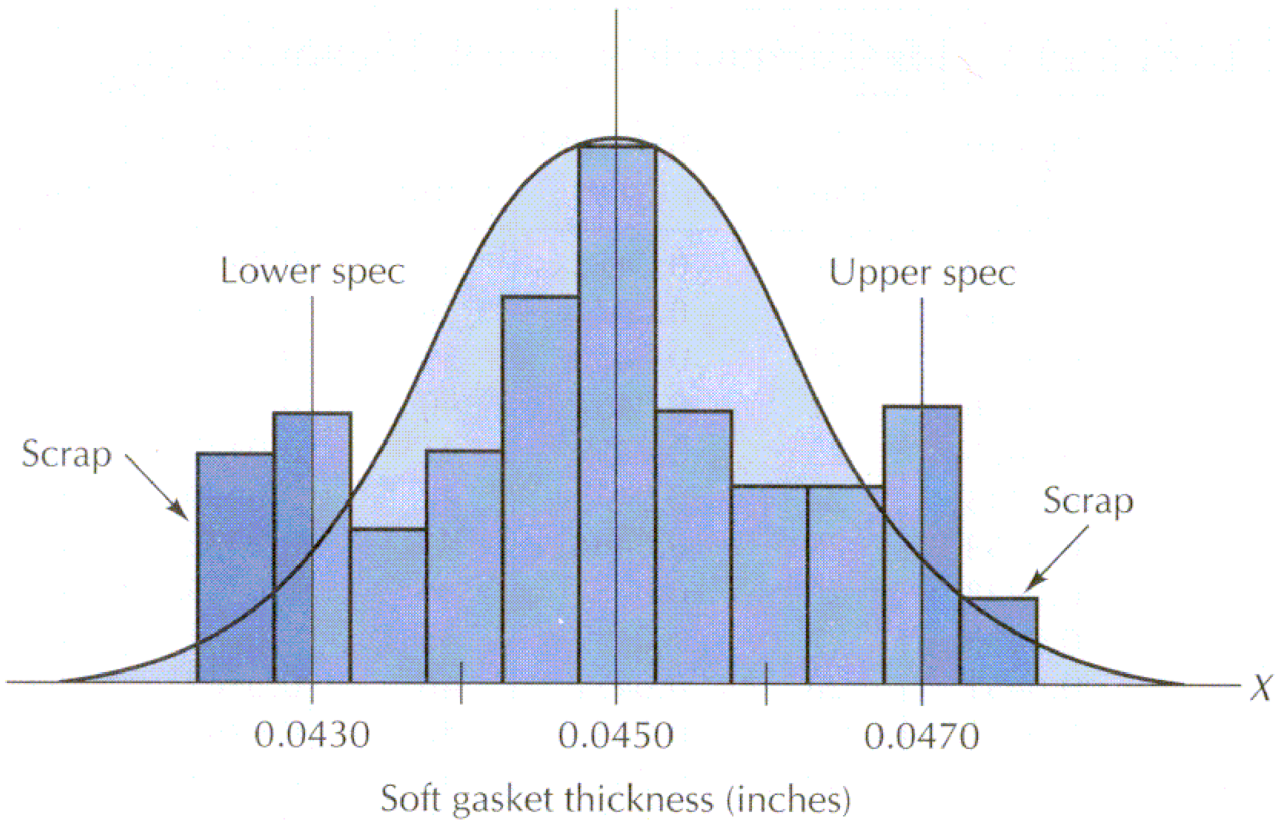
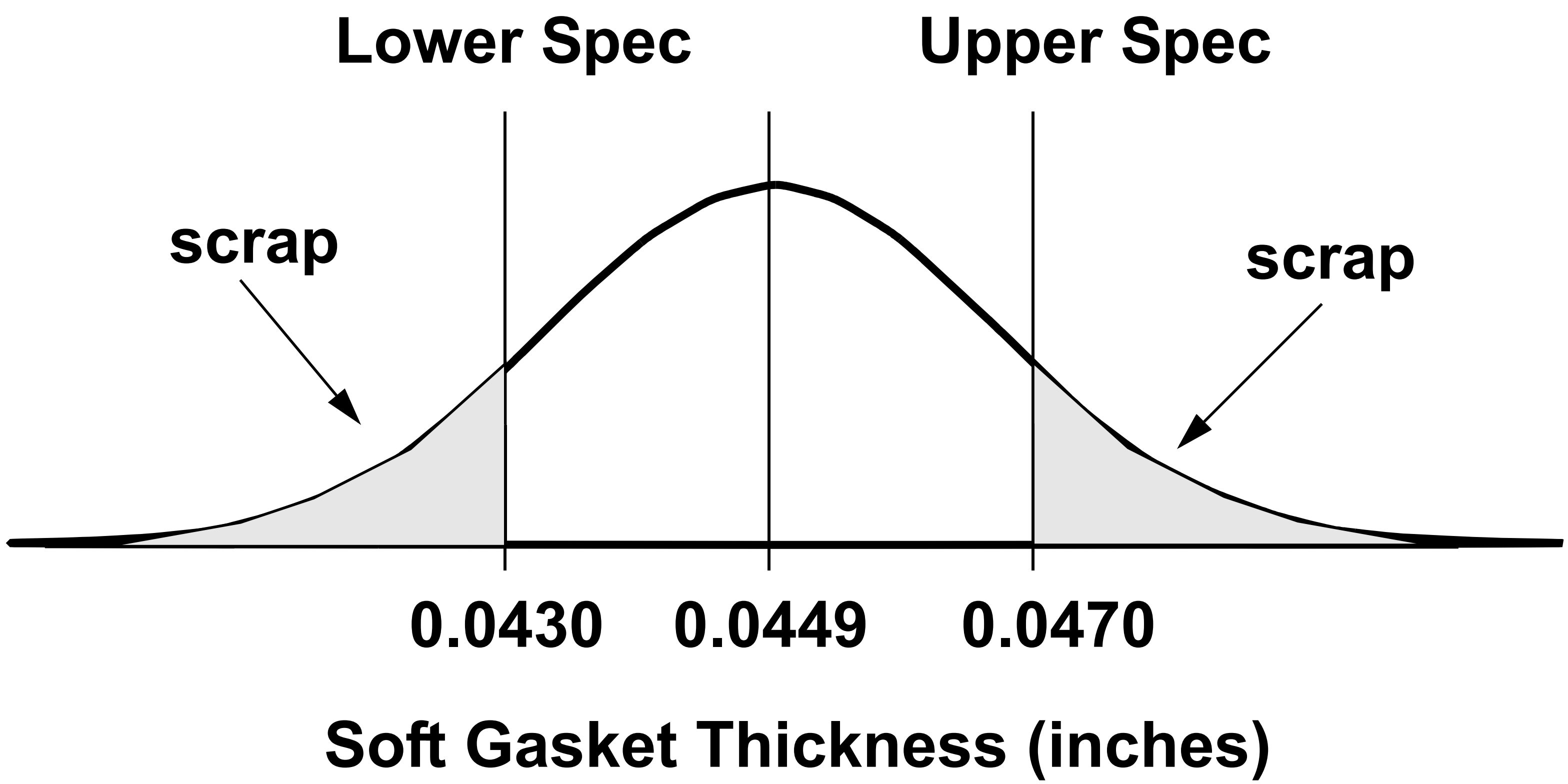


Figure 10.4 Histogram of Data from Initial Process Capability Study

72 sheets

X-double-bar = 0.0449 s_x = 0.0014



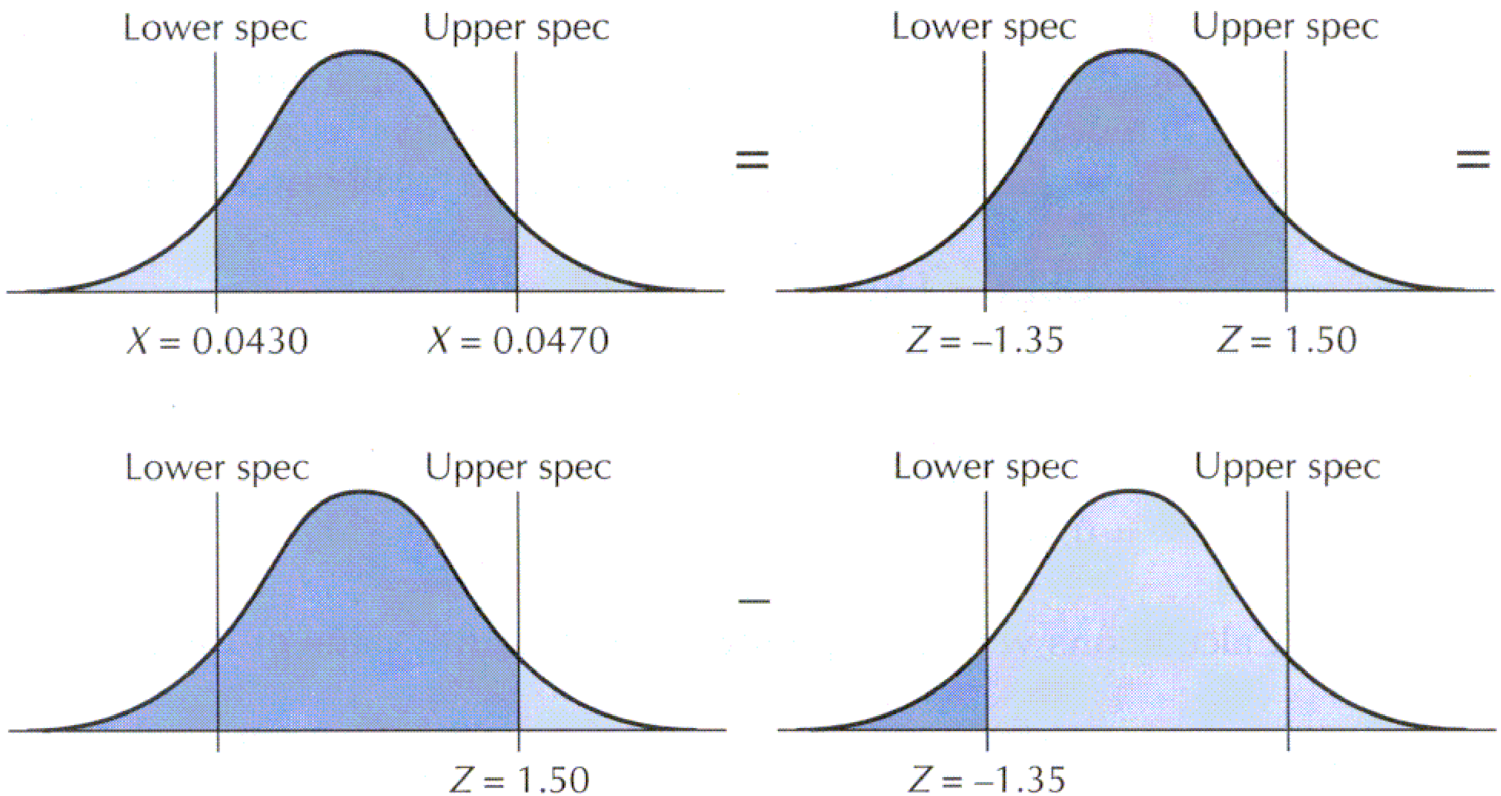


Figure 10.6 Graphical Representation of the Z Transformation and Probability Calculation

Cap = 84.6% **C_p = 0.477** **C_{pk} = 0.45**

Progress to Date

1. Product Control Way of Thinking
2. Tunnel Vision of the Engineers
3. Selective Assembly Process
4. Statistical Process Control / Process Capability Relationship

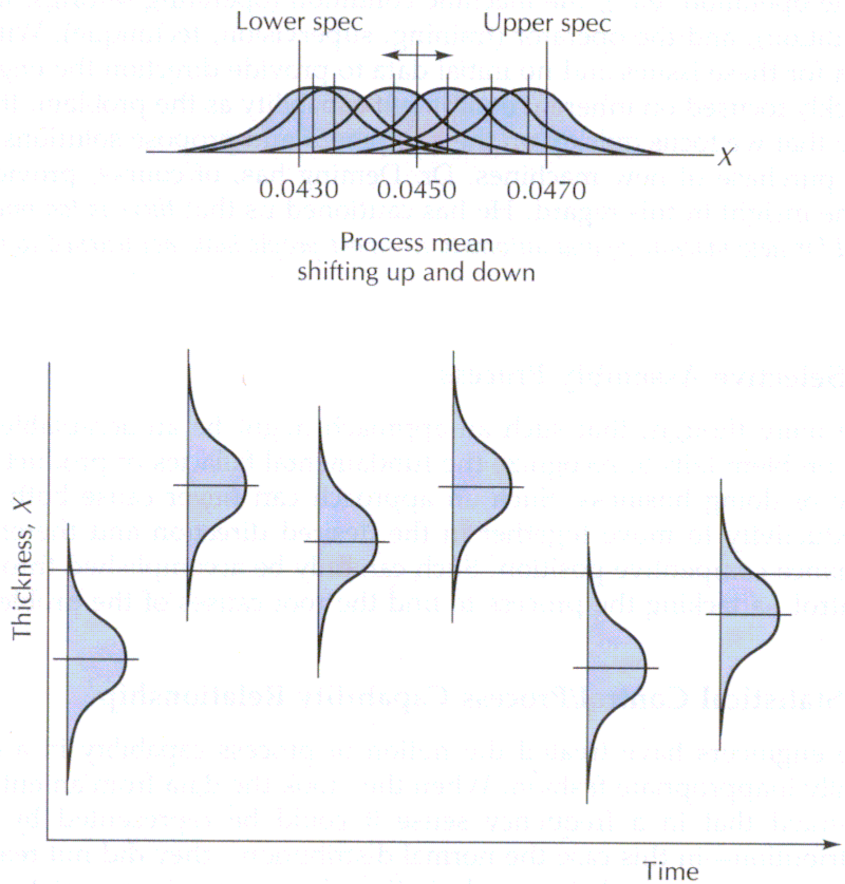


Figure 10.7 Time Behavior of a Process That Is Changing Mean Level in a Sporadic Fashion

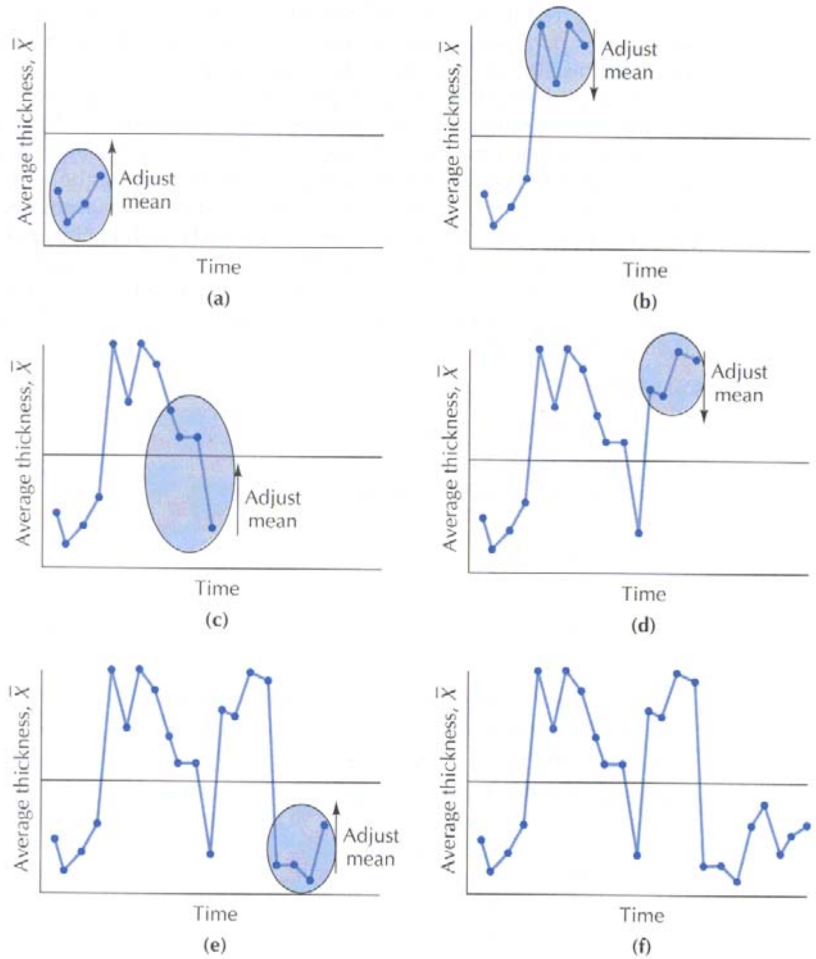


Figure 10.8 Process Adjustment History over One Shift

TABLE 10.2 Original Data Grouped into Subgroups of Size $n = 3$ (inches)

Subgroup	X_1	X_2	X_3	$\overline{X}$	R
1	0.0440	0.0446	0.0437	0.0441	0.0009
2	0.0438	0.0425	0.0443	0.0435	0.0018
3	0.0453	0.0428	0.0433	0.0438	0.0025
4	0.0451	0.0441	0.0434	0.0442	0.0017
5	0.0459	0.0466	0.0476	0.0467	0.0017
6	0.0449	0.0471	0.0451	0.0457	0.0022
7	0.0472	0.0477	0.0452	0.0467	0.0025
8	0.0457	0.0459	0.0472	0.0463	0.0015
9	0.0464	0.0457	0.0447	0.0456	0.0017
10	0.0451	0.0447	0.0457	0.0452	0.0010
11	0.0456	0.0455	0.0445	0.0452	0.0011
12	0.0448	0.0423	0.0442	0.0438	0.0025
13	0.0459	0.0468	0.0452	0.0460	0.0016
14	0.0456	0.0471	0.0450	0.0459	0.0021
15	0.0472	0.0465	0.0461	0.0466	0.0011
16	0.0462	0.0463	0.0471	0.0465	0.0009
17	0.0427	0.0437	0.0445	0.0436	0.0018
18	0.0431	0.0448	0.0429	0.0436	0.0019
19	0.0425	0.0442	0.0432	0.0433	0.0017
20	0.0429	0.0447	0.0450	0.0442	0.0021
21	0.0443	0.0441	0.0450	0.0445	0.0009
22	0.0443	0.0423	0.0447	0.0438	0.0024
23	0.0429	0.0427	0.0464	0.0440	0.0037
24	0.0448	0.0451	0.0428	0.0442	0.0023

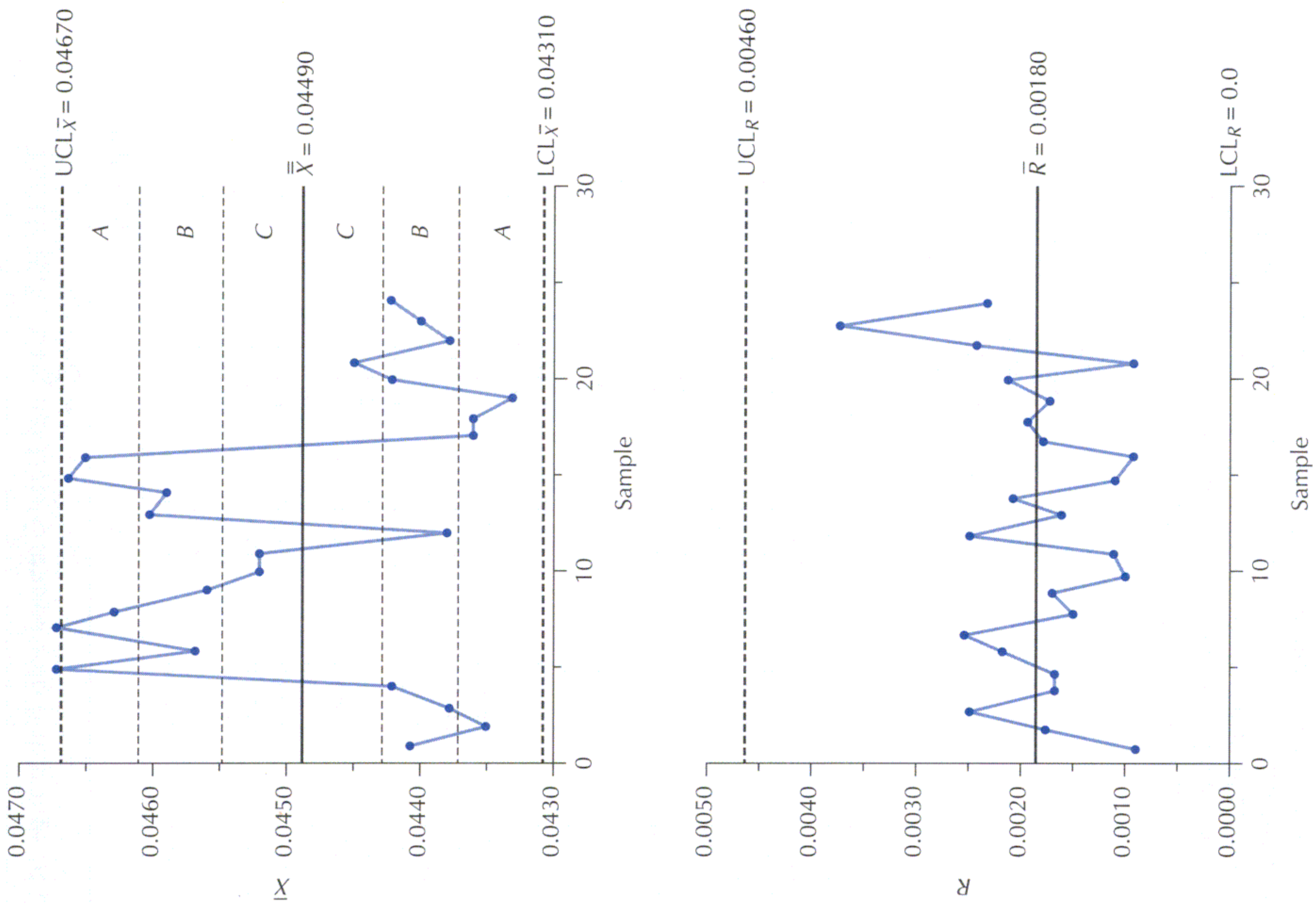


Figure 10.9 $\overline{X}$ and R Control Charts for the Roll Mill Process

TABLE 10.3 Zone Violations from Fig. 10.10, $\overline{X}$ Chart

Subgroup	Rule(s) Violated
4	1. Four out of five in zone *B* or beyond
5	1. Extreme point
7	1. Extreme point
	2. Two out of three in zone *A* or beyond
8	1. Two out of three in zone *A* or beyond
	2. Four out of five in zone *B* or beyond
	3. Eight in a row outside zone *C*
9	1. Eight in a row outside zone *C*
	2. Four out of five in zone *B* or beyond.
16	1. Two out of three in zone *A* or beyond
	2. Four out of five in zone *B* or beyond
18	1. Two out of three in zone *A* or beyond
19	1. Two out of three in zone *A* or beyond
	2. Eight in a row outside zone *C*
20	1. Four out of five in zone *B* or beyond
	2. Eight in a row outside zone *C*
22	1. Four out of five in zone *B* or beyond
23	1. Four out of five in zone *B* or beyond
24	1. Four out of five in zone *B* or beyond
	2. Run of eight below the centerline

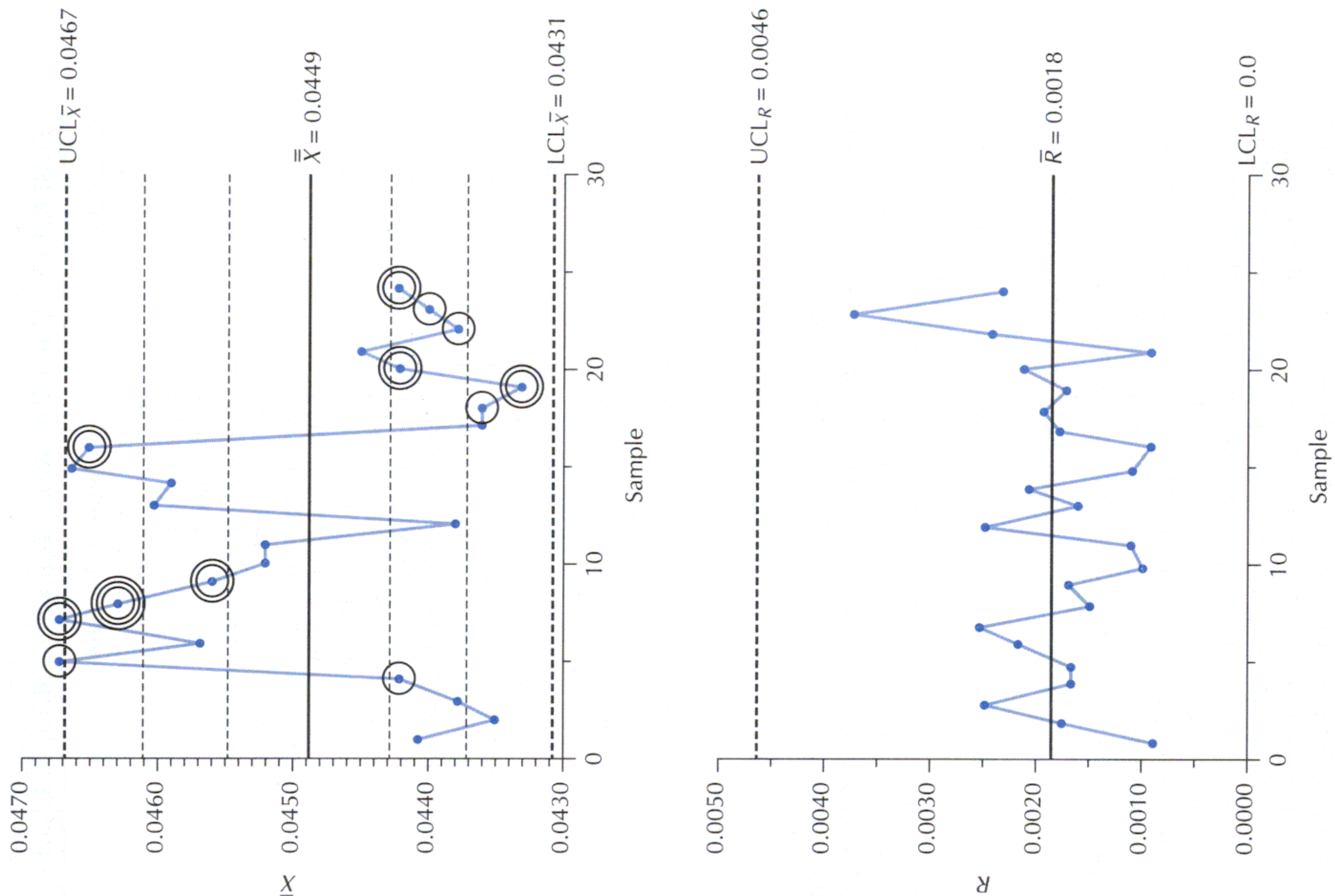


Figure 10.10 $\overline{X}$ and R Control Charts of Figure 10.9 Showing Out-of-Control Points

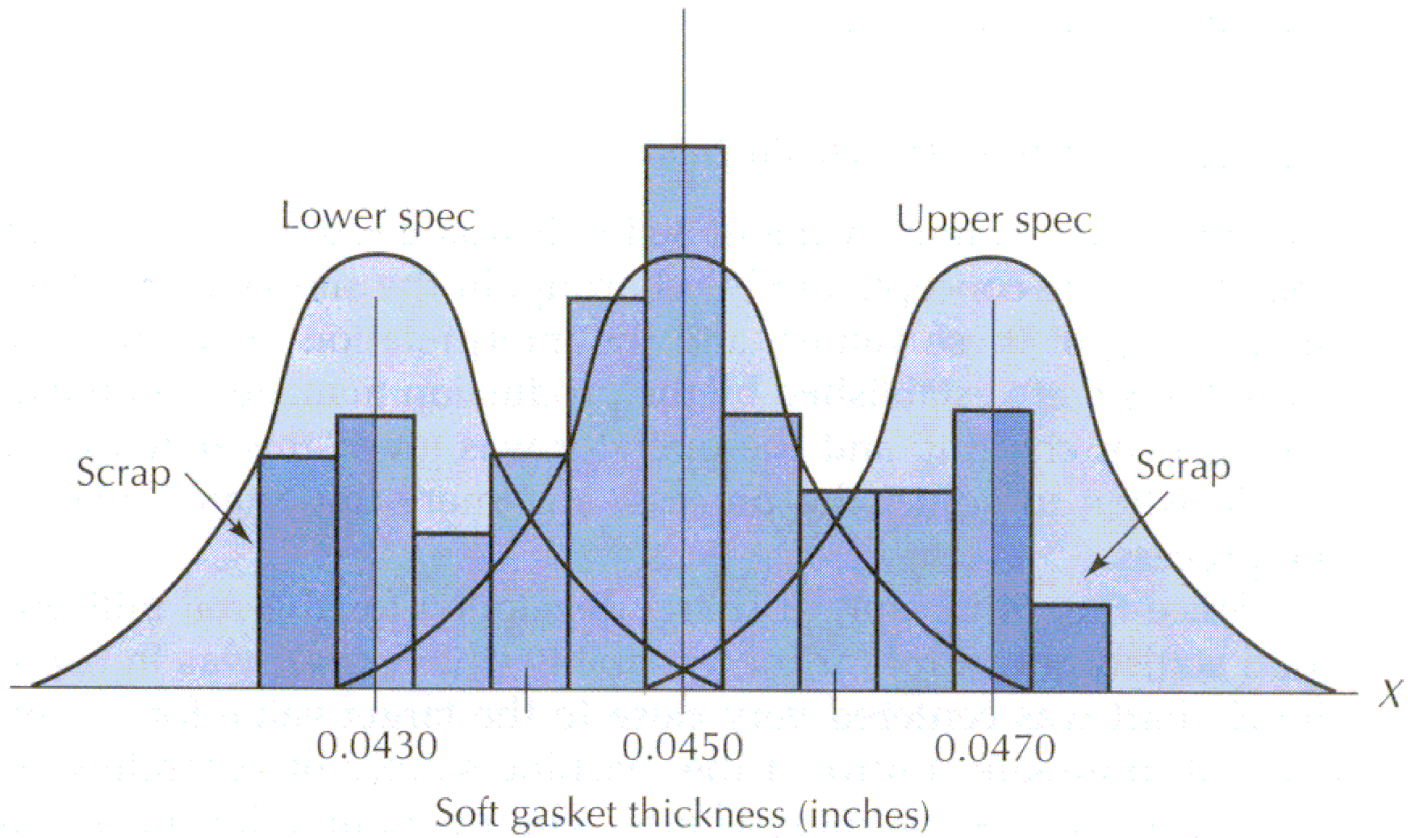


Figure 10.11 Consequences of Overcontrol on Process Performance