Lecture # 28

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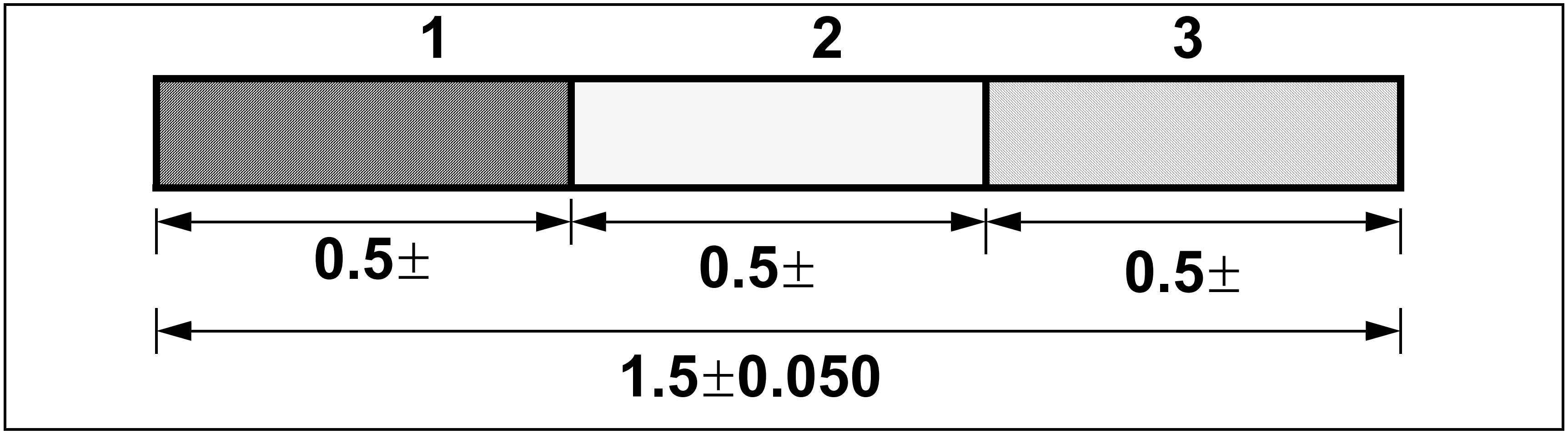
Another Example



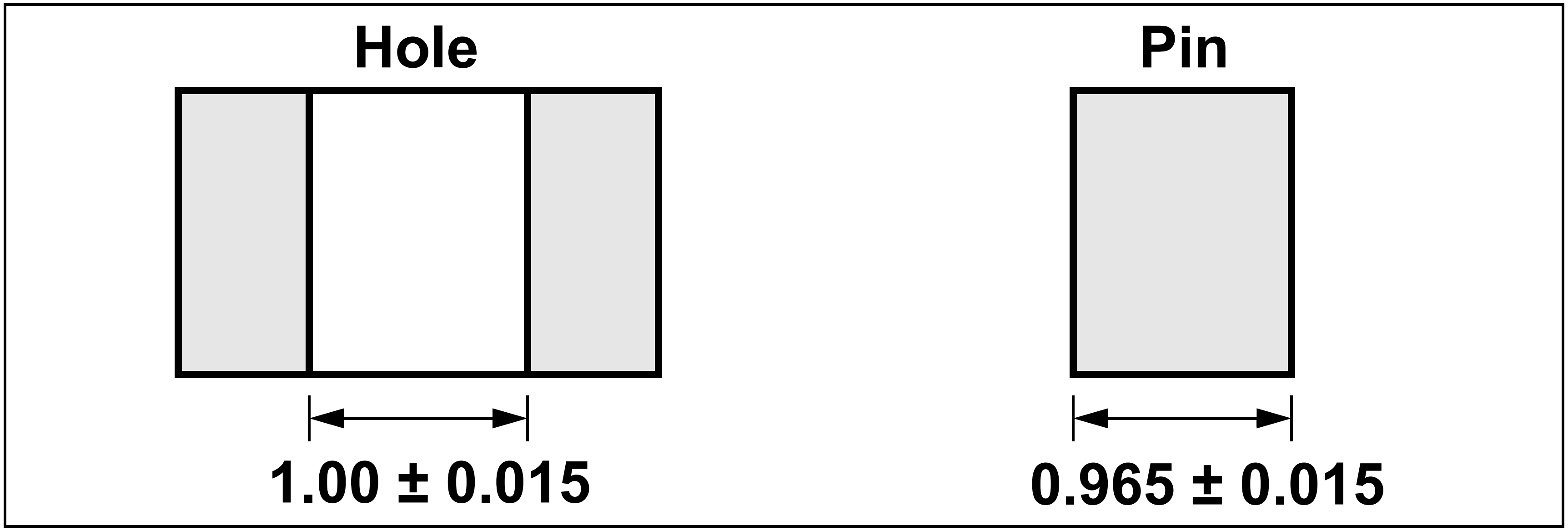
Three identical parts

Manufacturing process -- C_p = 1.5 (at least)

What will the individual process σ_x be, if tolerances are obtained by simple division?

Find (i) standard deviation for the processes
(ii) tolerances for individual parts

Another Example



For now, let's assume hole & pin producing processes are centered at the nominal values and that processes have Cp values of 1.0.

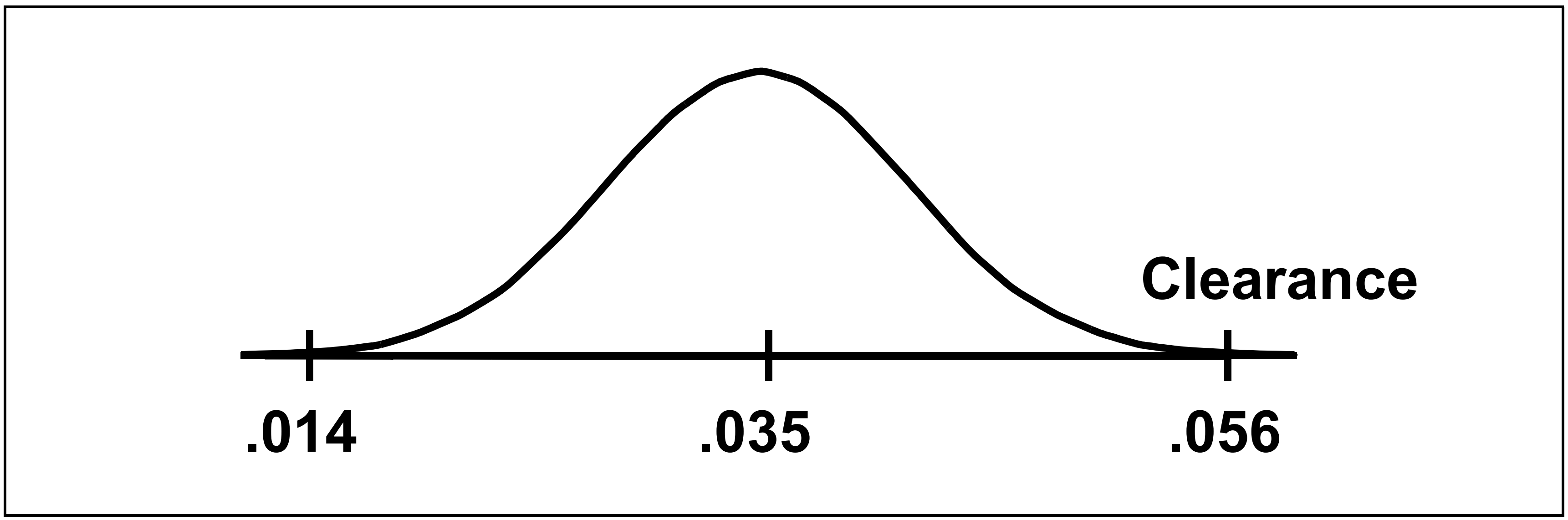
What does clearance dist. look like??

Clearance Distribution

C = H - P

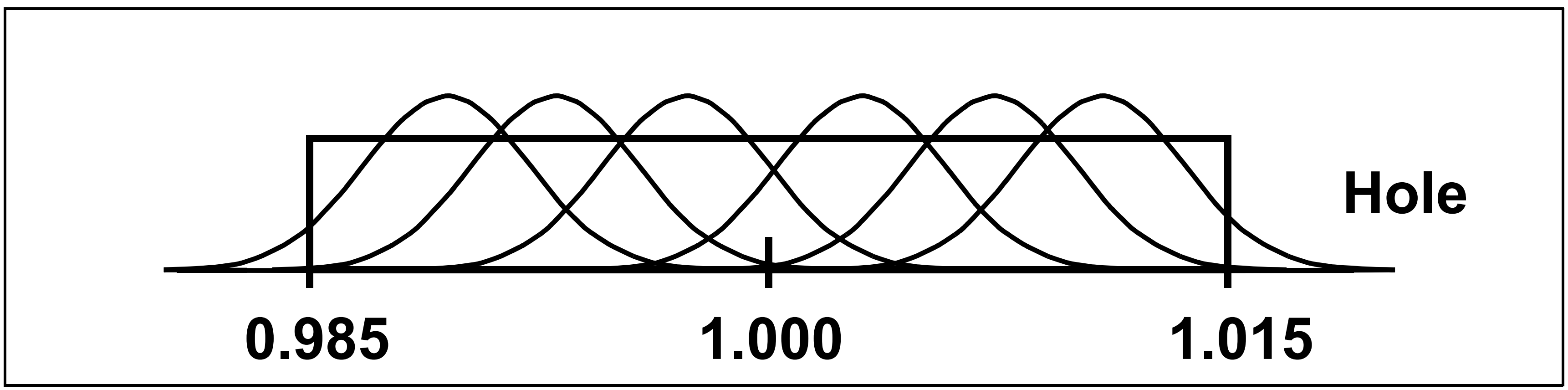
$\mu_C = \mu_H - \mu_P$ = 0.035

$\sigma_C^2 = \sigma_H^2 + \sigma_P^2$ ---- σ_c = 0.007

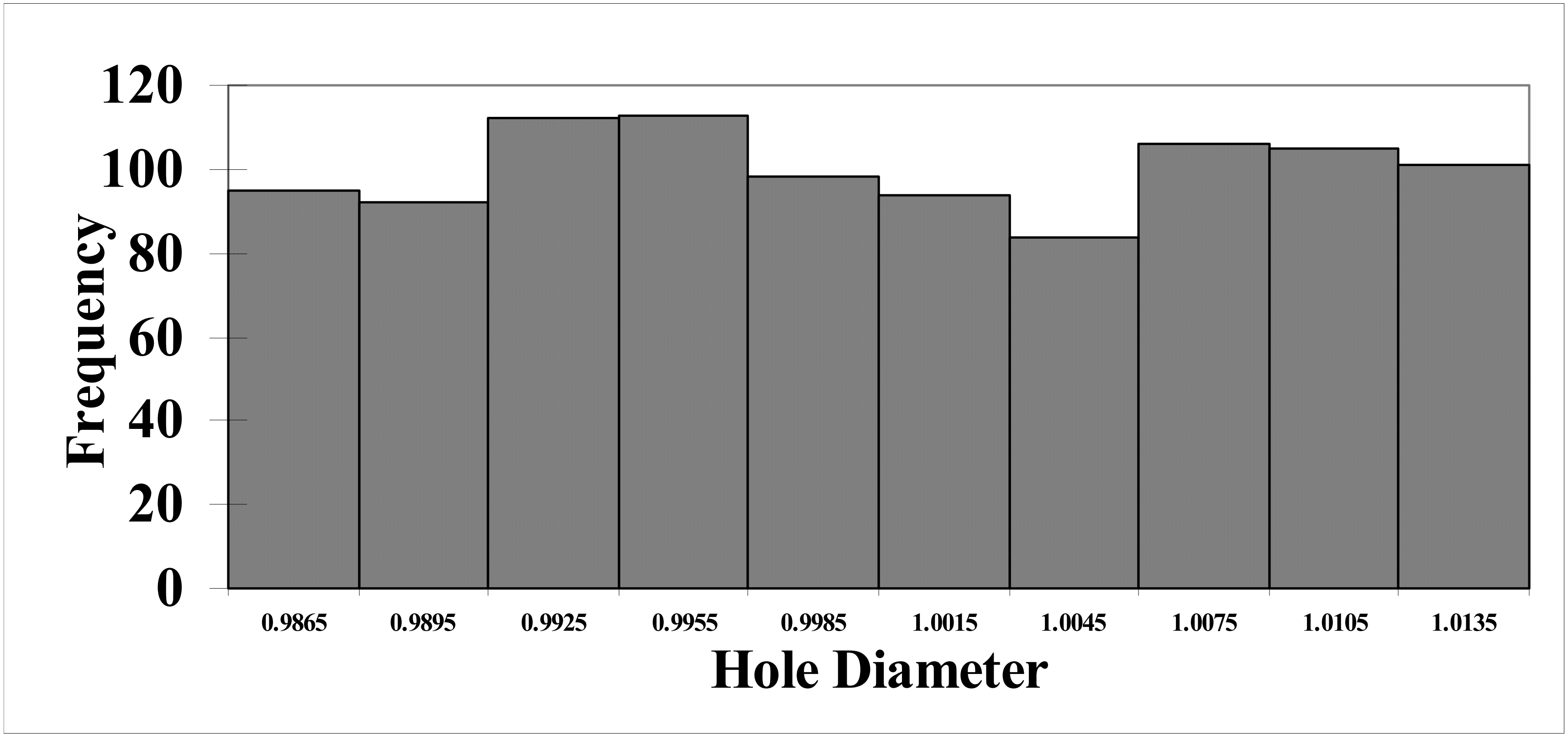


What can go Wrong?

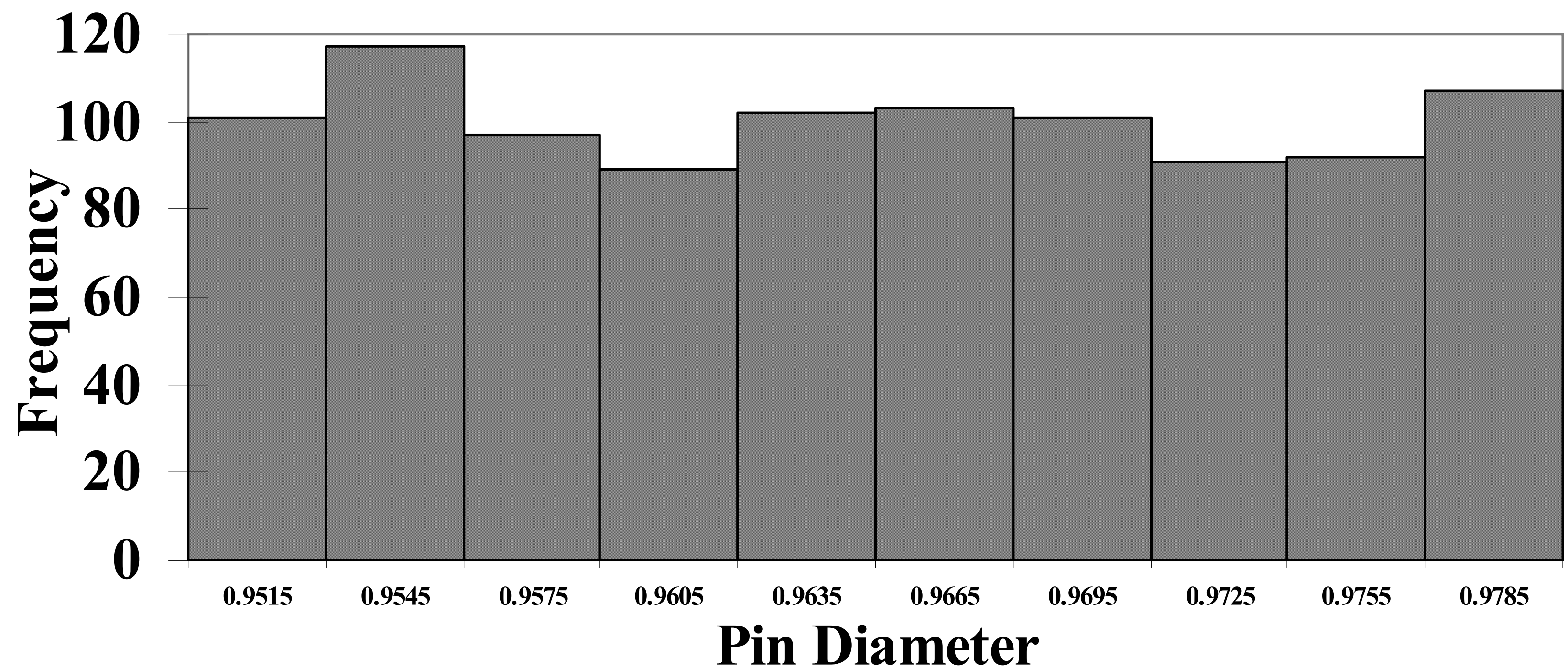
- We have already seen that if our individual processes (in this case pin and hole) do not remain centered - the results can be disastrous.
- What if our processes are not maintained in a state of statistical control?



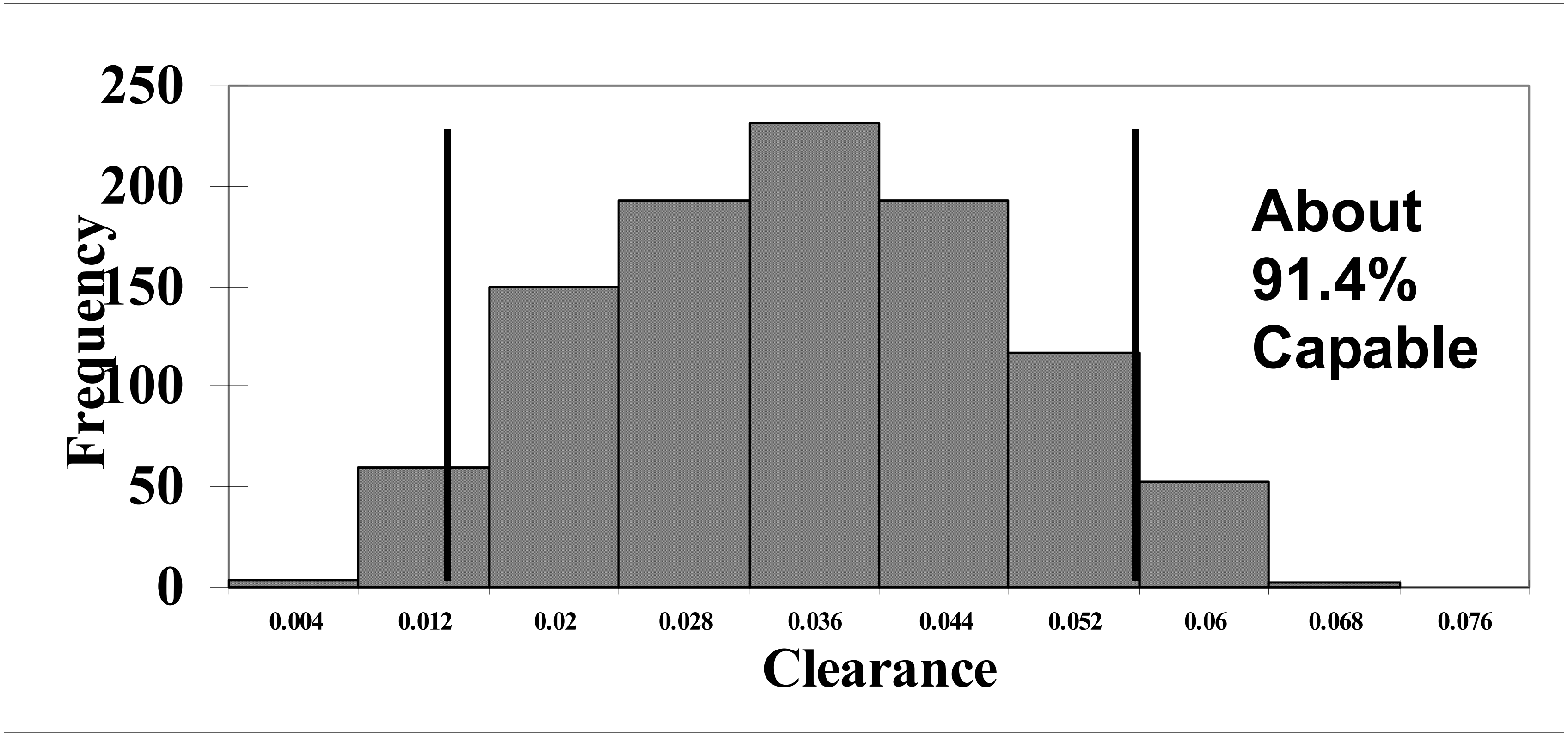
Histogram - Hole Dimension



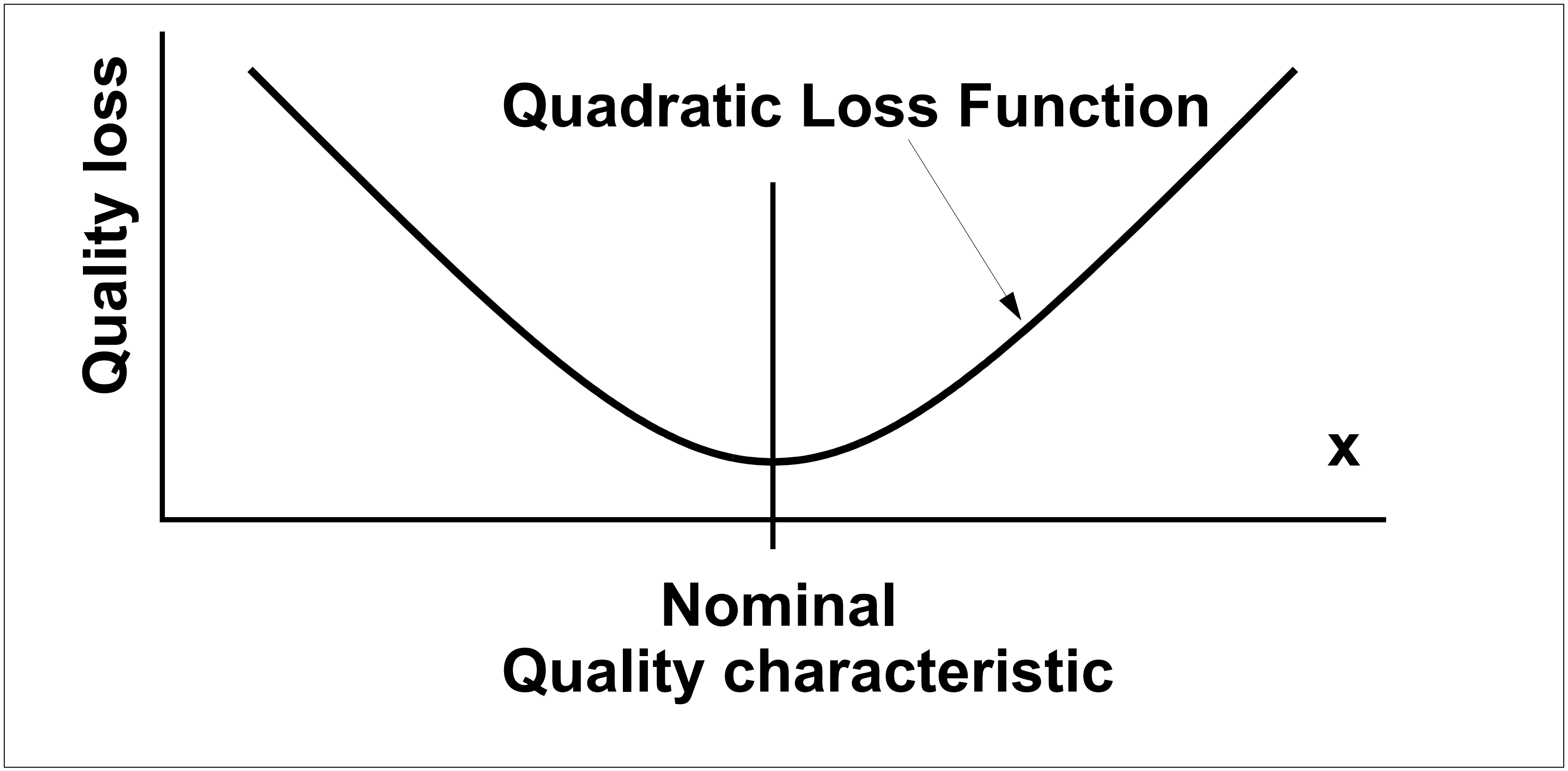
Histogram - Pin Dimension



Histogram - Clearance Dimension



Capability Assessment via a Loss Function



Defining the Loss Function

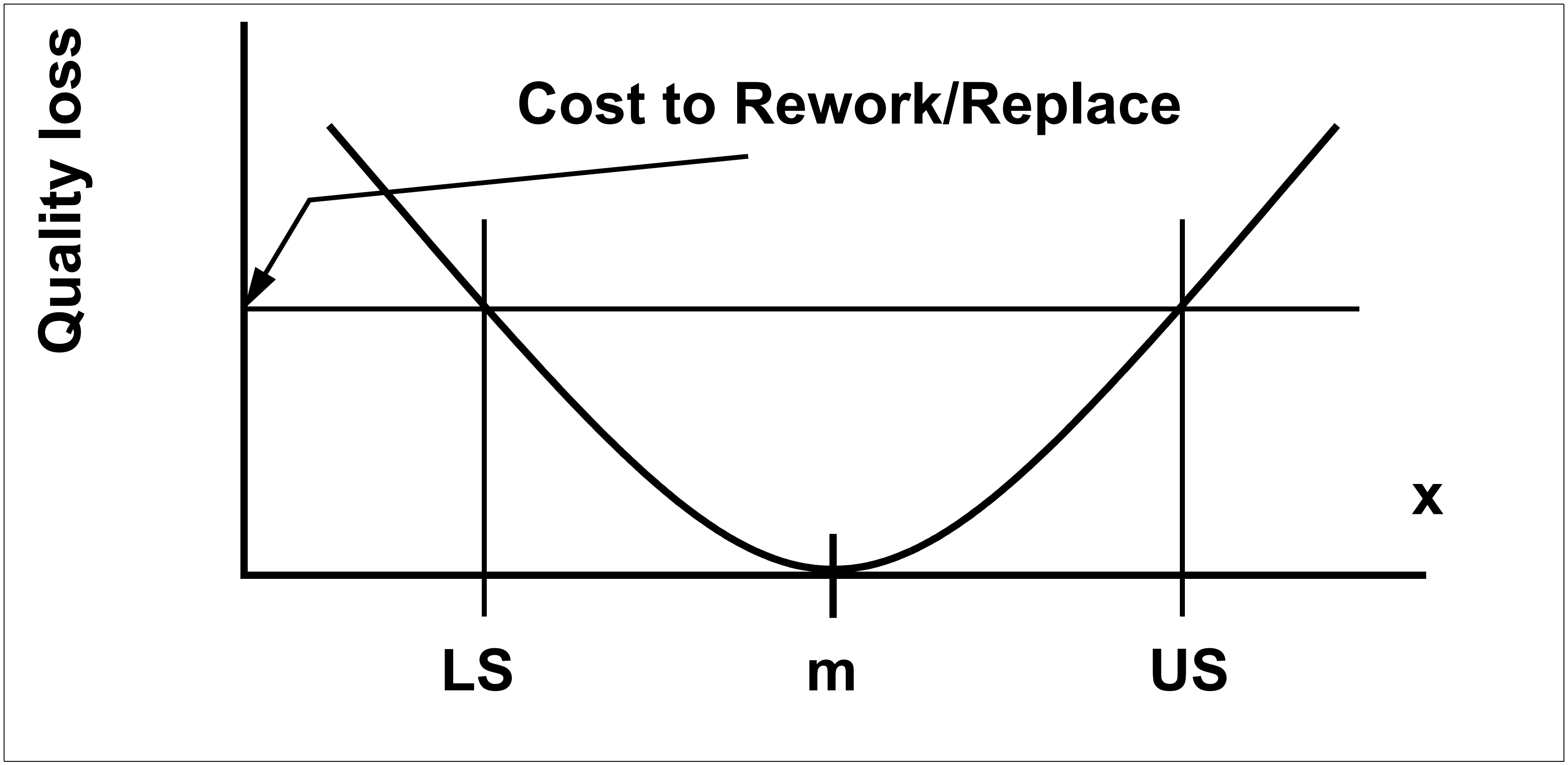
$$L = L_0 + k(x - m)^2$$

- **for an x value of 0, the loss, L, is \$0.**
  This means that m = 0 and L_0 = 0

- **when x= 1.5, the loss, L, is \$2.25**
  $L/(x-m)^2 = k$ $\qquad$ $k = \$2.25/(1.5 - 0)^2 = 1$

  The \$2.25 loss is obtained from the fact that when x=1.5, the probability of a \$45 complaint is 5%, and \$45 * (0.05) = \$2.25

Loss Function Interpretation of Engineering Specifications



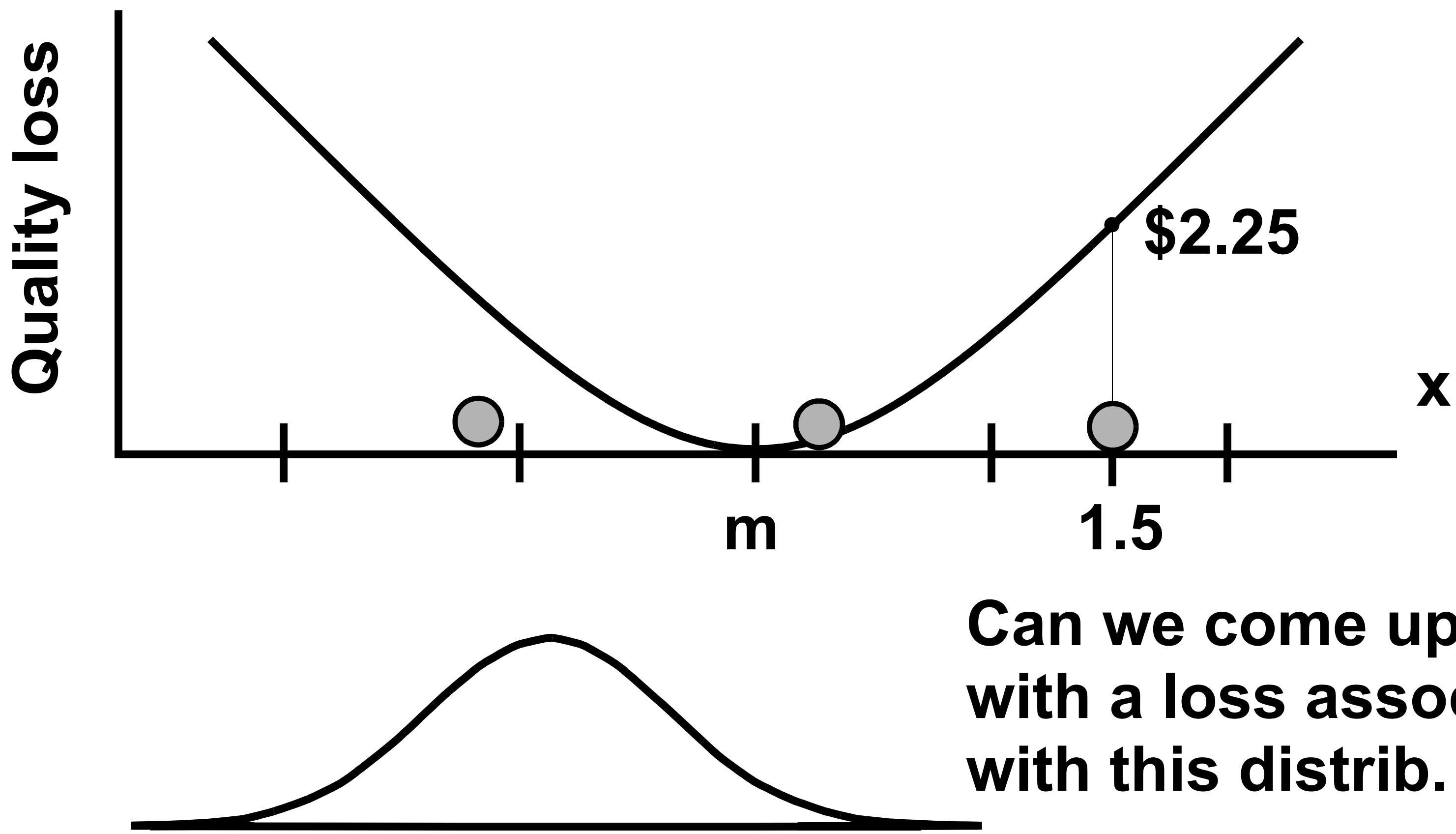
More on Loss / Specifications

So, we want to set our specifications at the point of indifference: where the loss = cost of replacement

If cost of replacement = $2 for our previous example,

$L = 1\ (x - 0)^2 = 2$ ----- $x = \pm 1.414$

More on the Loss Function



Can we come up with a loss assoc. with this distrib.

Expected Process Loss

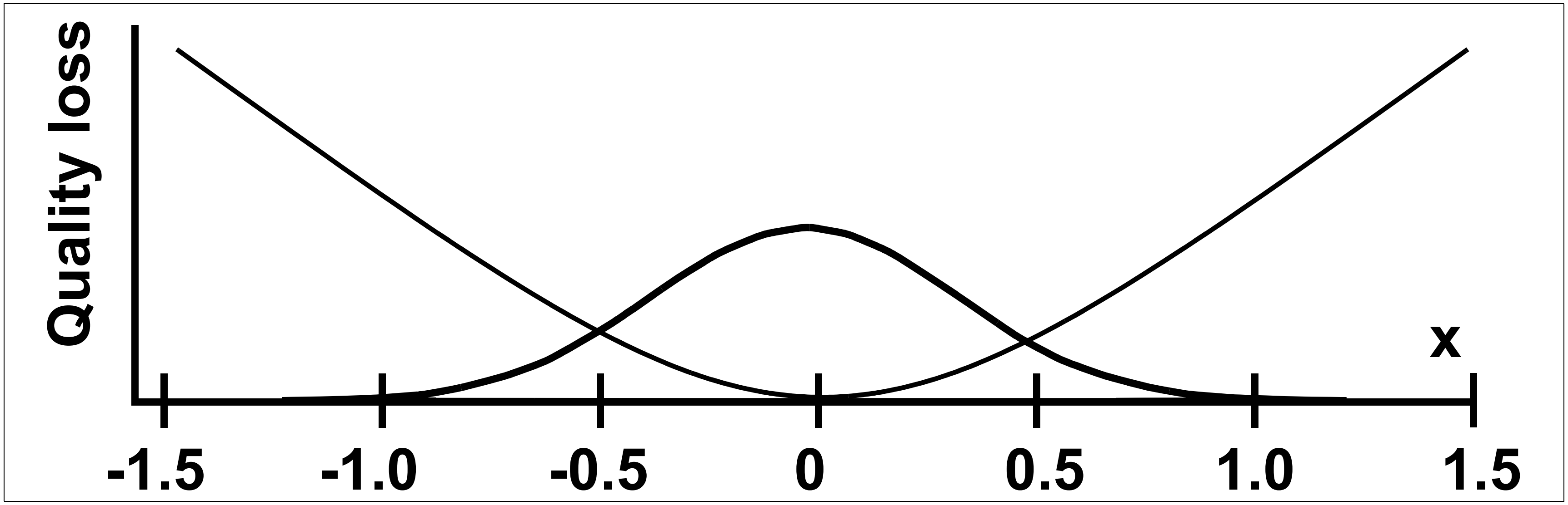
$$E[L(X)] = \int_{\text{all } X} L(t)f(t)dt$$ where,

$$L(x) = k(x - m)^2$$ and f(t) is the pdf

after simplification,

$$E[L(X)] = k\left\{(\mu_X - m)^2 + \sigma_X^2\right\}$$

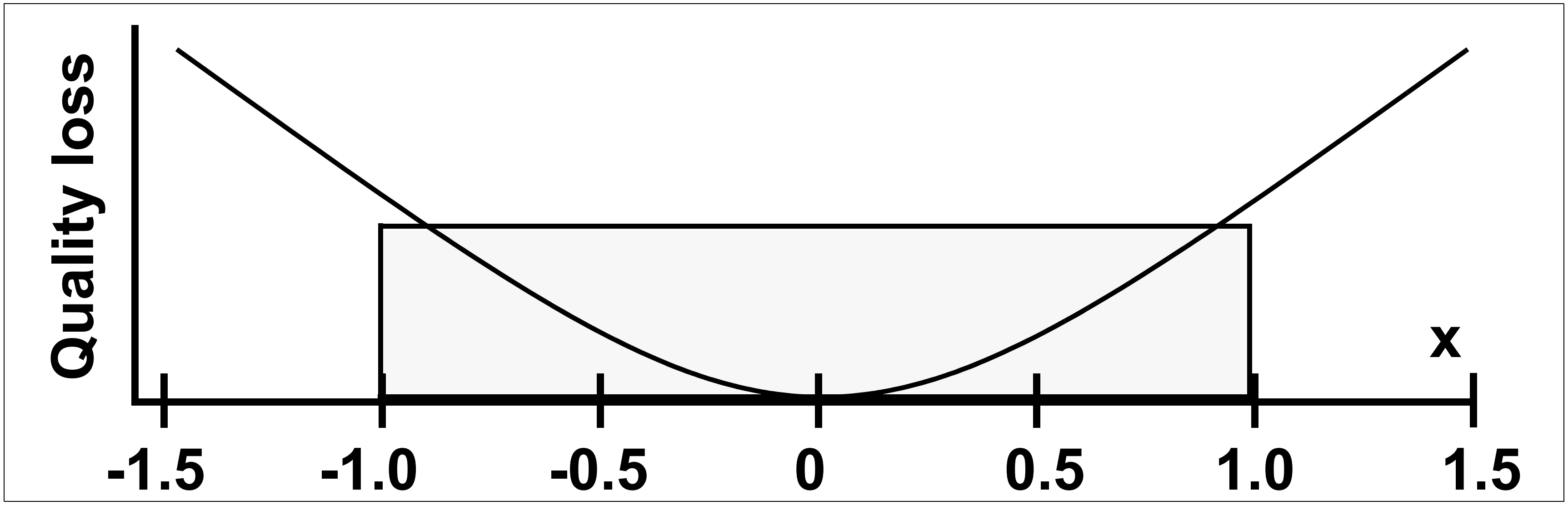
Loss Function: Example #1



m=0, k=$1.0, LSL=-1, USL=1,
X normal w/ mean 0 and σ_x = 0.33

$E[L] = 1\ [0.33^2 + (0-0)^2] = \$0.11$ How to interpret??

Loss Function: Example #2



$\sigma_x = 0.58$

$E[L] = 1\,[0.58^2 + (0\text{-}0)^2] = \$0.33$

Filling in a Past Blank

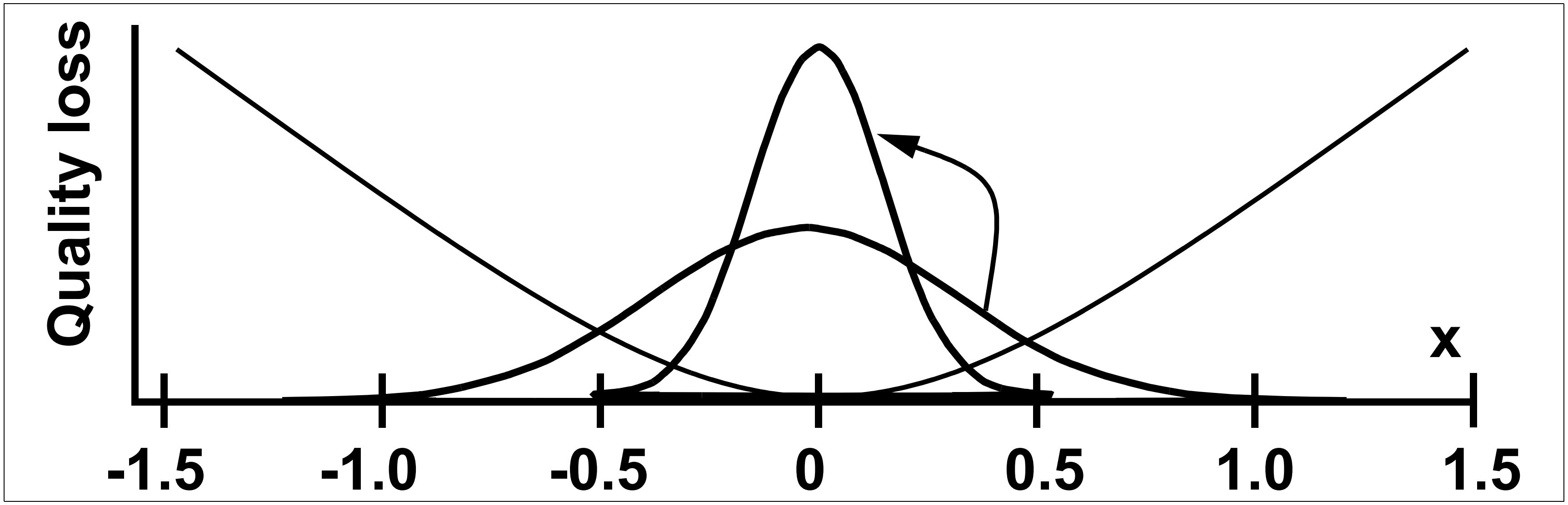
Should know from prior Statistics Class
For a Uniform Distribution between a & b

$$f(x) = 1/(b-a)$$

$$\mu_x = \int_a^b x f(x) dx = \int_a^b \frac{x}{(b-a)} dx = \left. \frac{x^2}{2(b-a)} \right|_a^b = \frac{(b+a)}{2}$$

$$\sigma_x^2 = \int_a^b (x-\mu_x)^2 f(x) dx = \frac{(b-a)^2}{12}$$

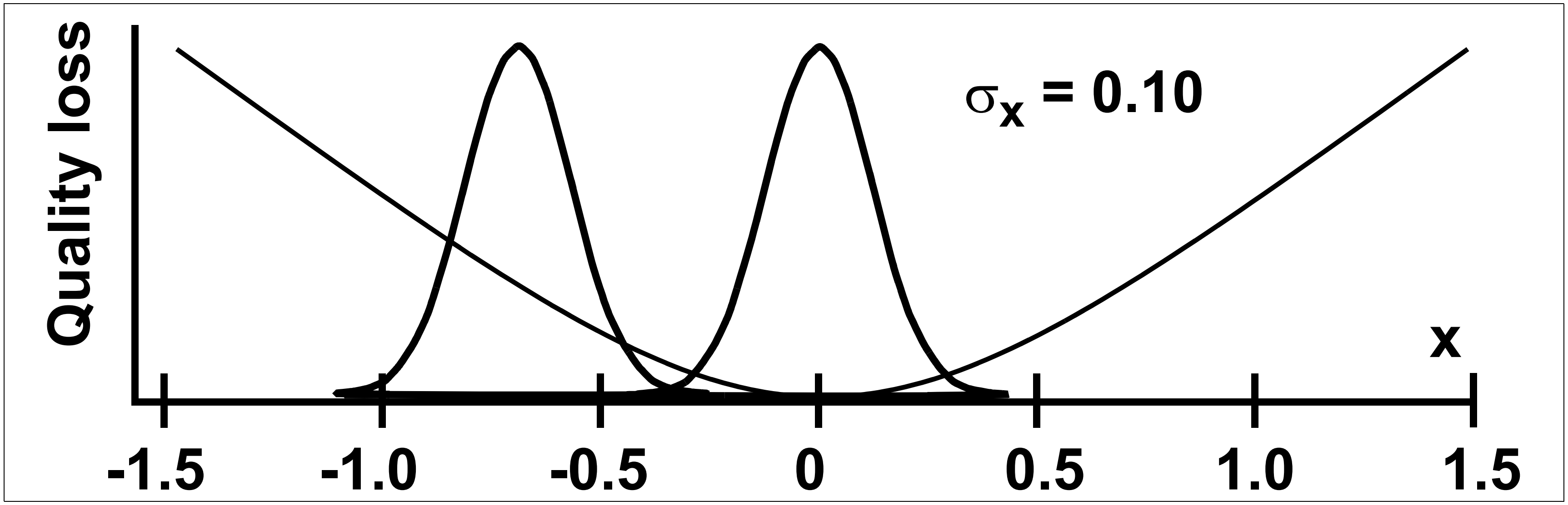
Loss Function: Example #3



Say we reduce $\sigma_x = 0.33$ to $\sigma_x = 0.10$

$E[L] = 1\ [0.10^2 + (0-0)^2] = \$0.01$

Loss Function: Example #4



Can save \$0.10 by reducing process mean from 0 to -0.7

$$E[L] = 1\ [0.10^2 + (-0.7 - 0)^2] = \$0.50 \quad \text{WOW!!}$$

Taguchi's comment: